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Distributed Formation Control of Double-Integrator Vehicles with Disturbance Rejection ⋆

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Abstract: This paper presents the development of a robust distributed and cooperative control algorithm for formation tracking by teams of vehicles modeled as double integrators, seeking its application to multirotor vehicles. Consensus-based protocols are considered to achieve coordination between the vehicles in a distributed manner, in the sense that only local information is either exchanged or perceived. A consensus protocol for agents modeled as triple integrators is proposed, and explicit necessary and sufficient conditions for convergence are provided. As a corollary, these conditions are particularized for the case of double integrators, yielding necessary and sufficient conditions that reduce the conservatism of existing sufficient conditions presented in the literature. The effect of constant disturbances on the system is then described, and finally, the third-order consensus protocol is used to incorporate integral action in a formation tracking controller used for double integrator vehicles.

Keywords: Formations, Consensus, Distributed control, Disturbance rejection, Multirotors.

1. INTRODUCTION

Formation control is an important topic of research in coordinated motion of multiple unmanned autonomous vehicles. Moving in formation can have several advantages on the overall system, such as increased redundancy and robustness, and reduced cost. This problem presents several challenges, mainly related to the lack of total information by each agent, but also to the desire to use a distributed or a decentralized approach. In decentralized approaches, each agent makes its own decisions independently from the others, therefore a central controller, coordinator or supervisor does not exist, making the problem more challenging. Despite the challenges, a decentralized approach is still the one that presents more potential applications, since it provides scalability and robustness to the system.

A survey on the topic of multi-agent formation control can be found in Oh et al. (2015). There, the authors distinguish between several proposed approaches according to the required sensing capabilities of the agents and the level of interaction necessary between them, categorizing the formation control methods into position-, displacement-, and distance-based. The position-based approach considers that each agent has access to measurements in the inertial frame (e.g. absolute position measurements). In this case, each agent can be equipped with a control law to drive its position to a desired position, thus achieving the prescribed formation without the need to interact with others. This is, however, the most demanding approach in terms of the sensing capability of each agent. The displacement-based approach considers that the agents can only measure relative quantities (e.g., measurement of the relative position or displacement to another agent), and that they have a common reference for orientation. However, more interactions between agents are required in order to overcome the reduced sensing capability. For agents modeled as single integrators, this approach is studied under directed interaction topologies, for example, in Ren et al. (2004), considering consensus-based protocols. For the case of agents modeled as double integrators, it was studied in Ren and Atkins (2007), under directed and undirected interaction topologies. As for the general case of agents with linear dynamics, it has been studied in Wen et al. (2012). Finally, in the distance-based approach, it is assumed that agents only have access to relative measurements and do not share a sense of orientation.

Fig. 1. Two Intel Aero Ready To Fly quadrotors flying.

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Formations are stabilized based only on the distance between the agents, not accounting for the orientation of the formation. This approach is the less demanding in terms of sensing capability of the agents. However, it requires the use of more elaborate control laws, and more interactions between the agents. It is commonly studied under the use of gradient control laws, which are defined using artificial potential fields. For single integrator modeled agents, it has been studied in Krick et al. (2008) and V. Dimarogonas and Johansson (2008), and for double integrator modeled agents in Oh and Ahn (2014) and Olfati-Saber and Murray (2002).

Considering the importance of developing robust control algorithms for the coordinated motion of multiple unmanned autonomous vehicles, this work aims to develop distributed control algorithms for formation tracking and apply the devised solutions to multirotor vehicles. Noting that it is usual for multirotors to have access to measurements of their orientation, a displacement-based approach is considered. This approach is studied using consensus based protocols for the case of agents modeled as triple integrators. Then the results are used to introduce integral action to the formation tracking controller for vehicles modeled as double integrators, thus enabling constant disturbance rejection.

2. NOTATION AND PROBLEM STATEMENT

The set of real numbers is denoted by $\mathbb{R}$, the subset of positive real numbers by $\mathbb{R}^+$, real numbers except zero by $\mathbb{R}_{\neq 0}$, and the set of complex numbers by $\mathbb{C}$. For a complex number $z \in \mathbb{C}$, $\mathrm{Re}(z)$ denotes its real part, and $\mathrm{Im}(z)$ its imaginary part. The m-dimensional Euclidean space is denoted by $\mathbb{R}^m$, with norm $\|\mathbf{x}\| = \sqrt{\mathbf{x}\cdot\mathbf{x}}$ for all $\mathbf{x} \in \mathbb{R}^m$, where $\mathbf{a}\cdot\mathbf{b} := \mathbf{a}^\top\mathbf{b}$ denotes the inner product between two vectors. The dot notation is used to define the time derivative (as in $\dot{\mathbf{x}}$), and the number of dots its order. The matrix $\mathbf{I}_n$ is used to denote the $n\times n$ identity matrix and $\mathbf{0}_{n\times m}$ an $n\times m$ matrix of zeros, whereas $\mathbf{0}$ is used when the size can be inferred from the context and $\mathbf{1}_n$ denotes an $n\times 1$ vector of ones.

