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RSS-based Byzantine Fault-tolerant Localization Algorithm under NLOS Environment

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Abstract**—**Localization is one of the most critical tasks in wireless sensor networks, but achieving a relatively accurate location estimation is challenging when there have Byzantine fault and non-line-of-sight (NLOS) bias simultaneously. In this context, a localization method, based on received signal strength (RSS), is proposed in this letter to mitigate the impact of Byzantine fault and NLOS bias on the localization accuracy of wireless sensor networks. The proposed method relies on a Byzantine fault-tolerant localization algorithm (BFLA), which converts the localization problem into a generalized trust-region subproblem (GTRS) by applying certain approximations. In order to obtain a feasible solution to the GTRS, a block-coordinate update (BCU) function with a regularization term is used to divide the localization problem into two subproblems. An iterative method, whose start-point is obtained by an unconstrained squared-range (USR) algorithm, is then used to obtain a solution. Numerical simulations are carried out to show the effectiveness of the proposed method, compared with the state-of-the-art approaches in different scenarios.

Index Terms**—Target localization, sensor networks, Byzantine fault, received signal strength (RSS), generalized trust region sub-problem (GTRS), non-line-of-sight (NLOS).**

I. INTRODUCTION

TARGET localization using radio-frequency (RF) is of paramount importance to military and industrial applications [1], [2], where only georeferenced data are significant [3]. Recently, an upsurge of interest has been drawn on received signal strength (RSS)-based localization methods since they are cost-effective and synchronization-free, compared to time of arrival (TOA), angle of arrival (AOA), and time difference of arrival (TDOA) based techniques [4].

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Extensive research has driven notable advances in RSS-based localization methods [4]–[12]. However, most of the research is studied under the assumption that anchors in the network operate as expected when locating the target. Unfortunately, the assumption may not always hold in practice [13], especially under malicious attacks [2], [13]. The anchors may send falsified measurements to the fusion center (FC) during malicious attacks, and this prevents FC from accurately locating the target. It has been concluded that FC may completely lose the ability to accurately locate the target when numerous malicious attacks occur [14]. A Byzantine fault occurs when an anchor node deviates from its expected operation, including sending falsified measurements. And the anchor that deviates from its expected operation is known as the Byzantine node [15]. Locating the target accurately can become a challenging problem, especially in the presence of Byzantine faults. Besides, the non-line-of-sight (NLOS) bias is another negative factor that prevents FC from accurately estimating the target's position [6], [7]. However, very few studies have been conducted on localization methods for wireless sensor networks that simultaneously experience NLOS bias and Byzantine fault.

In this context, the letter proposes an RSS-based localization method, which relies on a Byzantine fault-tolerant localization algorithm (BFLA), and is suitable for the network that experiences the NLOS bias and Byzantine fault simultaneously. The considered localization problem is transformed into a generalized trust-region subproblem (GTRS) by applying certain approximations. To avoid the GTRS to be ill-posed, a block coordinate update (BCU) function with a regularization term is used to divide the problem into two subproblems. An iterative method, whose start-point is obtained by an unconstrained squared-range (USR) algorithm, is then utilized to obtain the feasible solution. It should be noted that most existing localization algorithms detect and exclude Byzantine nodes before localization. However, BFLA locates the target without the need for detection and exclusion.

II. PROBLEM FORMULATION

Consider a two-Dimensional sensor network with N anchors at known locations and a target at an unknown location. Suppose the target's position $\boldsymbol{x} = [x_1, x_2]^T$ and the i^{th} anchor node's position $a_i = \left[a_{ix}, a_{iy}\right]^T$, where $i = 1, \cdots, N$, and T denotes the transpose.

Assume the target can transmit radio signals with RSS information to anchors. Such signals are modeled in [4]:

$$P_{ri} = P_s - PL\left(d_0\right) - \delta_i - 10\alpha \log_{10} \frac{\left\|\boldsymbol{x} - \boldsymbol{a}_i\right\|}{d_0} + \gamma_i, \qquad (1)$$

where P_{ri} is the power of the signal received by the i^{th} anchor

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node from the target, P_s is the transmit power of the target, α is the path loss exponent, $PL(d_0)$ is the loss in signal strength when reference distance $d_0 = 1$ m, δ_i is the positive NLOS bias that is expressed as δ by assuming the biases between the target and the anchors are the same, $\|\cdot\|$ is ℓ_2 norm, and γ_i is the noise modeled as Gaussian distribution $\gamma_i \sim \mathcal{N}(0, \sigma_i^2)$.

