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Decentralized Navigation Systems for Bearing-based Position and Velocity Estimation in Tiered Formations

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Abstract— **This paper presents a decentralized navigation system, capable of estimating positions and fluid velocities, for vehicle formations. Some vehicles have access to a measurement of their own position while the others have access to one or more bearing measurements and may have a depth measurement. Local observers with globally exponentially stable error dynamics are designed by obtaining an equivalent observable linear time-varying system using conveniently defined artificial outputs. The local observers rely on local measurements as well as limited communications between the vehicles. The stability of the system as a whole is obtained by studying the robustness of the local observers to exponentially decaying perturbations. Simulation results are presented to show the behaviour and convergence of the proposed solution.**

I. INTRODUCTION

There are many applications where the use of formations of vehicles is an advantage. In [1], [2], and [3] some applications of vehicle formations are studied for the purposes of minesweeping, surveillance, and localization, respectively.

While centralized control and navigation systems are conceptually simpler, the use of decentralized solutions have several advantages as, for example, the formation not depending on a central system, which, due to failure, may compromise the entire formation. A centralized system would need to be able to communicate with each element of the formation. However, in underwater applications, communications are limited. Decentralized solutions may help coping with this issue. In [4]–[7], examples of decentralized solutions for underwater applications are proposed.

The fact that GPS systems do not work underwater leads to the necessity of developing alternative navigation systems, such as the ones proposed in [8] and [9]. While the majority of the solutions available in the literature are based on range measurements, it is possible to develop navigation systems based on bearing measurements. A sensor that provides bearing measurements is developed and discussed in detail in [10]. Work on the observability issues of target motion analysis based on angle measurements can be found in [11]. Particle filters based on bearing measurements are proposed in [12] and [13], while in [14] a square cubature Kalman filter is presented.

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In this paper, a decentralized navigation system for vehicle formations operating underwater is designed. The formations are assumed to be tiered and acyclic, with the vehicles of the top tier having access to their own position due to, for example, GPS availability on the surface. In the rest of the tiers, vehicles measure bearings and have communication with vehicles in the upper tier.

Local state observers, capable of estimating the vehicle position and the surrounding fluid velocity, are designed. These local observers have access to bearing measurements and positions estimates communicated from the vehicles in the tier above. They also have access to several local measurements, such as attitude angles, velocity relative to the surrounding fluid and, in some cases, depth.

In [15], a continuous time navigation system based on one bearing measurement is proposed. The bearing measurements and the communications are not available at frequency high enough to consider a continuous-time system. Thus, the navigation system is designed considering discrete-time kinematics. In [16], a similar solution to the one proposed in [15] is developed, but in a discrete-time context instead. In [17], the previous work is extended to the case when multiple bearings are available.

The bearing measurement will lead to a system with nonlinear outputs. For such systems, the traditional solution is an extended Kalman filter (EKF), however, that does not offer any guarantee of stability. To obtain local observers with globally exponentially stable (GES) error dynamics, an artificial output based on the bearing measurement is used instead of the bearing itself.

Two different cases of local observers are considered: i) when one bearing and depth are available; and ii) when two or more bearings, but no depth, are available. More cases could be added but due to limitations on the available space they were left out.

The robustness of the local observers to exponentially decaying perturbations on the position estimates received through communication is analysed and the results obtained are used to ensure that the decentralized system as a whole will also have GES error dynamics. In [18], a similar solution is proposed, but for a continuous time range-based navigation system.

Simulation results are presented to show the behaviour and convergence of the proposed solution.

A. Notation

Throughout the paper, the symbol $\mathbf{0}$ denotes a matrix of 0s of proper dimensions and $\mathbf{I}_n$ denotes the $n \times n$

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identity matrix. A block diagonal matrix is represented by $\text{diag}(\mathbf{A}_1, ..., \mathbf{A}_n)$. The special orthogonal group is denoted by $SO(3) := \{\mathbf{X} \in \mathbb{R}^{3\times3} : \mathbf{X}^T\mathbf{X} = \mathbf{I}, \det(\mathbf{X}) = 1\}$, and the set of unit vectors is defined as $S(2) := \{\mathbf{x} \in \mathbb{R}^3 : \|\mathbf{x}\| = 1\}$. For $\mathbf{x} \in \mathbb{R}^3$, $\mathbf{x}^x$, $\mathbf{x}^y$ and $\mathbf{x}^z$ denote the first, second, and third component of $\mathbf{x}$, respectively. The transpose operator is defined as $(.)^T$.