The problem at hand consists in the development of a distributed and cooperative control algorithm for a team of n vehicles that is able to track a time varying formation. It is assumed that the desired position for each vehicle i is given as a function of time, i.e., a trajectory $\mathbf{p}_i^{\mathbf{d}}(t) \in \mathbb{R}^3$, is defined for all $t > 0$, with known and continuous first and second time derivatives. The goal is to apply it to multirotors, which are complex systems, subject to modeling errors that can be perceived as disturbances to the nominal system that is being considered. Consider that the vehicles are modeled by

$$\begin{cases}\dot{\mathbf{p}}_i = \mathbf{v}_i \\ \dot{\mathbf{v}}_i = \mathbf{u}_i + \mathbf{d}_i\end{cases}, \tag{1}$$

where $\mathbf{p}_i, \mathbf{v}_i \in \mathbb{R}^3$ denote the position and velocity of the i-th vehicle, respectively, $\mathbf{u}_i \in \mathbb{R}^3$ is the control input of the vehicle, in this case, its acceleration, and $\mathbf{d}_i \in \mathbb{R}^3$ is an unknown constant disturbance acting on the i-th vehicle. Let $\mathbf{p}_{ij} := (\mathbf{p}_i - \mathbf{p}_j)$ and $\mathbf{p}_{ij}^{\mathbf{d}} := (\mathbf{p}_i^{\mathbf{d}} - \mathbf{p}_j^{\mathbf{d}})$ denote the relative position and the desired relative position of vehicle i with respect to vehicle j, respectively. In order to track a prescribed formation, the goal is to have $\mathbf{p}_{ij}(t) - \mathbf{p}_{ij}^{\mathbf{d}}(t) \to \mathbf{0}$ as $t \to \infty$. Furthermore, each vehicle is considered to have limited information about the complete system. More specifically, it is assumed that each vehicle has access to the relative position and velocity of some of the other vehicles (its neighbors). The vehicles must also asymptotically achieve their desired position in space, i.e., $\mathbf{p}_i(t) - \mathbf{p}_i^{\mathbf{d}}(t) \to \mathbf{0}$ as $t \to \infty$.

3. CONSENSUS PROTOCOLS

In this section, some background on graph theory is provided, and the consensus protocols used for distributed coordination of the multiple agents are presented.

3.1 Preliminaires in graph theory

When working with networks of agents, the interaction topology is typically described using graph theory. A directed graph (or digraph) consists of a pair of sets $(\mathcal{V},\mathcal{E})$, where $\mathcal{V}$ is a non-empty finite set of nodes and $\mathcal{E} \in \mathcal{V}^2$ is a finite set of ordered pairs of nodes, called edges. If a digraph is weighted, the weight associated to an edge connecting node i to j is denoted by $k_{ij} \in \mathbb{R}^+$. If it is not weighted, all weights are considered to be one, and if there is no edge connecting i to j, $k_{ij} = 0$. When an edge connects i to j, i is the parent node and j the child node. The set of parent nodes of i, $\mathcal{N}_i \subset \mathcal{V}$, is called the neighborhood of i. A directed path is an ordered sequence of edges connecting two nodes. When there is at least one node that has a directed path to all others, the digraph is said to have a directed spanning tree. The matrix $\mathbf{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$, where n is the number of nodes, is defined as $l_{ij} = -k_{ji}$, for $i \neq j$, and $l_{ij} = \sum_{j\neq i} k_{ji}$ for $i = j$. This matrix has null row sum, meaning it has at least one null eigenvalue, with $\mathbf{1}_n$ as the associated eigenvector.

