Set-consensus for Multi-Agent Systems

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Set-Consensus using Set-Valued Observers

D. Silvestre, P. Rosa, J. Hespanha and C. Silvestre

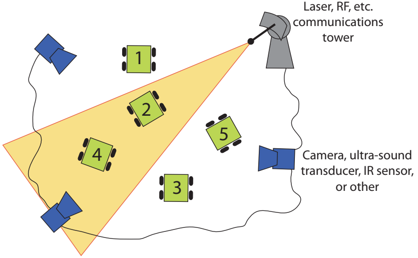
2015 American Control Conference
Chicago, Illinois, USA.

2nd July 2015

Outline

Motivation

- Distributed Sensing - Each node computes estimates and need to synchronize them before aggregating them.
- Robot Coordination - Fleet of robots wishes to agree on direction/speed or rendezvous point.
- Asynchronous Algorithms - Nodes acting independently changes the number of considered time steps as seen by each individual nodes.

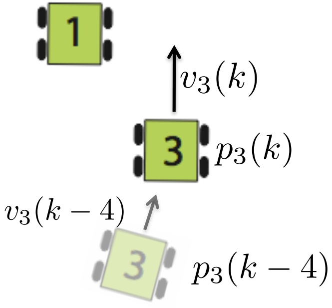


Set Consensus

- A group of n nodes is trying to achieve consensus.
- Nodes have neither sensing nor self-localization capabilities.
- A tower uses a directional antenna to transmit to the nodes their position and velocity.
- Two main issues: measurements are corrupted by noise and taken at different time instants.

Motivating Example

- Consider the case of two vehicles given in the figure.
- Node 1 receives the last measurement at time instant $k-4$.
- Due to sensor noise or disturbances, node 1 has only access to estimates.
- Then, the decision might result in a collision!



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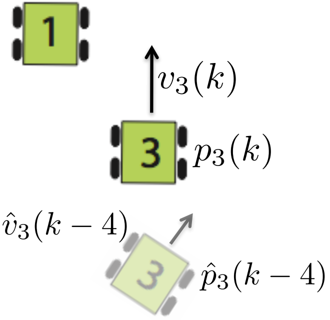

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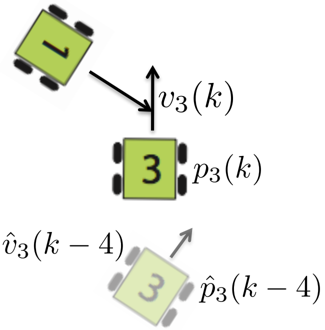


Problem Outline

- Take n nodes, where each node i has dynamics of the form

$$x_i(k+1) = A_i(k)x_i(k) + B_i(k)u_i(k) + E_i(k)d_i(k)$$

- $u_i(k)$ is the actuation signal and $d_i(k)$ possible disturbances.

Set-Consensus Problem

How to achieve position or velocity consensus when instead of knowing $x_i(k)$ only a set $X_i(k)$ is known such that $x_i(k) \in X_i(k)$.

Problem Model

- Each agent i has a system of the form

$$x_i(k+1) = \left(A_0 + \sum_{\ell=1}^{n_\Delta} \Delta_\ell(k) A_\ell\right) x_i(k) + B_i(k) u_i(k) + E_i d_i(k)$$

- Each S_i is a Linear Parameter-Varying (LPV) system
- n_Δ number of uncertainties
- $\Delta_\ell(k)$ are scalar uncertainties with $|\Delta_\ell(k)| \leq 1$
- A_ℓ are constant matrices

Proposed Solution

Broadcast Solution using Position

- Use Set-Valued Observers (SVOs) [1] to update the received $X_j(k - k_j)$ for each of the neighbors j;
- Compute the weighted average [2] of the updated $X_j(k)$;
- Compute the velocity vector to drive $X_i(k)$ to $X_{\text{avg}}(k)$.

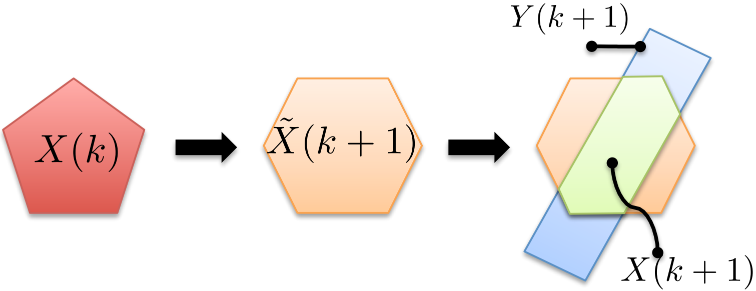
Unicast Solution using Estimation

- Node i receives sets $X_j(k - k_j)$ from a subset of its neighbors;
- Set $X_i(k)$ will include the concatenation of the updated $X_j(k)$ and disturbance terms to account for each node j actuation;
- The velocity vector will take into account the estimated position and velocity of the neighbors.