II. PROBLEM STATEMENT

Consider a formation of N vehicles, indexed from 1 to N. All the vehicles are evolving in a fluid whose velocity is assumed to have a time-invariant spatial distribution. Moreover, it is assumed that the velocity of the vehicles is small enough such that the fluid's velocity can be considered constant for each vehicle. Since the vehicles may be operating in different regions of the space, it is assumed that the velocity of the fluid may differ from vehicle to vehicle. As so, $\mathbf{v}_{fi}(t) \in \mathbb{R}^3$ denotes the fluid velocity around vehicle i, expressed in a local inertial frame, $\{I\}$. The position of the vehicle i, expressed in $\{I\}$, is denoted by $\mathbf{p}_i(t) \in \mathbb{R}^3$.

Each vehicle is moving with a velocity relative to the fluid, measured by a relative velocity sensor, such as a Doppler velocity log (DVL), and denoted by $\mathbf{v}_i(t) \in \mathbb{R}^3$, expressed in the body frame, $\{B_i\}$. Each vehicle is also equipped with an attitude and heading reference system (AHRS), which provides a rotation matrix, $\mathbf{R}_i(t) \in SO(3)$, from $\{B_i\}$ to $\{I\}$.

The kinematics of vehicle i are given by

$$\begin{cases} \dot{\mathbf{p}}_i(t) = \mathbf{v}_{fi}(t) + \mathbf{R}_i(t)\mathbf{v}_i(t) \\ \dot{\mathbf{v}}_{fi}(t) = \mathbf{0} \end{cases}.$$

The formation is assumed to be organized in a tiered topology, and each vehicle has access to either:

- An absolute position measurement, provided by, for example, GPS or a long baseline system, if they are in the first tier; or
- Bearing measurements and position estimates of one or more vehicles in the tier above and, in some cases, depth measurements.

The focus of this paper is on the second case, since the position is available in the first one. In the second case, the outputs are available at discrete-time and are given by

$$\begin{cases} \mathbf{d}_{ij}(k) = \mathbf{R}_i^T(t_k)\dfrac{\mathbf{p}_j(t_k) - \mathbf{p}_i(t_k)}{\|\mathbf{p}_j(t_k) - \mathbf{p}_i(t_k)\|}, & j \in D_i \\ h_i(k) = \mathbf{p}_i^z(t_k), & \textit{if depth available}, \end{cases},$$

where D_i is the set of vehicles to which vehicle i has bearing measurements.

From now on, and unless specified otherwise, it is considered

$$\mathbf{d}_{ij}(k) = \frac{\mathbf{p}_j(t_k) - \mathbf{p}_i(t_k)}{\|\mathbf{p}_j(t_k) - \mathbf{p}_i(t_k)\|}, \quad j \in D_i, \tag{1}$$

since this simplifies the computations. This is done without loss of generality since the matrix $\mathbf{R}_i(t_k)$ is available and invertible. For simulation purposes, the original bearing measurement is used.

Because the communication and the bearing measurements between vehicles are only available at low frequency, the system must be discretized, which leads to

$$\begin{cases} \mathbf{p}_i(t_{k+1}) = \mathbf{p}_i(t_k) + T\mathbf{v}_{fi}(t_k) + \mathbf{u}_i(k) \\ \mathbf{v}_{fi}(t_{k+1}) = \mathbf{v}_{fi}(t_k) \\ \mathbf{d}_{ij}(k) = \dfrac{\mathbf{p}_j(t_k) - \mathbf{p}_i(t_k)}{\|\mathbf{p}_j(t_k) - \mathbf{p}_i(t_k)\|}, & j \in D_i \\ h_i(k) = \mathbf{p}_i^z(t_k), & \textit{if depth available} \end{cases}, \tag{2}$$

where T is the sampling period and $\mathbf{u}_i(k)$ is given by

$$\mathbf{u}_i(k) = \int_{t_k}^{t_{k+1}} \mathbf{R}_i(t)\mathbf{v}_i(t)dt. \tag{3}$$

The problem addressed in this paper is that of designing a decentralized observer, with globally exponentially stable error dynamics, for the position and local fluid velocity of each vehicle, $\mathbf{p}_i$ and $\mathbf{v}_{fi}$ respectively. The decentralized observer is composed by local observers, each one with access to the local measurements described before.