The probability density function (PDF) of the observation vector $\boldsymbol{P} = [P_{\mathrm{r}i}]^T$ is given by

$$p(\boldsymbol{P}|\boldsymbol{x}) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left\{ \frac{\left(P_{ri} - P_0 + \delta + 10\alpha \log_{10} \frac{\|\boldsymbol{x} - \boldsymbol{a}_i\|}{d_0} \right)^2}{2\sigma_i^2} \right\}, \quad (2)$$

where $P_0 = P_s - PL(d_0)$.

By maximizing the PDF, the maximum likelihood estimator (MLE) of $\boldsymbol{x}$ could be derived as [16]

$$\arg\min_{\boldsymbol{x}} \sum_{i=1}^{N} \frac{\left(P_{\mathrm{r}i} - P_0 + \delta + 10\alpha \log_{10} \frac{\|\boldsymbol{x} - \boldsymbol{a}_i\|}{d_0} \right)^2}{2\sigma_i^2}. \quad (3)$$

Assume there are B Byzantine nodes in the network, the ratio of Byzantine nodes to the total number of anchors $\lambda = B/N$ is also the portion of falsified measurements. In the letter, Byzantine attacks occur during data transmission. The anchors typically compute measurements before transmitting them to FC. However, during transmission, attackers attempt to add interference noise to the measurements to prevent FC from locating the target accurately. Without loss of generality, the interference noise is assumed to be the non-Gaussian noise η [3]. The simultaneous occurrence of Byzantine fault and NLOS bias is challenging to a network because it is infeasible to determine if measurements contain inference noise (falsified measurements), and the localization problem in Eqn. (3) is highly non-convex, which is difficult to solve.

III. Byzantine Fault-Tolerant Localization

A. GTRS framework

First, a simple transformation based on Eqn. (1) is

$$\|\boldsymbol{x} - \boldsymbol{a}_i\| = d_0 10^{\frac{P_0 - P_{ri} - \delta}{10\alpha}} \cdot 10^{\frac{\gamma_i}{10\alpha}}. \quad (4)$$

When the noise is relatively small and $d_0 = 1$ m, the right side of Eqn. (4) can be approximated using the first-order Taylor series expansion [6], as given by

$$\|\boldsymbol{x} - \boldsymbol{a}_i\| \approx 10^{\frac{P_0 - P_{ri} - \delta}{10\alpha}} \cdot \left(1 + \frac{\ln 10}{10\alpha} \sigma_i \right). \quad (5)$$

The considered problem in Eqn. (3) is then converted into a non-linear least-square form by using squared range operation.

$$\arg\min_{\boldsymbol{x}} \sum_{i=1}^{N} \omega_i \left(\|\boldsymbol{x} - \boldsymbol{a}_i\|^2 - \hat{r}_i^2 \right)^2, \quad (6)$$

where $\hat{r}_i$ is the approximated distance between the i^{th} anchor node and the target, and ω_i is the weight that can be expressed as $\omega_i = 1 - \hat{r}_i / (\sum_{i=1}^{N} \hat{r}_i)$.

By expanding the squared norm part in Eqn. (6), the problem is further converted into the GTRS.

$$J(\boldsymbol{y}) = \underset{\boldsymbol{y}}{\text{minimize}} \ \|\boldsymbol{\omega}(\wp \boldsymbol{y} - \boldsymbol{\kappa})\|^2, \quad \text{subject to} \ \boldsymbol{y}^T \boldsymbol{D} \boldsymbol{y} + 2\boldsymbol{f}^T \boldsymbol{y} = 0, \quad (7)$$

where $\boldsymbol{y} = [\boldsymbol{x}^T, \|\boldsymbol{x}\|^2]^T$, $\boldsymbol{\omega} = \text{diag}([\omega_1^2, \cdots, \omega_N^2])$;

$$\wp = \begin{bmatrix} a_1^T & -1 \\ \vdots & \vdots \\ a_N^T & -1 \end{bmatrix}, \boldsymbol{\kappa} = \begin{bmatrix} \left(\|a_1\|^2 - r_1^2 \right) \\ \vdots \\ \left(\|a_N\|^2 - r_N^2 \right) \end{bmatrix}, \boldsymbol{D} = \begin{bmatrix} \boldsymbol{I}_2 & \boldsymbol{0}_{2\times 1} \\ \boldsymbol{0}_{1\times 2} & 0 \end{bmatrix}, \boldsymbol{f} = \begin{bmatrix} \boldsymbol{0}_{2\times 1} \\ -\frac{1}{2} \end{bmatrix},$$

where $\boldsymbol{I}$ and $\boldsymbol{0}$ represent identity matrix and zero matrix, respectively.