3.2 Single and double integrator dynamics

Consider a group of n agents modeled as single integrators, each described by $\dot{\alpha}_i = \mu_i$, with $\alpha_i, \mu_i \in \mathbb{R}$, where μ_i is the control input of the agent. The consensus protocol for this system is given by

$$\mu_i = -\sum_{j\in\mathcal{N}_i} k_{ji}\left(\alpha_i - \alpha_j\right). \tag{2}$$

The goal of protocol (2) is to drive the agents to a consensus, i.e., $|\alpha_i - \alpha_j| \to 0$ as $t \to \infty$. Note that each agent i only needs to know the difference between its state and the state of its neighbors, given by $(\alpha_i - \alpha_j)$, $j \in \mathcal{N}_i$. From the definition of $\mathbf{L}$, it is possible to write (2) in vector form as $\boldsymbol{\mu} = -\mathbf{L}\boldsymbol{\alpha}$, where $\boldsymbol{\alpha} = [\alpha_1 \cdots \alpha_n]^\top \in \mathbb{R}^n$ and $\boldsymbol{\mu} = [\mu_1 \cdots \mu_n]^\top \in \mathbb{R}^n$. As shown, for example, in Ren et al. (2004), the existence of a directed spanning tree on the digraph which describes the interaction topology is a necessary and sufficient condition for achieving consensus.

Consider now agents modeled as double integrators, i.e.,

$$\begin{cases}\dot{\alpha}_i = \beta_i \\ \dot{\beta}_i = \mu_i\end{cases}, \tag{3}$$

with $\alpha_i, \beta_i, \mu_i \in \mathbb{R}$, where μ_i is the control input. The following protocol was proposed in Ren and Atkins (2007),

$$\mu_i = -\sum_{j\in\mathcal{N}_i} k_{ji}\left[(\alpha_i-\alpha_j)+\gamma\left(\beta_i-\beta_j\right)\right], \tag{4}$$

with $\gamma \in \mathbb{R}^+$. The goal of protocol (4) is to drive $|\alpha_i - \alpha_j| \to 0$ as $t \to \infty$. In vector form, protocol (4) can be written as $\boldsymbol{\mu} = -\mathbf{L}\boldsymbol{\alpha} - \gamma\mathbf{L}\boldsymbol{\beta}$, with $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\mu} \in \mathbb{R}^n$. Unlike the single integrator case, it is shown in Ren and Atkins (2007) that the existence of a directed spanning tree is not a sufficient condition for reaching consensus. The same work then provides a sufficient but not necessary condition for reaching consensus.

3.3 Triple integrator dynamics

For agents modeled as triple integrators, i.e.,

$$\begin{cases}\dot{\vartheta}_i = \alpha_i\\ \dot{\alpha}_i = \beta_i\\ \dot{\beta}_i = \mu_i\end{cases}, \tag{5}$$

with $\vartheta_i, \alpha_i, \beta_i, \mu_i \in \mathbb{R}$, where μ_i is the control input of the agent, the following consensus protocol is proposed

$$\mu_i = -\sum_{j\in\mathcal{N}_i} k_{ji}\left[(\alpha_i-\alpha_j)+\gamma\left(\beta_i-\beta_j\right)+\zeta\left(\vartheta_i-\vartheta_j\right)\right], \tag{6}$$

where $\gamma, \zeta \in \mathbb{R}_{\neq 0}$. To achieve consensus, the goal is to have $|\vartheta_i - \vartheta_j| \to 0$ as $t \to \infty$. Note that, in vector form, (6) can be written as $\boldsymbol{\mu} = -\mathbf{L}\boldsymbol{\alpha} - \gamma\mathbf{L}\boldsymbol{\beta} - \zeta\mathbf{L}\boldsymbol{\vartheta}$, with $\boldsymbol{\vartheta}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\mu} \in \mathbb{R}^n$, hence, the feedback actuated system becomes

$$\begin{bmatrix}\dot{\boldsymbol{\vartheta}}\\ \dot{\boldsymbol{\alpha}}\\ \dot{\boldsymbol{\beta}}\end{bmatrix} = \mathbf{H}\begin{bmatrix}\boldsymbol{\vartheta}\\ \boldsymbol{\alpha}\\ \boldsymbol{\beta}\end{bmatrix}, \tag{7}$$

with

$$\mathbf{H} = \begin{bmatrix}\mathbf{0}_{n\times n} & \mathbf{I}_n & \mathbf{0}_{n\times n}\\ \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} & \mathbf{I}_n\\ -\zeta\mathbf{L} & -\mathbf{L} & -\gamma\mathbf{L}\end{bmatrix}.$$