III. LOCAL OBSERVER DESIGN

Depending on the available measurements, the design of a local observer for (2) differs. Due to limitations on the available space, only two cases are discussed: i) when one bearing and depth are available; and ii) when two or more bearings are available. The first case is analysed in detail while, in the second, results obtained in [17] are used. From now on, the study will be focused on the design of the local observer for vehicle i. To simplify the notation, the index i will be omitted from this point onward, resulting in the system

$$\begin{cases} \mathbf{p}(t_{k+1}) = \mathbf{p}(t_k) + T\mathbf{v}_f(t_k) + \mathbf{u}(k) \\ \mathbf{v}_f(t_{k+1}) = \mathbf{v}_f(t_k) \\ \mathbf{d}_j(k) = \dfrac{\mathbf{p}_j(t_k) - \mathbf{p}(t_k)}{\|\mathbf{p}_j(t_k) - \mathbf{p}(t_k)\|}, & j \in D \\ h(k) = \mathbf{p}^z(t_k), & \textit{if depth available} \end{cases}. \tag{4}$$

A. Artificial output

The dynamic system (4) is nonlinear due to the bearing outputs. To address this issue, and obtain a linear time-varying (LTV) system, the bearing outputs will be replaced by artificial ones. First, note that

$$\mathbf{d}_j(k)\mathbf{d}_j^T(k)\mathbf{d}_j(k) = \mathbf{d}_j(k)$$

since $\mathbf{d}_j(k)$ is a unit vector, from which it is possible to write

$$(\mathbf{I} - \mathbf{d}_j(k)\mathbf{d}_j^T(k))\mathbf{d}_j(k) = \mathbf{0}. \tag{5}$$

Substituting the last term $\mathbf{d}_j(k)$ of (5) using the third equation of (4) leads to

$$(\mathbf{I} - \mathbf{d}_j(k)\mathbf{d}_j^T(k))(\mathbf{p}_j(t_k) - \mathbf{p}(t_k)) = \mathbf{0}.$$

From this, $\mathbf{z}_j(k) \in \mathbb{R}^3$ is defined as

$$\mathbf{z}_j(k) := (\mathbf{I} - \mathbf{d}_j(k)\mathbf{d}_j^T(k))\mathbf{p}_j(t_k) = (\mathbf{I} - \mathbf{d}_j(k)\mathbf{d}_j^T(k))\mathbf{p}(t_k).$$

This quantity is known since $(\mathbf{I}-\mathbf{d}_j(k)\mathbf{d}_j^T(k))\mathbf{p}_j(t_k)$ can be computed using known measurements. Also, because $\mathbf{d}_j(k)$ is a known measurement, $\mathbf{z}_j(k)$ is linear on the state $\mathbf{p}(k)$. Replacing $\mathbf{d}_j(k)$ by $\mathbf{z}_j(k)$ in (4) yields

$$\begin{cases}\mathbf{p}(t_{k+1}) = \mathbf{p}(t_k) + T\mathbf{v}_f(t_k) + \mathbf{u}(k)\\ \mathbf{v}_f(t_{k+1}) = \mathbf{v}_f(t_k)\\ \mathbf{z}_j(k) = (\mathbf{I} - \mathbf{d}_j(k)\mathbf{d}_j^T(k))\mathbf{p}(t_k),\ j \in D\\ h(k) = \mathbf{p}^z(t_k),\ \textit{if depth available}\end{cases}. \quad (6)$$

