Biologically Inspired Stochastic Hybrid Control of Multi-Robot Systems

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Abstract

This paper describes a biologically inspired approach to the modeling and control of multi-agent populations composed by a large number of agents. Individual agents are modeled based on a deterministic Hybrid Automata endowed with input events and continuous-valued outputs. The complexity of population modeling is avoided by assuming a stochastic approach, under which the agents distribution over the state space is modeled. This is based on a Stochastic Hybrid Automaton, which results from inputting a stochastic event sequence to the individual model. The dynamics of the state probability density functions is determined and the results applied to the mission control of a simulated robotic population.

1. Introduction

Multi-agent systems (MAS), concerning both virtual [8] or real (robotic) [9] agent populations, are currently a subject of major interest in the literature. One of the most relevant topics in MAS is the modeling and control of large-size agent populations. Under the current state-of-the-art, it seems that results for small-sized populations do not scale necessarily well for large-scale ones. Therefore, progresses towards the mathematical modeling of large-size agent populations are welcome. Such models can be used to predict the evolution of the population and subsequently design controllers or supervisors capable of changing the population behavior by the suitable adjustment of appropriate parameters.

One approach with large potential for this purpose, followed in this paper, is based on recent results on the mathematical modeling of biological systems [6, 7]. In fact, our work has been originally developed for biological experiments modeling. However, we have found that such an approach also provides results of potential interest for the MAS community. The paper starts by motivating a biological approach to the modeling and control of large size multi-agent populations in Section 2. Individual agents are modeled based on a deterministic Hybrid Automata in Section 3. The complexity of population modeling is avoided by using a Stochastic Hybrid Automata model for the population in Section 4. The dynamics of the state probability density functions is determined (Section 4) and the results applied to the mission control of a simulated robotic population in Section 5.

2. Motivation

The problem that motivated this work is related to the modeling of a population of T-Cells[1][2]. Figure 3 presents a population of T-Cells surrounded by Antigen Presenting Cells (APC). The interaction between T-Cells and APCs is one of the most important reactions of the immune system. This reaction is called T-Cell Receptor (TCR) triggering and leads to the production of effector cells, which kill antigens. The interaction between T-Cells and APC produces changes in the amount of

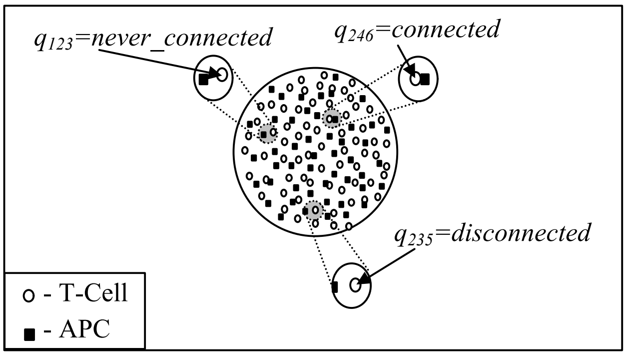

Fig. 1. The T-Cells population surrounded by APCs. q_i is the discrete state of the ith T-Cell.

TCR (TCR dynamics) in a T-cell. Before interaction starts, the T-Cell should be connected to an APC. However, simultaneously some of the T-Cells will disconnect from APCs, and others will connect again (Fig. 1).

The T-Cell population is definitely a complex system. To follow the complete dynamics of the population, the TCR dynamics and the motion dynamics, which leads to the connection or disconnection of each T-Cell to APCs, should be followed. If we assume a 3D model of motion we need 6 state variables per T-Cell just to describe the position and velocity of a T-Cell. We need also at least one state for TCR dynamics and at least one discrete state variable that contains information on whether the T-Cell is connected or disconnected. In total this means, at least, 8 variables per T-Cell. A population of 1000 T-Cells has a state vector of dimension 8000. Although the simulation of the population would not be impossible with current computational power, the dynamics of the average value and the variance of the TCRs in the population are typically sought by biologists. These moments are particularly important when the population observed data is to be matched to the individual TCR dynamics. These facts motivated a more general approach to the modeling of a multi-agent population, which is described in the sequel.

2. Micro Agent Individual Model

The aim of this section is to introduce the population building block, designated as Micro Agent. The prefix "Micro" is used because this building block describes the population behavior at the micro level i.e. at the level of the individual behavior. A Micro Agent is a single-input multi-output hybrid system. The input to a Micro Agent is a continuous time discrete event sequence. The output of a Micro Agent is a continuous time real vector. The output of a Micro Agent is a function of the hybrid system state. This hybrid system state is a function of the discrete event time sequence at the system input.