3.4 Convergence analysis

The convergence properties of the proposed consensus protocol (6) are provided in this section. Here, some results on the convergence of this protocol are presented, including: i) the final consensus values, ii) necessary and sufficient conditions for convergence given by a graphical condition and bounds on the parameters γ and ζ of protocol (6), and iii) the effect of disturbances on the convergence of the agents. To this end, the eigenvalues of $\mathbf{H}$ are studied. These are the solutions of $\det(\lambda\mathbf{I}_{3n} - \mathbf{H}) = 0$. Noting that all the blocks of $\lambda\mathbf{I}_{3n} - \mathbf{H}$ commute, the result described in Theorem 1 in Kovács et al. (1999) can be used to conclude that

$$\det(\lambda\mathbf{I}_{3n} - \mathbf{H}) = \det\big(\lambda^3\mathbf{I}_n + (\gamma\lambda^2+\lambda+\zeta)\mathbf{L}\big). \tag{8}$$

Since the determinant of a matrix is equal to the product of its eigenvalues, it is possible to conclude, expanding (8), that

$$\det(\lambda\mathbf{I}_{3n} - \mathbf{H}) = \prod_{i=1}^{n} g_i(\lambda),$$

with

$$g_i(\lambda) \coloneqq \lambda^3 + (\gamma\lambda^2 + \lambda + \zeta)\eta_i,$$

where $g_i(\lambda)$ are the eigenvalues of $\lambda^3\mathbf{I}_n + (\gamma\lambda^2+\lambda+\zeta)\mathbf{L}$ and η_i, $i = 1, \ldots, n$, are the eigenvalues of $\mathbf{L}$ (see Lutkepohl (1996)). By definition, $\zeta \neq 0$, hence it is possible to conclude that $\lambda = 0$ is a root of $g_i(\lambda)$ if and only if $\eta_i = 0$. Therefore, for each $\eta_i = 0$, there are exactly three null eigenvalues in $\mathbf{H}$. The following result can be considered an extension of Lemma 4.1 in Ren and Atkins (2007).

Lemma 1. The consensus protocol (6) for agents modeled as triple integrators reaches consensus if and only if the matrix $\mathbf{H}$ has exactly three null eigenvalues and the remaining eigenvalues have negative real part. Moreover, when reaching consensus (for large t), $\boldsymbol{\beta}(t) \to \mathbf{1}_n\mathbf{r}^\top\boldsymbol{\beta}(0)$, $\boldsymbol{\alpha}(t) \to \mathbf{1}_n\mathbf{r}^\top\boldsymbol{\alpha}(0) + \mathbf{1}_n\mathbf{r}^\top\boldsymbol{\beta}(0)t$ and $\boldsymbol{\vartheta}(t) \to \mathbf{1}_n\mathbf{r}^\top\boldsymbol{\vartheta}(0) + \mathbf{1}_n\mathbf{r}^\top\boldsymbol{\alpha}(0)t + \mathbf{1}_n\mathbf{r}^\top\boldsymbol{\beta}(0)\frac{t^2}{2}$, where $\mathbf{r}$ is a non-negative left eigenvector of $\mathbf{L}$ associated to the null eigenvalue, and is such that $\mathbf{1}_n^\top\mathbf{r} = 1$.

Motivated by the result presented in Lemma 1, the following result from Ren et al. (2004) is used to conclude that the digraph must have a directed spanning tree in order for the agents to achieve consensus.

Lemma 2. (from Ren et al. (2004)). The matrix $\mathbf{L}$ associated to a digraph has a single null eigenvalue and all other eigenvalues have positive real part if and only if the digraph has a directed spanning tree.

As stated in Lemma 1, the protocol (6) achieves consensus when $\mathbf{H}$ has exactly three null eigenvalues, which implies that $\mathbf{L}$ has exactly one null eigenvalue. Then, the necessity for a spanning tree follows from Lemma 2. Conditions on the parameters γ and ζ for which all non-null eigenvalues of $\mathbf{H}$ have negative real part are then determined, yielding the following result where necessary and sufficient conditions for convergence are explicitly described as bounds to be imposed on these parameters.

Theorem 3. The third-order consensus protocol (6) achieves consensus asymptotically if and only if the associated digraph has a directed spanning tree and

$$\begin{cases}\gamma > \max\limits_{\eta_i\neq 0}\sqrt{\dfrac{1-\xi_i^2}{\xi_i\omega_{n_i}}}\\ 0 < \zeta < \min\limits_{\eta_i\neq 0}\left[\dfrac{\omega_{n_i}}{\xi_i}\left(\gamma - \sqrt{\dfrac{1-\xi_i^2}{\xi_i\omega_{n_i}}}\right)\right]\end{cases},$$

where $\omega_{n_i} = |\eta_i|$ and $\xi_i = \mathrm{Re}(\eta_i)/\omega_{n_i}$ represent the natural frequency and damping coefficient, respectively, associated with the i-th eigenvalue of $\mathbf{L}$.