This is an LTV system and can be written in the form

$$\begin{cases}\mathbf{x}(k+1) = \boldsymbol{\mathcal{A}}_k\mathbf{x}(k) + \boldsymbol{\mathcal{B}}_k\mathbf{u}(k)\\ \mathbf{y}(k) = \boldsymbol{\mathcal{C}}_k\mathbf{x}(k)\end{cases},$$

where

$$\mathbf{x}(k) = [\mathbf{p}^T(t_k)\ \mathbf{v}^T(t_k)]^T \in \mathbb{R}^6,$$

and

$$\mathbf{y}(k) = [\mathbf{z}_1^T(k)\ ...\ \mathbf{z}_L^T(k)\ h(k)]^T \in \mathbb{R}^{3L+1},$$

or

$$\mathbf{y}(k) = [\mathbf{z}_1^T(k)\ ...\ \mathbf{z}_L^T(k)]^T \in \mathbb{R}^{3L},$$

with L representing the number of elements in D.

B. Observability

When studying the system with only one bearing available, the index j will be omitted in $\mathbf{d}_j(k)$. The state matrices when depth and only one bearing measurement are available are thus given by

$$\boldsymbol{\mathcal{A}}_k = \begin{bmatrix}\mathbf{I}_3 & T\mathbf{I}_3\\ \mathbf{0} & \mathbf{I}_3\end{bmatrix} \in \mathbb{R}^{6\times6}, \quad \boldsymbol{\mathcal{B}}_k = \begin{bmatrix}\mathbf{I}_3\\ \mathbf{0}\end{bmatrix} \in \mathbb{R}^{6\times3},$$

$$\boldsymbol{\mathcal{C}}_k = \begin{bmatrix}\mathbf{I}_3 - \mathbf{d}(k)\mathbf{d}^T(k) & \mathbf{0}\\ \mathbf{e}_3^T & \mathbf{0}\end{bmatrix} \in \mathbb{R}^{4\times6},$$

with $\mathbf{e}_3 := [0\ 0\ 1]^T$. The following theorem addresses the observability of this system.

Theorem 1: The system (6) with depth and only one bearing available is observable on the interval $[k_a, k_a+2]$ if and only if $\mathbf{d}^z(k_a) \neq 0$ and $\mathbf{d}^z(k_a+1) \neq 0$.

The proof of this theorem is not presented due to space limitations but is done by showing that the rank of the observability matrix is equal to the number of states.

When more than one bearing is available but there is no depth measurement, the state matrices are given by

$$\boldsymbol{\mathcal{A}}_k = \begin{bmatrix}\mathbf{I}_3 & T\mathbf{I}_3\\ \mathbf{0} & \mathbf{I}_3\end{bmatrix} \in \mathbb{R}^{6\times6}, \quad \boldsymbol{\mathcal{B}}_k = \begin{bmatrix}\mathbf{I}_3\\ \mathbf{0}\end{bmatrix} \in \mathbb{R}^{6\times3},$$

$$\boldsymbol{\mathcal{C}}_k = \begin{bmatrix}\mathbf{I}_3 - \mathbf{d}_1(k)\mathbf{d}_1^T(k) & \mathbf{0}\\ \vdots & \vdots\\ \mathbf{I}_3 - \mathbf{d}_L(k)\mathbf{d}_L^T(k) & \mathbf{0}\end{bmatrix} \in \mathbb{R}^{3L\times6},$$

where L is the number of vehicles in D. This system has been studied in [17], from where the following theorem can be used.

Theorem 2: The system (6) with more than one bearing and no depth measurement is observable on the interval $[k_a, k_a+2]$ if and only if there exists $m, n, l, p \in \{1, ..., L\}$ such that

$$\mathbf{d}_m(k_a) \neq \alpha_1\mathbf{d}_n(k_a)$$

and

$$\mathbf{d}_l(k_a+1) \neq \alpha_2\mathbf{d}_p(k_a+1)$$

for all $\alpha_1, \alpha_2 \in \mathbb{R}$.