B. BFLA

However, Byzantine fault and NLOS bias may cause this problem to become ill-posed, which decreases the stability of the solution. In this case, a regularization function of $\boldsymbol{\omega}$ is developed in GTRS.

$$J(\boldsymbol{y}, \boldsymbol{\omega}) = \underset{\boldsymbol{y}, \boldsymbol{\omega}}{\text{minimize}} \ \sum_{i=1}^{N} \boldsymbol{\omega}_i (\wp_i \boldsymbol{y} - \boldsymbol{\kappa}_i)^2 + g(\boldsymbol{\omega}_i), \quad \text{subject to} \ \boldsymbol{y}^T \boldsymbol{D} \boldsymbol{y} + 2\boldsymbol{f}^T \boldsymbol{y} = 0, \quad (8)$$

where $g(\boldsymbol{\omega}_i)$ is the regularization function, $\boldsymbol{\omega}_i$, $\wp_i$ and $\boldsymbol{\kappa}_i$ represent the i^{th} row of the corresponding matrix.

Inspired by [17], we propose to use the regularization function $g(\boldsymbol{\omega}_i)$ as

$$g(\boldsymbol{\omega}_i) = \sum_{i=1}^{N} \varepsilon^2 \boldsymbol{\omega}_i - \frac{1}{\boldsymbol{\omega}_i}, \quad (9)$$

where ε is the robust factor.

Further, the GTRS is separated into two sub-problems as

$$\begin{aligned} &\text{(I)} \quad \boldsymbol{y}^{k+1} = \arg\min_{\boldsymbol{y}} \ J(\boldsymbol{y}, \boldsymbol{\omega}^k), \\ &\qquad \text{subject to} \ \boldsymbol{y}^T \boldsymbol{D} \boldsymbol{y} + 2\boldsymbol{f}^T \boldsymbol{y} = 0, \\ &\text{(II)} \quad \boldsymbol{\omega}^{k+1} = \arg\min_{\boldsymbol{\omega}} \ J(\boldsymbol{y}^{k+1}, \boldsymbol{\omega}), \\ &\qquad \text{subject to} \ \boldsymbol{\omega}_i > 0, \ \forall i, \end{aligned} \quad (10)$$

where k is the number of iterations.

The problem of (II) in Eqns. (10) is strictly convex; thus, its global minimizer is

$$\boldsymbol{\omega}_i^k = \frac{1}{\sqrt{(\wp_i \boldsymbol{y}^k - \boldsymbol{\kappa}_i)^2 + \varepsilon^2}}. \quad (11)$$

However, the problem of (I) in Eqns. (10) is non-convex due to the constrained condition; thus, we develop a BCU function with a Lipschitz continuity constant l to the aforementioned function.

$$\boldsymbol{y}^k = \arg\min_{\boldsymbol{y}} \ \left\langle \nabla_{\boldsymbol{y}} J(\hat{\boldsymbol{y}}^k, \boldsymbol{\omega}^{k-1}), \boldsymbol{y} - \hat{\boldsymbol{y}}^k \right\rangle + l^k \|\boldsymbol{y} - \hat{\boldsymbol{y}}^k\|^2, \quad \text{subject to} \ \boldsymbol{y}^T \boldsymbol{D} \boldsymbol{y} + 2\boldsymbol{f}^T \boldsymbol{y} = 0, \quad (12)$$

where

$$\hat{\boldsymbol{y}}^k = \boldsymbol{y}^{k-1} + \xi^k \left(\boldsymbol{y}^{k-1} - \boldsymbol{y}^{k-2} \right), \quad (13)$$

and ξ^k ($\xi^k \neq \boldsymbol{\omega}_i^k$) is the weight related to l^k, i.e., $\xi^k = \left(\sqrt{l^{k-1}/l^k}\right)/12$, and $l^0 = 2\|\wp^T \boldsymbol{\omega}^k \wp\|$.