Definition 1 [3]. *A hybrid automata H is a collection $H = (Q, X, Init, f, Inv, E, G, R)$ where:*

- *Q is a finite set of discrete states* (1)
- *X is $\mathbf{R}^n$ is the continuous state space* (2)
- *$Init \subseteq Q \times X$ is the set of initial states* (3)
- *$f: X \times Q \to TX$ assigns to each $q \in Q$ a vector field $f(x,q)$* (4)
- *$Inv: Q \to 2^X$ assigns to each $q \in Q$ an invariant set* (5)
- *$E \subseteq Q \times Q$ is a collection of discrete transitions* (6)
- *$G: E \to 2^X$ assigns to $e \in E$ a guard set, representing the collection of the discrete transitions allowed by the state space vector* (7)
- *$R: X \times E \to 2^X$ assigns to $e \in E$ and $x \in X$ a reset map, describing jumps in the continuous state space due to event e.* (8)

Definition 2. *A Micro Agent μA is a single-input multi-output hybrid automaton. It is a collection $\mu A = (H, U, \tau, Y)$ where :*

- *H is a Hybrid automata $H = (Q, X, Init, f, Inv, E, G, R)$ satisfying properties:* (9)
  - *$Inv: X, \ \forall Q$* (10)
  - *$R(e,x) = x, \ \forall (e \in E \wedge x \in X)$* (11)
- *U is a finite set of input discrete events including the nil event ε* (12)
- *$\tau: U \times Q \to E$, assigns to the discrete event $u \in U$ and discrete state $q \in Q$ the transition $e = (q, q') \in E$, where $\tau(\varepsilon, q) = (q, q)$* (13)
- *X is R^n, the state space of the continuous piece of H* (14)
- *Y is R^m, is the output state, a μA output $y \in Y$ is a function of the continuous state x, $y = g(x)$* (15)

Remark1. *The Micro Agent state, called micro state, is a pair $(x,q) \in X \times Q$. This couple consists of continuous $x \in X$ and discrete state $q \in Q$ parts.*

Properties (10) and (11) in Definition 2 mean that, for hybrid system H, discrete and continuous dynamics could evolve in a free manner. However, jumps in the continuous state space part are not allowed. It should also be underlined that a Micro Agent is a *deterministic* system.

3. Stochastic Micro Agent

The Micro Agent model is deterministic since it is based on a deterministic Hybrid System. Here, a Stochastic Micro Agent model will be introduced.

Definition 3 [5] *(Micro Agent Stochastic Execution) A stochastic process $(x(t), q(t)) \in X \times Q$ is called a Micro Agent Stochastic Execution iff a Micro Agent stochastic input event sequence $e(t_n), n \in N$, $t_0 = 0 \leq t_1 \leq t_2 \leq \ldots$ generates transitions such that in each interval $[t_n, t_{n+1})$, $n \in N$, $q(t) \equiv q(t_n)$.* ***Remark 1***. *The $x(t)$ of a Stochastic Execution is a continuous time function since the transition changes only the discrete state of a Micro Agent.*

Definition 4. *(Micro Agent Continuous Time Markov Process Execution) A Micro Agent Stochastic Execution (x(t),q(t)) Î X ´ Q is called a Micro Agent Continuous Time Markov Process Execution iff the input stochastic event sequence* $e(t_n), n$ Î N, $t_0 = 0$ £ t_1 £ t_2 £... *generates transitions whose conditional probability satisfies:*

$$P[q(t_{k+1})=q_{k+1}|\ q(t_k)=q_k,\ q(t_{k-1})=q_{k-1},\ldots,\ q(t_0)=q_0]=P[q(t_{k+1})=q_{k+1}|\ q(t_k)=q_k] \qquad (16)$$

Remark 1. *The q(t) of a Micro Agents Continuous Markov Process Execution is a Continuous Time Markov chain.*

Definition 5. *(Stochastic Micro Agent, SmA) A Stochastic Micro Agent is a pair SmA=(mA,e(t)) where mA is a Micro Agent and e(t) is a Micro Agent stochastic input event sequence such that the stochastic process (x(t),q(t)) Î X ´ Q is a Micro Agent Stochastic Execution.*

Definition 6. *(Continuous Time Markov Process Micro Agent, CTMPmA) A Stochastic Micro Agent is called a Continuous Time Markov Process Micro Agent iff (x(t),q(t)) Î X ´ Q is a Micro Agent Continuous Time Markov Process Execution.*

Previous definitions aimed at making clear that a Stochastic Micro Agent is a Stochastic Hybrid Automaton based on a Micro Agent, which is a deterministic system. In the sequel, a Stochastic Micro Agent will be used as a model of Micro Agents populations.