Remark 1: The conditions of the first theorem are easy to achieve, considering the tiered topology of the formations. If the tiers are related to the vertical spacial distribution, being any tier deeper than the upper tier, then these conditions will be always met. For the second theorem, even if only two bearings are available, it is enough that the vehicle of the system and the two vehicles to which the bearings are measured are not aligned. Even though this might not be the case, both conditions are not hard to achieve. If the conditions are not met during a finite interval of time, the observers may diverge, but will converge again once they are met again.

With the observability studied, the design of a Kalman filter is the obvious choice since it is applied to a system that is linear in the state. This is due to the fact that $\mathbf{d}_j$ is known. The Kalman filter yields globally exponentially stable error dynamics if the system is shown to be uniformly completely observable [19]. Here, only observability was shown due to space limitations but the proof of uniform complete observability, while tedious, follows similar steps considering uniform bounds in time.

IV. Decentralized System

A. Filter Robustness

The conditions for stability of the local observers have already been established. However, these local observers have access to the position of the vehicles to which bearings are measured. When the observers are put together into a decentralized system, they will only have access to position estimates, which can be written as

$$\hat{\mathbf{p}}_j(t_k) = \mathbf{p}_j(t_k) + \mathbf{e}_j(k),$$

where $\hat{\mathbf{p}}_j(t_k)$ is an estimate of the position of vehicle j and $\mathbf{e}_j(k)$ is a term with GES dynamics representing the estimation error of $\mathbf{p}_j(t_k)$. This will alter the value of the artificial output, which will be given by

$$\mathbf{z}_j(k) = (\mathbf{I} - \mathbf{d}_j(k)\mathbf{d}_j^T(k))\mathbf{p}(t_k) + \bar{\mathbf{e}}_j(k), \quad (7)$$

where $\bar{\mathbf{e}}_j(k)$ is defined as

$$\bar{\mathbf{e}}_j(k) = (\mathbf{I} - \mathbf{d}_j(k)\mathbf{d}_j^T(k))\mathbf{e}_j(k).$$

Since $\mathbf{e}_j(k)$ decays exponentially and $(\mathbf{I} - \mathbf{d}_j(k)\mathbf{d}_j^T(k))$ is bounded, $\bar{\mathbf{e}}_j(k)$ will also decay exponentially.

As so, the effect of not having the true position of the other vehicles can be regarded as an exponentially decaying perturbation on the outputs of system (6). This will not alter the dynamics of the Kalman filter covariance matrix

$$\begin{cases}\mathbf{P}_{k|k-1} = \boldsymbol{\mathcal{A}}_k\mathbf{P}_{k-1|k-1}\boldsymbol{\mathcal{A}}_k^T + \mathbf{Q}\\ \mathbf{K}_k = \mathbf{P}_{k|k-1}\boldsymbol{\mathcal{C}}_k^T(\mathbf{R} + \boldsymbol{\mathcal{C}}_k\mathbf{P}_{k|k-1}\boldsymbol{\mathcal{C}}_k^T)^{-1}\\ \mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\boldsymbol{\mathcal{C}}_k)\mathbf{P}_{k|k-1}(\mathbf{I} - \mathbf{K}_k\boldsymbol{\mathcal{C}}_k)^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T\end{cases},$$

where $\mathbf{Q}$ and $\mathbf{R}$ are the process and output noise covariance matrices, respectively. $\mathbf{P}$ is the estimation error covariance and $\mathbf{K}$ is the observer gain. Since these equations are not affected by the perturbation, they will remain bounded. The estimates will be given by

$$\hat{\mathbf{x}}(k+1) = \boldsymbol{\mathcal{A}}_k\hat{\mathbf{x}}(k) + \boldsymbol{\mathcal{B}}_k\mathbf{u}(k) + \mathbf{K}_k(\mathbf{z}(k) - \boldsymbol{\mathcal{C}}_k\hat{\mathbf{x}}(k)).$$

Considering (7), the exponentially decaying perturbation will be multiplied by a bounded matrix, $\mathbf{K}$, which will cause an exponentially decaying error on the estimate of the state $\hat{\mathbf{x}}$.

B. Chain Propagation

All the local observers of the vehicles of the second tier receive true information of the position of the vehicles of the tier above, as it is assumed that the first tier has access to its own position. Therefore, they will produce estimates of their own position with GES error dynamics. As shown before, all the vehicles receiving position estimates with GES error dynamics will also produce positions estimates with GES error dynamics of their own. As so, the observers of all the tiers will converge, since the errors that are propagated will always have GES error dynamics.