Assumption 1: $\nabla_{\boldsymbol{y}} = J(\hat{\boldsymbol{y}}^k, \boldsymbol{\omega}^{k-1})$ has a Lipschitz continuity constant l^k in respect of $\boldsymbol{y}^k$ with

$$\left\|\nabla_{y} J\left(\boldsymbol{u}, \boldsymbol{\omega}^{k-1}\right)-\nabla_{y} J\left(\boldsymbol{v}, \boldsymbol{\omega}^{k-1}\right)\right\| \leq l^{k}\|\boldsymbol{u}-\boldsymbol{v}\|, \ \forall \boldsymbol{u}, \boldsymbol{v}. \quad (14)$$

Remark: In BCU, the variables are determined based on blocks. The blocks in the sub-problems in Eqns. (10) are x_1, x_2, and $\|\boldsymbol{x}\|^2$. Besides, the GTRS in Eqn. (8) has a global minimum that is obtained using the bisection method. Therefore, the function of $J(\boldsymbol{y}, \boldsymbol{\omega})$ is lower bounded.

Proposition: Let $\{\boldsymbol{y}\}^k$ be the sequence obtained by Eqn. (12), $\boldsymbol{y}_j^k$ be the j^{th} block at k^{th} iteration, l_j^k be the Lipschitz continuity constant of the j^{th} block at k^{th} iteration, and ξ_j^k be the weight of the j^{th} block at k^{th} iteration with $0 \leq \xi_j^k \leq \chi\sqrt{l_j^{k-1}/l_j^k}$ for $0 < \chi < 1$.If *Assumption 1* is satisfied, we have

$$\sum_{k=1}^{\infty}\left\|\boldsymbol{y}^{k-1}-\boldsymbol{y}^{k}\right\|^{2}<\infty. \quad (15)$$

Proof: From the Lipschitz continuity of the function $\nabla_{\boldsymbol{y}_j}{=}J(\boldsymbol{y}^k, \boldsymbol{\omega}^{k-1})$ about $\boldsymbol{y}_j$, it holds by a direct application of the Lemma 2.1 in [18] that

$$\begin{aligned} J\left(\boldsymbol{y}^{k}, \boldsymbol{\omega}^{k-1}\right) \leq & J\left(\boldsymbol{y}^{k-1}, \boldsymbol{\omega}^{k-1}\right)+\left\langle\nabla_{y_j} J\left(\boldsymbol{y}^{k-1}, \boldsymbol{\omega}^{k-1}\right), \boldsymbol{y}_{j}^{k}-\boldsymbol{y}_{j}^{k-1}\right\rangle \\ & +\frac{l^{k}}{2}\left\|\boldsymbol{y}_{j}^{k}-\boldsymbol{y}_{j}^{k-1}\right\|^{2}. \end{aligned} \quad (16)$$

Then, we have

$$\begin{aligned} & J\left(\boldsymbol{y}_{j}^{k-1}, \boldsymbol{\omega}^{k-1}\right)-J\left(\boldsymbol{y}_{j}^{k}, \boldsymbol{\omega}^{k-1}\right) \\ & \geq \frac{l_{j}^{k-1}}{2}\left\|\hat{\boldsymbol{y}}_{j}^{k-1}-\boldsymbol{y}_{j}^{k}\right\|^{2}+l_{j}^{k-1}\left\langle\hat{\boldsymbol{y}}_{j}^{k-1}-\boldsymbol{y}_{j}^{k-1}, \boldsymbol{y}_{j}^{k}-\hat{\boldsymbol{y}}_{j}^{k-1}\right\rangle \\ & \geq \frac{l_{j}^{k-1}}{2}\left\|\boldsymbol{y}_{j}^{k-1}-\boldsymbol{y}_{j}^{k}\right\|^{2}+\frac{l_{j}^{k-2}}{2} \chi^{2}\left\|\boldsymbol{y}_{j}^{k-2}-\boldsymbol{y}_{j}^{k-1}\right\|^{2}. \end{aligned} \quad (17)$$