The following result is a direct consequence of Theorem 3.

Corollary 4. The consensus protocol (4), for agents modeled as double integrators, reaches consensus asymptotically if and only if the digraph describing the interaction topology of the agents has a directed spanning tree and

$$\gamma > \max_{\eta_i\neq 0}\sqrt{\frac{1-\xi_i^2}{\xi_i\omega_{n_i}}},$$

where $\omega_{n_i} = |\eta_i|$ and $\xi_i = \mathrm{Re}(\eta_i)/\omega_{n_i}$ represent the natural frequency and damping coefficient, respectively, associated with the i-th eigenvalue of $\mathbf{L}$.

Note that Corollary 4 can now replace the result previously presented in Ren and Atkins (2007), since it provides not only a sufficient but also a necessary condition for consensus.

The following result describes the effect on the overall convergence of the system of constant disturbances acting on each agent. Considering these disturbances, the model for the agents is now described as $\dot{\beta}_i = \mu_i + d_i$.

Proposition 5. In the conditions of Theorem 3, the consensus protocol (6) achieves consensus for the state variables $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in the presence of a constant disturbance, $\mathbf{d} = [d_1 \ \cdots \ d_n]^\top \in \mathbb{R}^n$. Moreover, the consensus protocol (4) achieves consensus for the state variables $\boldsymbol{\beta}$ in the conditions of Corollary 4. If all entries of $\mathbf{d}$ are equal, consensus is achieved for all variables.

4. FORMATION TRACKING CONTROLLER

Consider the system of n vehicles with the dynamics described in (1), ignoring for the moment the effect of the disturbance $\mathbf{d}_i$. Let $\tilde{\mathbf{p}}_i \coloneqq \mathbf{p}_i - \mathbf{p}_i^{\mathbf{d}}$ be the trajectory tracking error. Then,

$$\begin{cases} \dot{\tilde{\mathbf{p}}}_i = \dot{\mathbf{p}}_i - \dot{\mathbf{p}}_i^{\mathbf{d}} \coloneqq \tilde{\mathbf{v}}_i \\ \dot{\tilde{\mathbf{v}}}_i = \dot{\mathbf{v}}_i - \ddot{\mathbf{p}}_i^{\mathbf{d}} \coloneqq \tilde{\mathbf{u}}_i \end{cases} . \tag{9}$$

Note that $\mathbf{p}_{ij} - \mathbf{p}_{ij}^{\mathbf{d}} = \tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j$, meaning that the goal of achieving formation can be written as $\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j \to \mathbf{0}$. The dynamics, described by (9), are decoupled, and so the controllers can be designed independently for each axis. Comparing the dynamics over each axis with the ones described in (3), it is possible to conclude they are the same. Also, note that the control objective is the same as the one described for protocol (4). Therefore, protocol (4) can be used to achieve formation tracking, and the control input $\tilde{\mathbf{u}}_i$ for the error dynamics becomes

$$\tilde{\mathbf{u}}_i = -\sum_{j\in\mathcal{N}_i} k_{ji} \left[(\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j) + \gamma (\tilde{\mathbf{v}}_i - \tilde{\mathbf{v}}_j) \right], \tag{10}$$

which is guaranteed to drive the vehicles into formation under the conditions of Corollary 4. The control input for the i-th vehicle, based on relative measurements p_{ij} and v_{ij}, can then be recovered, yielding

$$\mathbf{u}_i = \ddot{\mathbf{p}}_i^{\mathbf{d}} - \sum_{j\in\mathcal{N}_i} k_{ji} \left[\left(\mathbf{p}_{ij} - \mathbf{p}_{ij}^{\mathbf{d}}\right) + \gamma \left(\mathbf{v}_{ij} - \dot{\mathbf{p}}_{ij}^{\mathbf{d}}\right) \right]. \tag{11}$$

4.1 Integral action

When performing formation tracking using the control law described in (11), the distributed multi-vehicle system is able to track the prescribed formation under the conditions of Corollary 4. However, real systems are susceptible to a number of non-idealities, such as disturbances, modeling errors, and actuator dead-zones. In the system described in (1), a constant disturbance is considered. It follows from Proposition 5 that (11) is not able to achieve the goal in the presence of this disturbance. To this end, the use of integral action is proposed, which can be achieved by augmenting the state vector to include the integral of the position tracking error, with model given by $\dot{\tilde{\mathbf{g}}}_i = \tilde{\mathbf{p}}_i$. Note that the system including this new state becomes a triple integrator system. It is then straightforward to conclude that the consensus protocol (6) can be used, and

$$\tilde{\mathbf{u}}_i = -\sum_{j\in\mathcal{N}_i} k_{ji} \left[(\tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_j) + \gamma (\tilde{\mathbf{v}}_i - \tilde{\mathbf{v}}_j) + \zeta (\tilde{\mathbf{g}}_i - \tilde{\mathbf{g}}_j) \right]. \tag{12}$$