V. Simulation Results

Simulations were performed to study the behaviour and convergence of the proposed solution. In this section, the results are presented.

A. Setup

To perform the simulations, the formation depicted in Fig. 1 was used. The vehicles in Tier 1 measure depth and bearing to one of the vehicles in Tier 0. The vehicle in Tier 2 measures bearing to all four vehicles of Tier 1 and has no access to depth measurements.

All the vehicles performed the same type of trajectory but with different starting points. The trajectory was generated with way points, which are described in Table I. The acceleration was limited to 0.01 m/s^2, which resulted in the curve presented in Fig. 2. The fluid velocities were chosen with different values for each vehicle. The starting points and the fluid velocities can be seen in Table II. The fluid velocity for the first two vehicles was not specified since the observers for the upper tier were not simulated. This is done without loss of generality since the observers of Tier 0 do not depend the rest of the formation.

A sampling period of 1s is assumed for both the bearing measurements and the communications between the vehicles, while all the other measurements are assumed to be available at 100Hz. Azimuth and inclination are measured, from which the bearing is obtained as $\mathbf{d} = [\sin(\theta)\cos(\phi)\ \sin(\theta)\sin(\phi)\ \cos(\theta)]^T$, where ϕ and θ are, respectively, the azimuth and inclination angles to another vehicle. Zero-mean white Gaussian noise with a standard deviation of 1° was added to both angles. For the vehicles in Tier 0, the position is available but zero-mean white Gaussian noise was added with a standard deviation of 0.1m in each component. Some correlation was added, resulting in the covariance matrix

$$0.01 \times \begin{bmatrix} 1 & 0.1 & 0.1 \\ 0.1 & 1 & 0.1 \\ 0.1 & 0.1 & 1 \end{bmatrix}.$$

Zero-mean white Gaussian noise with a standard deviation of 0.1m was added to the depth measurements. For the Euler angles used to obtain the rotation matrix, uncorrelated zero-mean white Gaussian noise was added with a standard deviation of 0.01° for the pitch and roll angles and 0.03° for the yaw angle. Finally, the relative velocity to the fluid was

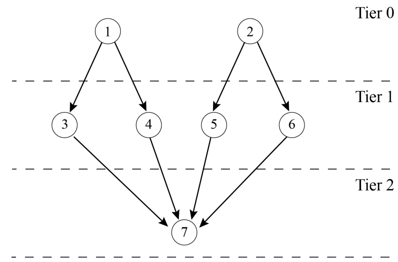

Fig. 1. Formation graph

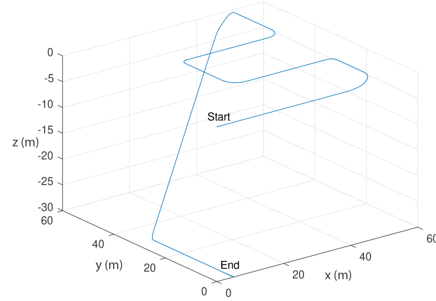

Fig. 2. Trajectory for vehicle 1

Time (s)	Position (m)
0	[0 0 0]
100	[50 0 0]
200	[50 20 0]
300	[20 20 0]
400	[20 40 0]
500	[50 40 0]
600	[50 60 0]
800	[5 30 -30]
1000	[5 0 -30]

TABLE I
Trajectory waypoints for vehicle 1

Vehicle	Initial Position (m)	Fluid Velocity (m/s)
1	[0 0 0]	—
2	[100 100 0]	—
3	[1 1 -50]	[0.19 0.13 0.30]
4	[0 10 -60]	[0.20 0.10 0.30]
5	[110 100 -40]	[0.18 0.11 0.28]
6	[90 90 -50]	[0.21 0.10 0.27]
7	[50 50 -100]	[0.21 0.12 0.27]

TABLE II
Initial positions and fluid velocities used in the simulations.