As for the whole blocks, Eqn. (17) still holds. Hence,

$$\begin{aligned} & J\left(\boldsymbol{y}^{k-1}, \boldsymbol{\omega}^{k-1}\right)-J\left(\boldsymbol{y}^{k}, \boldsymbol{\omega}^{k-1}\right) \\ & =\sum_{j=1}^{3} J\left(\boldsymbol{y}_{j}^{k-1}, \boldsymbol{\omega}^{k-1}\right)-J\left(\boldsymbol{y}_{j}^{k}, \boldsymbol{\omega}^{k-1}\right) \\ & \geq \sum_{j=1}^{3}\left(\frac{l_{j}^{k-1}}{2}\left\|\boldsymbol{y}_{j}^{k-1}-\boldsymbol{y}_{j}^{k}\right\|^{2}+\frac{l_{j}^{k-2}}{2} \chi^{2}\left\|\boldsymbol{y}_{j}^{k-2}-\boldsymbol{y}_{j}^{k-1}\right\|^{2}\right). \end{aligned} \quad (18)$$

Suppose the total number of iterations is K, we sum the above inequality and obtain

$$\begin{aligned} & J\left(\boldsymbol{y}^{0}, \boldsymbol{\omega}^{k-1}\right)-J\left(\boldsymbol{y}^{K}, \boldsymbol{\omega}^{k-1}\right) \\ & \geq \sum_{k=1}^{K} \sum_{j=1}^{3}\left(\frac{l_{j}^{k-1}}{2}\left\|\boldsymbol{y}_{j}^{k-1}-\boldsymbol{y}_{j}^{k}\right\|^{2}+\frac{l_{j}^{k-2}}{2} \chi^{2}\left\|\boldsymbol{y}_{j}^{k-2}-\boldsymbol{y}_{j}^{k-1}\right\|^{2}\right) \\ & \geq \sum_{k=1}^{K} \sum_{j=1}^{3}\left(\frac{\left(1-\chi^{2}\right) l_{j}^{k-1}}{2}\left\|\boldsymbol{y}_{j}^{k-1}-\boldsymbol{y}_{j}^{k}\right\|^{2}\right) \\ & \geq \sum_{k=1}^{K}\left(\frac{\left(1-\chi^{2}\right) l_{j}}{2}\left\|\boldsymbol{y}^{k-1}-\boldsymbol{y}^{k}\right\|^{2}\right). \end{aligned} \quad (19)$$

The function $J(\boldsymbol{y}, \boldsymbol{\omega})$ is lower bounded; thus, $K \to \infty$ completes the proof. □

Under *Assumption 1*, for any limit point $\overline{\boldsymbol{y}}$ of $\{\boldsymbol{y}\}^k$ in line with *Proposition 1*, there exists a subsequence that converges to $\overline{\boldsymbol{y}}$ using the Theorem 1 in [19].

Definition 1: $q{:}\,\mathbb{R}^n \to \mathbb{R}$ and $c{:}\,\mathbb{R}^n \to \mathbb{R}$ are the quadratics, and $\{\tau \in \mathbb{R}^n{:}\, c(\tau) = 0\}$ is not empty. If

$$v \neq 0, v^{T} C v=0 \Rightarrow v^{T} Q v>0, \quad (20)$$

where $Q = \nabla^2 q$, $C = \nabla^2 c$, then the optimization problem $\min\{q(\tau){:}\, c(\tau) = 0\}$ has a global minimizer.

Under *Definition 1*, we can easily verify that Eqn. (20) holds for the considered problem. Therefore, a global minimizer of the solution of (I) in Eqns. (10) is acquired. And $\boldsymbol{y}^k$ is an optimal solution if there is a multiplier ς that satisfies the Kuhn-Tucker condition, i.e.,

$$\begin{aligned} & \left(l^{k} \boldsymbol{I}_{3}+\varsigma \boldsymbol{D}\right) \boldsymbol{y}^{k}=-\wp^{T} \boldsymbol{\omega}^{k-1}\left(\wp \hat{\boldsymbol{y}}^{k}-\boldsymbol{\kappa}\right)+l^{k} \hat{\boldsymbol{y}}^{k}-\varsigma \boldsymbol{f}, \\ & \left(\boldsymbol{y}^{k}\right)^{T} \boldsymbol{D} \boldsymbol{y}^{k}+2 \boldsymbol{f}^{T} \boldsymbol{y}^{k}=0, \\ & \varsigma \geq \max \left\{-l^{k},-\frac{1}{\vartheta}\right\}. \end{aligned} \quad (21)$$

where ϑ is the largest eigenvalue of $(\wp^T\wp)^{-\frac{1}{2}}\boldsymbol{D}(\wp^T\wp)^{-\frac{1}{2}}$.