4. Mathematical Analysis

The connection between the individual micro dynamics and the population macro dynamics is strongly related to the area of statistical physics [10] where behavior and properties of mechanical bodies made up of a very large number of separate particles are studied. In this framework the connection between the micro and macro dynamics is established through the pdf of system particles over a state space.

Concerning the Micro Agent population we assume that:

-The interaction between individuals is modeled as a Micro Agent (17)

-The complexity of the interactions among individuals in the population produces the Micro Agent Stochastic Execution of a Micro Agent in the population. (18)

The previous assumptions bring us directly to similar problems in statistical physics and the following conclusion: *The individual Micro Agent dynamics and the dynamics of the Micro Agents population measurements are connected through the probability density function of a Stochastic Micro Agent state which represents the population state.*

Different kinds of Stochastic Micro Agents could be considered. In sequel we will interested in CTMPμAs. The following theorem concerns the probability density function of a CTMCμA over the state space.

Theorem 1. *For a* CTMPμA *with N discrete states and discrete state probability satisfying*

$$\dot{\mathbf{P}}(t) = L^T\,\mathbf{P}(t) \qquad (19)$$

where $\mathbf{P}(t)=[P_1(t)\ P_2(t)\ldots P_N]^T$, P_i *is the probability of discrete state i,* $L=[l_{ij}]^T_{N´N}$*, is transition rate matrix and* l_{ij} *is a transition rate from discrete state i to discrete state j, the vector of state probability density functions* $r(x,\ t)=[r((x,1),\ t),\ r((x,2),\ t),\ldots,\ r((x,N),\ t)]^T$ *where* $r((x,i),\ t)$ *is the pdf of state (x,i) at time t, satisfies the following equation:*

$$\frac{\partial\ r(x,t)}{\partial t} = L^T\ r(x,t) - \begin{bmatrix} \nabla\cdot\left(f(x,1)\,r((x,1),t)\right) \\ \nabla\cdot\left(f(x,2)\,r((x,2),t)\right) \\ \vdots \\ \nabla\cdot\left(f(x,N)\,r((x,N),t)\right) \end{bmatrix} \qquad (20)$$

where f(x,i) is the vector field value at state (x,i).

Proof. The state space *X´Q* of the Stochastic Micro Agent is presented in Fig. 2. Transition between the discrete states is a Continuous Time Markov Chain stochastic process and *x(t)* is a continuous time function i.e $x(t^-)=x(t^+)=x(t)$. The probability $p_{V,i}$ that the Micro Agent state *(x,q)∈{(x,q) | x Î V, q=i}* is given by

$$p_{V,i}(t) = \int_V r\left((x,i),t\right)dV \qquad (21)$$

where *r(x,i)* is the probability density function of the state *(x,i)* and arbitrary chosen volume *V* in *X*. The time derivative of $p_{V,i}$ is:

$$\dot{p}_{V,i}(t) = \int_V \frac{\partial r\left((x,i),t\right)}{\partial t}dV \qquad (22)$$

Using Fig. 2 the time derivative of $p_{V,i}$ could be written as:

$$\dot{p}_{V,i}(t) = \lim_{Dt\to 0}\frac{1}{Dt}\left[Dp_{V_I} + \sum\nolimits_{S,DS\to 0} Dp_{DSDx}\right] \qquad (23)$$

where Dp_V and Dp_{DSDx} are probability changes in the volumes V_I and $DV_B=DSDx$, respectively, and $V_B=\Sigma_{S,DS®0}\ DSDx$. Due to the continuity of $x(t)$

$$\lim_{Dt\to 0}\frac{1}{Dt}Dp_{V_I} = \sum\nolimits_{k=1}^{N} l_{ki}\int_{V_I} r(x,i)dV \qquad (24)$$