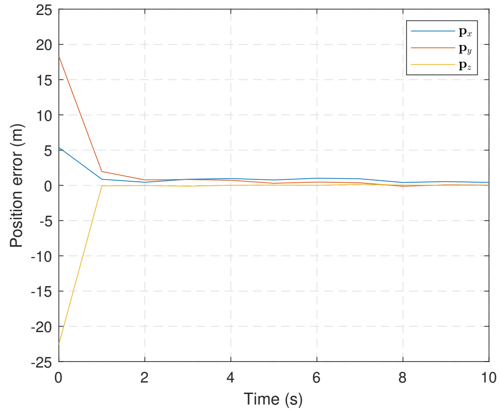

Fig. 3. Vehicle 3: Position estimate transient error.

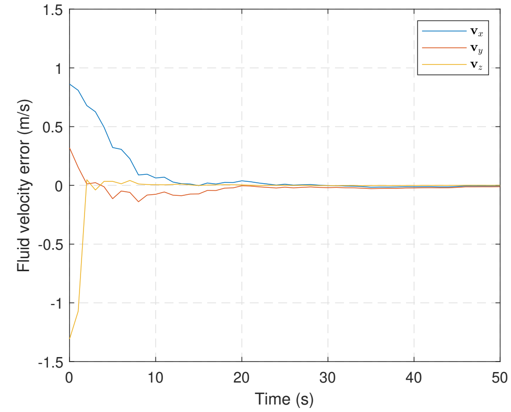

Fig. 5. Vehicle 3: Fluid velocity estimate transient error.

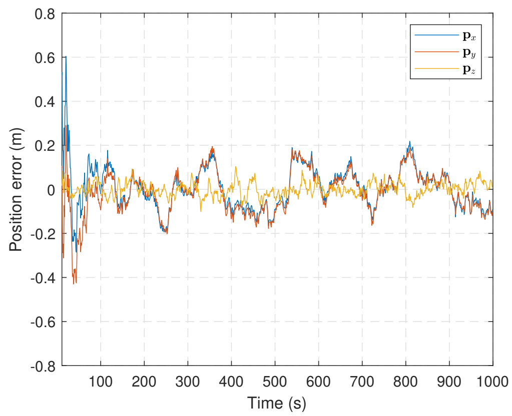

Fig. 4. Vehicle 3: Position estimate steady error.

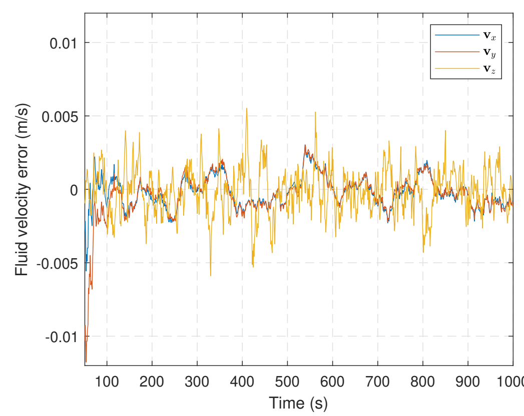

Fig. 6. Vehicle 3: Fluid velocity estimate steady error.

corrupted by uncorrelated zero-mean white Gaussian noise with standard deviation of 0.01 m/s. The integral in (3) was computed using the trapezoidal rule.

B. Results

The developed solution consists of a Kalman filter for (6) for each vehicle. The state covariance matrices were set to $\mathrm{diag}(0.01^2\mathbf{I}; 0.001^2\mathbf{I})$, while the output covariance matrices were set to $\mathrm{diag}(0.1^2; 10\mathbf{I})$ or $10\mathbf{I}$, depending on the availability of depth measurements. The initial state estimate was generated by a Gaussian distribution centred on the true state and with covariance $\mathrm{diag}(10^2\mathbf{I}; \mathbf{I})$.

The estimates converged for all vehicles, but due to space limitations only the results for vehicles 3 and 7 are presented. The results for vehicle 3 can be seen in Fig. 3 and Fig. 4 for the position estimate and in Fig. 5 and Fig. 6 for the velocity fluid estimate. The estimates converge rapidly to the true state, with the position converging faster. There is no visible bias and the error on the z component is smaller, as expected, since a depth measurement is available.