The initiation should be defined in BFLA; otherwise, the solution is inaccurate. In the letter, we utilize the USR algorithm to obtain the iteration start-point, where $\boldsymbol{y}^0 = (\wp^T\wp)^{-1}\wp^T\boldsymbol{\kappa}$ and $\boldsymbol{\omega}^0 = \text{diag}([\boldsymbol{I}_N])$.

C. Cramer-Rao lower bound (CRLB)

CRLB is used to be the benchmark for the estimators [20]. The CRLB can be indicated as the trace of the inverse of the Fisher information matrix (FIM) when the noise is Gaussian.

$$CRLB \triangleq \operatorname{Tr}\left(FIM^{-1}\right)=\operatorname{Tr}\left[\left(\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{x}}\right) \Sigma^{-1}\left(\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{x}}\right)^{T}\right]^{-1}, \quad (22)$$

where Σ denotes $\text{diag}(\sigma_1, \cdots, \sigma_N)$, $\text{Tr}(\cdot)$ is the trace of a matrix;

$$\frac{\partial \boldsymbol{P}}{\partial \boldsymbol{x}}=\left(\begin{array}{c} \psi \cdot \dfrac{x_{1}-a_{1x}}{\left\|\boldsymbol{x}-\boldsymbol{a}_{1}\right\|^{2}}, \ldots, \psi \cdot \dfrac{x_{1}-a_{Nx}}{\left\|\boldsymbol{x}-\boldsymbol{a}_{N}\right\|^{2}} \\ \psi \cdot \dfrac{x_{2}-a_{1y}}{\left\|\boldsymbol{x}-\boldsymbol{a}_{1}\right\|^{2}}, \ldots, \psi \cdot \dfrac{x_{2}-a_{Ny}}{\left\|\boldsymbol{x}-\boldsymbol{a}_{N}\right\|^{2}} \end{array}\right), \quad (23)$$

with $\psi = \dfrac{10\alpha}{\ln 10}$.

However, the attack (non-Gaussian noise) makes the closed-form expressions of the FIM unavailable. In this case, we exploit a Monte Carlo simulation to obtain the closed-form expression.

$$FIM=\psi^{2} \cdot I \cdot\left[\begin{array}{cc} \sum_{i=1}^{N} \dfrac{\left(x_{1}-a_{ix}\right)^{2}}{\left\|\boldsymbol{x}-\boldsymbol{a}_{i}\right\|^{4}} & \sum_{i=1}^{N} \dfrac{\left(x_{1}-a_{ix}\right)\left(x_{2}-a_{iy}\right)}{\left\|\boldsymbol{x}-\boldsymbol{a}_{i}\right\|^{4}} \\ \sum_{i=1}^{N} \dfrac{\left(x_{2}-a_{iy}\right)\left(x_{1}-a_{ix}\right)}{\left\|\boldsymbol{x}-\boldsymbol{a}_{i}\right\|^{4}} & \sum_{i=1}^{N} \dfrac{\left(x_{2}-a_{iy}\right)^{2}}{\left\|\boldsymbol{x}-\boldsymbol{a}_{i}\right\|^{4}} \end{array}\right], \quad (24)$$

where $I \approx \dfrac{1}{N_C} \sum_{sample=1}^{N_C} \dfrac{\left[\nabla_{\eta} p\left(\gamma_{i}, \eta\right)^{sample}\right]^{2}}{p^{2}\left[\left(\gamma_{i}, \eta\right)\right]^{sample}}$.

IV. Numerical Results

Numerical simulations are carried out in Matlab to verify the effectiveness of the proposed method (BFLA), compared with other state-of-the-art methods, including fault-tolerant