Noting that $\tilde{\mathbf{g}}_i - \tilde{\mathbf{g}}_j = \int_{t_0}^{t} \left(\mathbf{p}_{ij} - \mathbf{p}_{ij}^{\mathbf{d}}\right) dt$, the control input for the vehicle, again based on relative measurements, is recovered, yielding

$$\begin{aligned} \mathbf{u}_i = \ddot{\mathbf{p}}_i^{\mathbf{d}} - \sum_{j\in\mathcal{N}_i} k_{ji} \Big[& \left(\mathbf{p}_{ij} - \mathbf{p}_{ij}^{\mathbf{d}}\right) \\ & + \gamma \left(\mathbf{v}_{ij} - \dot{\mathbf{p}}_{ij}^{\mathbf{d}}\right) + \zeta \int_{t_0}^{t} \left(\mathbf{p}_{ij} - \mathbf{p}_{ij}^{\mathbf{d}}\right) dt \Big]. \end{aligned} \tag{13}$$

It follows from Proposition 5, that this controller is able to track the formation in the presence of a constant disturbance, in the conditions of Theorem 3.

4.2 Goal-seeking term

Now that a controller that achieves formation tracking has been designed, the positions of all vehicles must be driven to their desired positions. This is accomplished using goal seeking terms, which take the form of a trajectory tracking controller. This could be added to all the vehicles, as in Ren and Atkins (2007), but then all vehicles would need access to their own state. Note however that it only needs to be added to one vehicle. Consider adding goal seeking terms to the consensus protocol (6) to drive the errors of all the vehicles to zero. To do so, $\boldsymbol{\mu}$ must be designed such that the feedback actuated system becomes $\dot{\mathbf{x}} = \mathbf{H_G}\mathbf{x}$, where $\mathbf{x} = \left[\boldsymbol{\vartheta}^\top \ \boldsymbol{\alpha}^\top \ \boldsymbol{\beta}^\top\right]^\top$, and $\mathbf{H_G}$ is a stable matrix. To add a goal seeking term, the following is added to μ_i

$$\mu_i^G = -k_i^G \left[K_\alpha \alpha_i + \zeta K_\vartheta \vartheta_i + \gamma K_\beta \beta_i \right], \tag{14}$$

where $K_\alpha, K_\beta, K_\vartheta \in \mathbb{R}^+$ are the gains of the goal seeking term, and $k_i^G \in \{0, 1\}$ is a variable controlling whether the i-th agent has a goal seeking term. Only one vehicle is assumed to use this term. The following result is intuitive, and has probably been introduced before, however, since it was not found in the literature, it is hereby presented.

Proposition 6. The vehicles reach their desired positions only if the vehicle that has a goal seeking term has a directed path to all others.

In the scope of this work, a leader vehicle is assumed to exist. This is a vehicle that only has outgoing links in the digraph that describes the interaction topology, corresponding to a line of zeros in $\mathbf{L}$. Then, if there is a directed spanning tree on the digraph, this is the only vehicle with a directed path to all others. The controller for this leader vehicle can be designed independently of the formation tracking controller. If both the controller for the leader vehicle and the formation tracking controller are stable, all vehicles are able to achieve their desired position. Let the leader vehicle be denoted by vehicle 1. Its consensus seeking term is null, and this is the only vehicle which considers a goal seeking term. Hence, the control input for the error dynamics associated to this vehicle becomes $\tilde{\mathbf{u}}_1 = -K_P\tilde{\mathbf{p}}_1 - K_V\tilde{\mathbf{v}}_1 - K_I\tilde{\mathbf{g}}_1$, where $K_P = K_\alpha$, $K_I = \zeta K_\vartheta$ and $K_V = \gamma K_\beta$ are the proportional, integral, and derivative gains, respectively. Then, its control input is given by

$$\mathbf{u}_1 = \ddot{\mathbf{p}}_1^{\mathbf{d}} - K_p\tilde{\mathbf{p}}_1 - K_v\tilde{\mathbf{v}}_1 - K_i \int_{t_0}^{t} \tilde{\mathbf{p}}_1 \, dt. \tag{15}$$

Note that the control law for the leader vehicle, which can be independently designed taking only into account its own state vector, consists on the feedback of all the error states associated to this vehicle. This can then be described by an LQR feedback control law, to tune the gains for this PID trajectory tracking controller.