The results for vehicle 7 can be seen in Fig. 7 and Fig. 8 for the position estimate and in Fig. 9 and Fig. 10 for the fluid velocity estimate. The same conclusions can be drawn.

VI. Conclusions

The communication bandwidth is very limited in underwater scenarios, rendering centralized navigation solutions impossible to implement. This paper presents a cooperative, decentralized navigation solution for formations of underwater vehicles based on bearing measurements. Two cases of interest are analyzed: i) in the first, a vehicle has access to its depth and bearing to another vehicle of the formation; and ii) in the second, the vehicle has access to bearings to at least two other vehicles of the formation. In order to cope with the nonlinear nature of the outputs, artificial outputs are employed that render the dynamics linear, thus allowing for the design of local Kalman filters with GES errors dynamics. Then, the error dynamics of the formation as a whole are also shown to be GES. Finally, simulation results are presented to show the behaviour of the solution.

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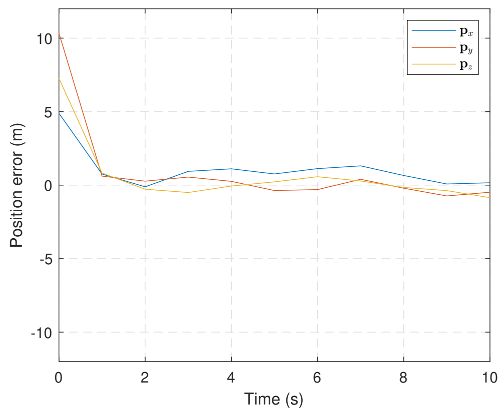

Fig. 7. Vehicle 7: Position estimate transient error.

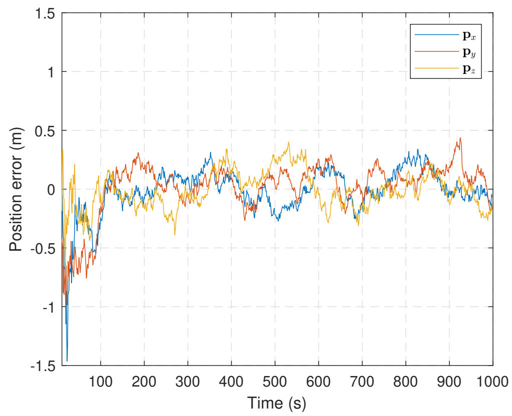

Fig. 8. Vehicle 7: Position estimate steady error.

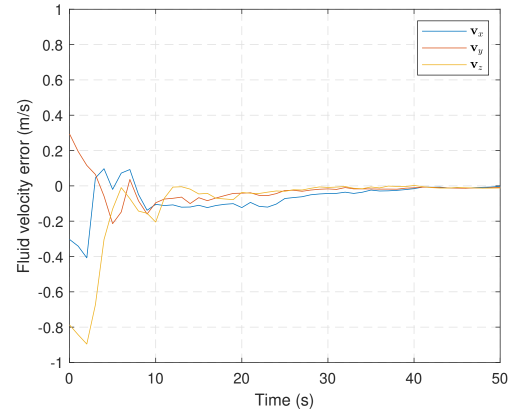

Fig. 9. Vehicle 7: Fluid velocity estimate transient error.

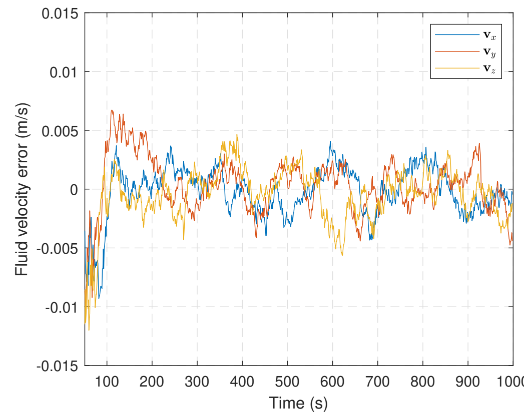

Fig. 10. Vehicle 7: Fluid velocity estimate steady error.

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