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maximum likelihood (FTML) [15], weighted least squares (WLS) with an RSS-only in the non-cooperative scheme in [7], squared range weighted least squares (SRWLS) in [6], USR in [5], USR-based majorization-minimization (USRMM) in [8] and CRLB. The target and anchors are deployed randomly in the square area, with length $Grid = 25$ m, during each Montel Carlo trial (MCT). The remaining fixed simulation parameters are set as $P_0 = -35$ dBm, $d_0 = 1$ m, $\alpha = 3$, and $MCT = 200$. It is worth noting that the precise NLOS bias cannot be directly determined in practice. However, the maximum possible error can be determined, and NLOS bias can be obtained from a specific distribution [6], [21]. Inspired by [6], the NLOS bias is assumed to be drawn from an exponential distribution, and the rate is drawn from a uniform distribution $bias \sim \mathrm{Exp}(\mathrm{U}[0, \delta_{\max}])$. It is also assumed that noise from attacks obeys a uniform distribution, i.e., $\varphi(\eta) \sim \mathrm{U}\left[-\sqrt{2} \times Grid, \sqrt{2} \times Grid\right]$. Noise with uniform distributions has been used to simulate malicious attacks in many studies [2], [3], [22]. To ensure accurate statistics, the estimator is made 95% asymptotically efficient through selecting an appropriate factor ε [23]. In this study, the robust factor $\varepsilon = 1.34\sqrt{3}\sigma_i$, according to [17]. Root mean square error (RMSE) is used to evaluate the performance of the methods: $RMSE = \sqrt{\sum_{mct=1}^{MCT} (\|\hat{\boldsymbol{x}}_{mct} - \boldsymbol{x}_{mct}\|)/MCT}$, where $\boldsymbol{x}_{mct}$ and $\hat{\boldsymbol{x}}_{mct}$ denote the exact position and the estimate in the mct^{th} trial, respectively.

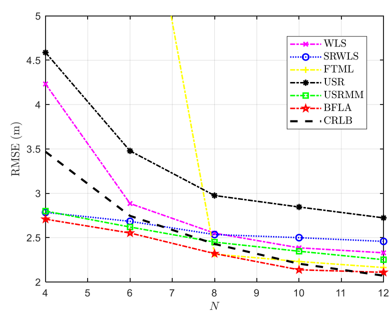

Fig. 1. RMSE versus variable N with $\delta_{\max} = 5$ dB, $\sigma_i = 5$ dB, $\lambda = 0.5$, and $N_{\mathrm{NLOS}} = N$.

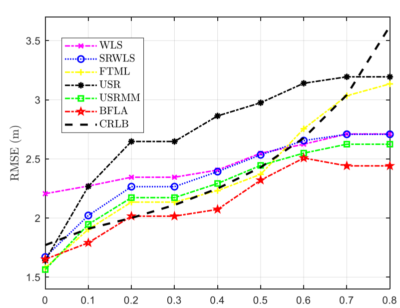

Fig. 2. RMSE versus variable λ with $\delta_{\max} = 5$ dB, $\sigma_i = 5$ dB, and $N_{\mathrm{NLOS}} = N = 8$.

In some scenarios, all anchors suffer the NLOS links, i.e., the number of NLOS nodes (N_{NLOS}) is equal to the total number of anchors N, referred to as Fig. 1, Fig.2, Fig. 3, Fig.4, and Fig.6.

A comparison between RMSE and the total number of anchors N is illustrated in Fig. 1. It is worth noting that the available information for localization increases while N grows. Thus, the performance is improved for all methods in Fig. 1. Although the performance of some approaches is relatively close when $N = 8$, the proposed method, BFLA, performs better than others, and the margin is sizeable when N increases.

A comparison of the RMSE (m) versus variable λ is presented in Fig. 2. As expected, the localization error of all considered algorithms increases as λ grows. However, the localization error of BFLA is the lowest than others, which illustrates the outperformance of the proposed method when existing Byzantine nodes.

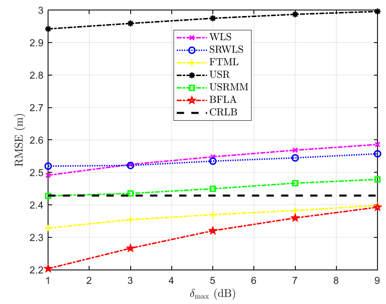

Fig. 3. RMSE versus variable $\delta_{\max}$ with $\lambda = 0.5$, $\sigma_i = 5$ dB, and $N_{\mathrm{NLOS}} = N = 8$.

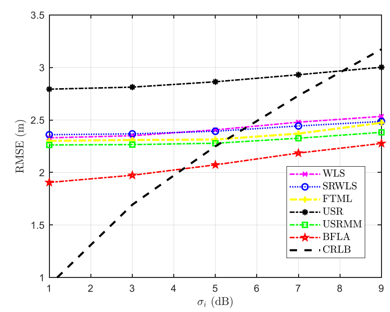

Fig. 4. RMSE versus variable σ_i with $\lambda = 0.5$, $\delta_{\max} = 5$ dB, and $N_{\mathrm{NLOS}} = N = 8$.