5. EXPERIMENTAL VALIDATION

5.1 Experimental setup

In order to validate the proposed approach, experiments were conducted with multirotor vehicles in an indoor environment, using a motion capture system to acquire position data, which was then sent to the multirotors using WiFi. The Intel Aero Ready To Fly quadrotor was used, equipped with the PX4 autopilot. The capabilities of the autopilot were used for sensor fusion between the several onboard sensors and the motion capture position data and for interaction with the platform. In order to control the multirotor in acceleration, this was transformed in attitude and thrust commands, which were then sent to the multirotor. The controller was implemented in a single computer, which receives data from the multirotors and sends commands through WiFi. Due to space constraints, the experiments were conducted using two multirotors, while other multirotors were simulated in-the-loop, using the Gazebo simulator. The Robot Operating System (ROS) was used as middleware for communication with the autopilot. The flight procedure considers the takeoff for all vehicles simultaneously, and only after all the multirotors are flying, control is switched to the controller to test.

5.2 Results

Experimental results obtained using the previously described setup are now presented. When presenting the results, circles are used to represent the current positions of the vehicles and squares their initial positions. The shaded areas in the figures used to present the evolution of the altitude with time represent the takeoff and landing parts of the flight, where the PX4 autopilot controls the multirotors, and the remaining area is when the formation tracking controller to test is active.

In the presented experiment, the goal is to show the convergence of the vehicles from their initial positions to the desired formation. For this, a virtual leader was considered. The virtual leader is not an actual vehicle, but some other entity of interest, such as a target to follow. The goal of the vehicles is then to control their position with respect to each other and to the virtual leader. An actual object, with known position and velocity, was used as the virtual leader. The position and velocity of this object along the Z-axis were set to zero, ensuring that it only influences the movement of the vehicles in the horizontal plane. Initially, this object is placed at the origin of the inertial frame. It is moved after the vehicles reach the prescribed formation. The goal formation prescribed to the vehicles is a square shaped formation, at an altitude of one meter relative to the virtual leader (recall that the virtual leader is set to have null altitude).

The interaction topology that is considered is described by the digraph of Fig. 2. For this digraph, the eigenvalues of the associated matrix $\mathbf{L}$ contain an imaginary part, meaning that some oscillations are expected in the movement of the vehicles. The connection weights were set to $k_{ij} = 1.4$, the derivative gain is $\gamma = 1.2\,\mathrm{s}^{-1}$, and when considering integral action, $\zeta = 0.15\,\mathrm{s}^{-3}$, which can be shown to verify the conditions of Theorem 3. Vehicle 1 is the virtual leader, as previously described. When the virtual leader moves to a position other than the origin, all vehicles are expected to move in the same way. Disturbances were not introduced artificially since these are expected to occur in practice. The physical multirotors used were vehicles 2 and 3. A video presenting the described experiment can be visualized in `https://youtu.be/1Rp_FBXtdY4` .

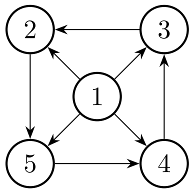
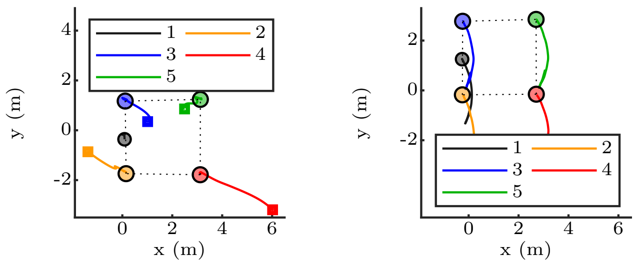


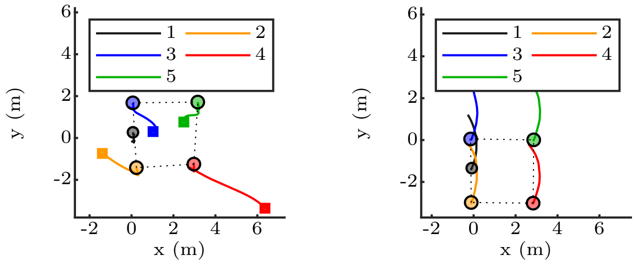
Fig. 2. Interaction topology used in the presented experiment.