SVOs

Given the previous set $X(k)$:

- Using SVOs, the algorithm predicts $\tilde{X}(k+1)$ using the dynamics;
- Then, the set is intersected with the measurement set $Y(k+1)$.



Algorithm

- Node i computes:

$$X_i(k+1) = \alpha X_i(k) + (1-\alpha)\frac{1}{|N_i|}\sum_{j \in N_i} X_j(k)$$

- Velocity vector v can be found through:

$$\begin{aligned} v = \text{arg min} \quad & \max_{x,y}(||(v+x)-y||) \\ \text{subject to} \quad & x \in X_i(k) \\ & y \in X_i(k+1), \end{aligned}$$

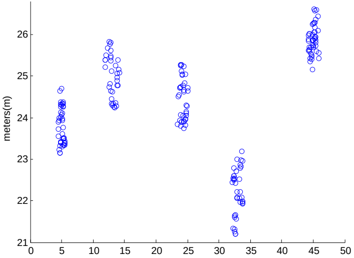
Properties

- Nodes position converge to a ball of radius equal to the maximum uncertainty in the measurement sets;
- For the case of unicast communication and using estimates, uncertainty is higher as there are added disturbances and dynamics uncertainties in the update of the estimates;
- Convergence for a single cluster depends on the allocation of transmissions by the various directions.

Simulation Results (1/2)

Setup: 200-node network randomly distributed over a 50m×50m square and round-robin service using an offset to cover 10 partitions of the terrain.

- In a typical run nodes converge to a smaller number of clusters (5 in the example).
- Nodes *aligned* themselves along the partitions.
- Figure depicts the evolution of the maximum distance between any two nodes.
- Convergence to a cluster can be identified when there is little



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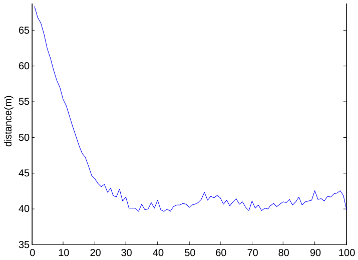
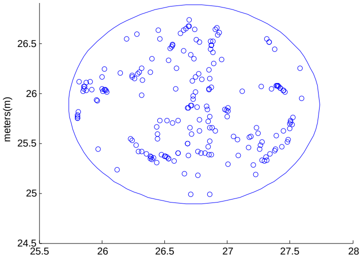


Simulation Results (2/2)

Setup: 200-node network randomly distributed over a 50m×50m square with two antennae (length and width) used in a periodic scheduling. Amround-robin service is used for each antenna using an offset to cover 10 partitions of the terrain.

- A typical run achieves consensus for a single cluster.
- The maximum ball around the nodes has radius equal to the maximum uncertainty $\epsilon_{\max}$.
- The maximum difference between two nodes converges to a value smaller than $\epsilon_{\max}$.



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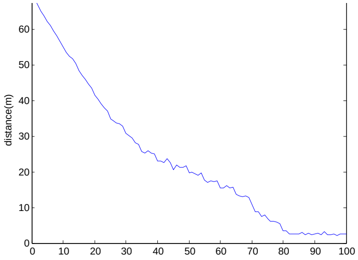

Concluding Remarks

Contributions:

- the use of SVOs to update the set representing the uncertainty about the position of the nodes;
- Two scenarios are addressed:
  - Broadcast - nodes use the positions for the other nodes;
  - Unicast - nodes obtain information in the shared medium and estimate the position for the other nodes.
- the positions of the nodes are shown to converge to the vicinity of the remaining nodes dependent on a measure of the uncertainty.
- In Simulation, it is observed that the policy for the communication influences the number of clusters.

References

D. Silvestre, P. Rosa, R. Cunha, J. P. Hespanha, and C. Silvestre, "Gossip average consensus in a byzantine environment using stochastic set-valued observers," in *52nd IEEE Conference on Decision and Control.*, 2013, Florence, Italy.

D. Antunes, D. Silvestre, and C. Silvestre, "Average consensus and gossip algorithms in networks with stochastic asymmetric communications," in *Decision and Control, 2011. CDC 2011. 50th IEEE Conference on*, 2011, pp. 2088 –2093.

The end

- Thank you for your time.

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