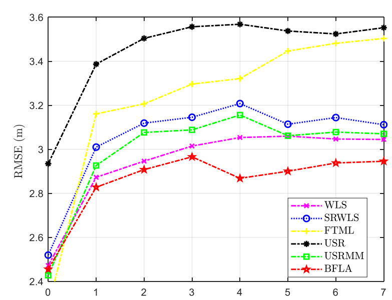

Fig. 5. RMSE versus variable number of NLOS nodes with $\sigma_i = 5$ dB, $\lambda = 0.5$, $\delta_{\max} = 10$ dB, and $N = 8$.

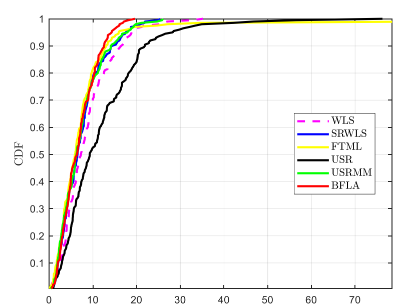

Fig. 6. CDF of $\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|$ with $\sigma_i = 5$ dB, $\lambda = 0.5$, $\delta_{\max} = 10$ dB, and $N_{\mathrm{NLOS}} = N = 6$.

The RMSE (m) versus variable $\delta_{\max}$ and variable σ_i are

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presented in Fig. 3 and Fig. 4, respectively. Interestingly, although the performance of the algorithms deteriorates as $\delta_{\max}$ and σ_i grow, the rate of deterioration is relatively low. Among them, the performance of the proposed method, BFLA, seems to be better than others. Besides, the performance of CRLB degrades dramatically as the rise in λ and σ_i, and even worse than others in some scenarios. This is because when λ is large, the PDF of γ_i deviates from the Gaussian model, and the intrinsic accuracy I is small, leading to deteriorated performance of the estimator. It is also worth noting that the bias gradient matrix is zero due to the known NLOS bias. Therefore, the biased CRLB that we conduct is equivalent to the unbiased one. In other words, the performance of CRLB is stable no matter what the bias is, referred to as Fig. 3. It has been proven that allowing a small amount of the bias into an estimator can improve the performance, which sometimes is better than the unbiased CRLB [24]. From Fig. 1, Fig. 2, and Fig. 4, we can see that when $\delta_{\max} = 5$ dB, some of the biased estimators outperform the CRLB. Nevertheless, the performance of these biased estimators would be worse than that of the CRLB if the NLOS bias is too large. We have carried out several simulations with the same conditions in Fig. 3 and find that the performance of these biased estimators is worse than that of the CRLB if the maximum bias exceeds 12dB. CRLB performance continues to reduce as σ_i increases, provided λ is determined. On the contrary, with the good quality of the initial guess, the biased estimator, BFLA, can mitigate the bias through iteration.

The RMSE (m) versus the variable number of NLOS nodes is depicted in Fig.5. RMSE is similar for most localization algorithms at $N_{\text{NLOS}} = 0$. However, the performance becomes divergent when parts of anchors suffer NLOS links. From Fig. 5, we can see that all considered algorithms show relatively good robustness to the ratio of LOS/NLOS links, wherein the proposed method performs better than the rest. Fig. 6 shows the cumulative distribution function (CDF) of the algorithms. From Fig. 6, we can see that the proposed method, BFLA, to some extent, beats the rest of the methods. The performance of BFLA achieves $\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|< 20$ m at 100%, whereas others attain the same probability in the case of $\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|> 25$ m or more.

V. Conclusion

In this letter, an RSS-based localization method is proposed to mitigate the impact of Byzantine fault and NLOS bias on the localization accuracy of wireless sensor networks. The proposed method relies on BFLA, which studies the localization problem in a GTRS framework by exploiting certain approximations. To avoid the GTRS to be ill-posed, a BCU function with a regularization term is developed to divide the problem into two subproblems. An iterative method, whose start-point is obtained by USR, is then used to obtain a solution. Numerical simulations are executed to demonstrate the outperformance of BFLA against other state-of-the-art approaches in different scenarios. However, this letter assumes that NLOS bias follows a specific distribution with a known maximum possible error. Future studies may discard the aforementioned assumption and develop an adaptive bias estimator.

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