The movement of the vehicles on the horizontal plane without considering integral action is presented in Fig. 3. In Fig. 3a, the convergence of the vehicles to the desired formation is illustrated, while the virtual leader remains static. After the initial convergence, the virtual leader is moved, as depicted in Fig. 3b, and the vehicles maintain the formation, tracking the movement of the leader. The same data is presented in Fig. 4, now considering integral action. Some distinctions in between Fig. 4a and Fig. 3a are to be noted. Note that there is still an observable error regarding the desired square formation after the initial 10 seconds, most apparent in vehicle 4. This seems likely to have been caused by actuator saturation. Note that this is the vehicle that starts farther away from the desired formation, therefore, it is expected to have an high control input during the initial instants, which causes a saturation. For this reason, the controller is not able to decrease the error at the desired rate, causing the integrator to wind up. A possible solution consists



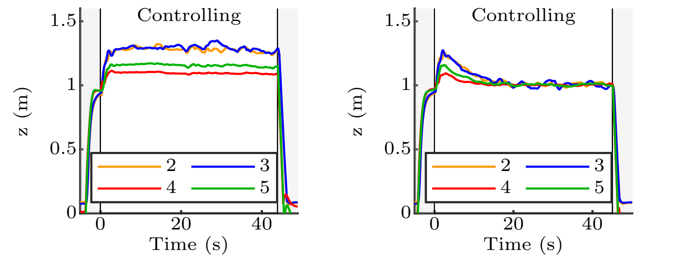
(a) Movement at $t = 10\,\mathrm{s}$. (b) Movement at $t = 30\,\mathrm{s}$.

Fig. 3. Movement on the plane without integral action.



(a) Movement at $t = 10\,\mathrm{s}$. (b) Movement at $t = 30\,\mathrm{s}$.

Fig. 4. Movement on the plane with integral action.



(a) Without integral action.

(b) With integral action.

Fig. 5. Evolution of the altitude with time.

of adding anti-windup action to the integrator. Figure 4b shows the movement of the vehicles, tracking the object while maintaining the formation. It is also possible to see that the integrator accumulated error has disappeared, as the vehicles achieved the prescribed square formation.

In Fig. 5, the time evolution of the altitude of the vehicles is presented. As it can be seen, when no integral action is used, the physical vehicles 2 and 3 both climb to an higher altitude, consequently dragging the simulated vehicles with them. This is evidence of unknown constant disturbances acting on the multirotors. When integral action is used, it is possible to observe that the vehicles were able to achieve consensus on the altitude of the formation and reach the prescribed altitude of one meter, thus rejecting these disturbances.

5.3 Discussion

The goal of this experiment was to assess the ability of the proposed integral action to reject disturbances, and verify that it provides increased capabilities when working with multirotor vehicles. These vehicles are susceptible to modeling errors, which can be interpreted as disturbances to the nominal system that is being considered. For this reason, when applying controllers to this type of vehicles, it is important to keep in mind that these must be designed with some robustness to these errors. It was shown that the ability of these vehicles to reach the prescribed formation is considerably influenced by these disturbances. These effects would be even more evident when considering an increased number of vehicles. In the described experiment, only two physical multirotors were used. Still, considerable improvements were observed when adding integral action. It is clear that the proposed integral action has a positive effect when performing formation control with multirotor vehicles. The effect of disturbances in multirotor vehicles is clearly more apparent on the vertical axis, as evidenced by the presented experiment. Since the system of double integrator agents previously described is decoupled, a choice could be made to add integral action to the vertical axis alone, as this is where most of the effect is visible. However, if flights were to be made in an open space, an interesting movement for the vehicles would be to move in formation with a constant velocity. In this case, if the velocity was high enough for the drag to have a considerable effect, it would probably prove to be useful to add integral action to the controller on the other axis as well. It was shown that the proposed algorithm is able to reject disturbances acting on the vehicles, while following a decentralized approach, and considering a limited amount of information.

6. CONCLUSION

This paper proposed solutions to enhance the existing algorithms for vehicles modeled as double integrators with the feature of integral action, which enabled disturbance rejection and proved to be useful when working with multirotor vehicles. New theoretical results were achieved regarding the consensus-based protocols used, obtaining criteria for the convergence of the proposed third-order consensus protocol, and enhancing the existing criteria provided in the literature for the second-order consensus protocol. The algorithms were then tested on physical multirotor vehicles and experimental results were obtained, allowing to successfully validate the proposed approach using several vehicles.

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