since, in the time interval *[t,t+Dt), x*(*t*) does not leave

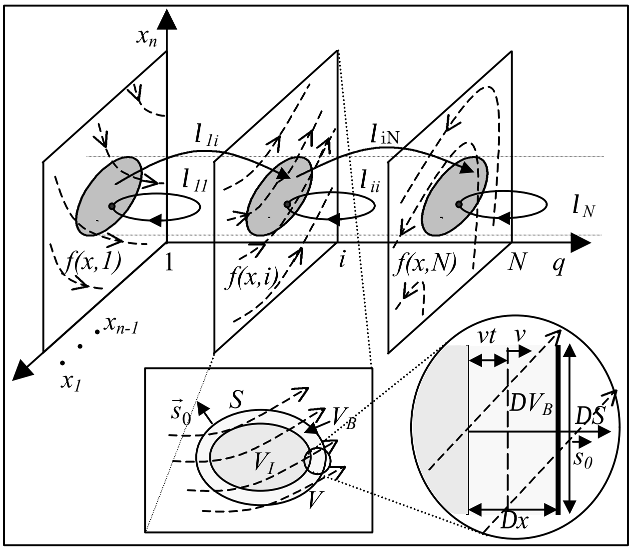

Fig. 2. x_i-state of continuous space, q-state of discrete space, *f(x,i)*-vector field for *q=i*, *V*-trajectory volume, V_I - volume of trajectories not crossing the surface *S* in the time interval *[t,t+Dt)*, V_B - volume of trajectories crossing surface *S* in the time interval *[t,t+Dt)*, DV_B - element of the volume V_B, *DS* - element of the surface *S*, *v* - projection of the vector field *f(x,i)* onto the surface vector $\vec{s}_0$, *Dx*-length *v*Δt.

volume V_I and probability in V_I changes due to the Markov Chain transitions. During the same time interval, the increase of probability in the volume DV_B=*DSDx* is

$$Dp_{DSDx}(t) = -DS\left[\int_t^{t+Dt} vr\big((x,i),t\big) + \dot{r}\big((x,i),t\big)\big(Dx - vt\big)dt\right] \quad (25)$$

where $x \in DV_B$. Taking into account the Markov Chain transitions in the volume DV_B and equation (24) we have

$$\lim_{Dt\to 0}\frac{Dp_{DSDx}(t)}{Dt} = -DSvr\big((x,i),t\big) + Dx\sum_{k=1}^{N} l_{ki}\, r\big((x,k),t\big) \quad (26)$$

Replacing (24) and (26) in (23) gives

$$\dot{p}_{V,i}(t) = \sum_{k=1}^{N} l_{ki}\int_{V_I} r(x,i)dV + \sum_{S,DS\to 0}\left[-DSvr\big((x,i),t\big) + DSDx\sum_{k=1}^{N} l_{ki}\, r\big((x,k),t\big)\right] \quad (27)$$

i.e.

$$\dot{p}_{V,i}(t) = \sum_{k=1}^{N} l_{ki}\int_{V} r(x,i)dV - \oint_{S} f(x,i)\, r\big((x,i),t\big)dS \quad (28)$$

With the use of Gauss' theorem

$$\dot{p}_{V,i}(t) = \int_{V}\left[\sum_{k=1}^{N} l_{ki}\, r(x,i) - \nabla\cdot\big(f(x,i)\, r\big((x,i),t\big)\big)\right]dV \quad (29)$$

Taking the small volume limit of the equations (22) and (29) we have

$$\frac{\partial r\big((x,i),t\big)}{\partial t} = \sum_{k=1}^{N} l_{ki}\, r(x,i) - \nabla\cdot\big(f(x,i)\, r\big((x,i),t\big)\big) \quad (30)$$

Using r(x, *t*)=[$r((x,1), t)$, $r((x,2), t)$,…, $r((x,N), t)$]^T the equation system (30) becomes

$$\frac{\partial\, \mathrm{r}(x,t)}{\partial t} = L^T\, \mathrm{r}\,(x,t) - \begin{bmatrix} \nabla\cdot\big(f(x,1)\, r((x,1),t)\big) \\ \nabla\cdot\big(f(x,2)\, r((x,2),t)\big) \\ \vdots \\ \nabla\cdot\big(f(x,N)\, r((x,N),t)\big) \end{bmatrix} \quad (31)$$

Q.E.D
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The partial differential equation (20) has the form of the Convection-Diffusion equation[4]. This type of equation is used for description of incompressible fluids. The solution of this equation is the pdf of *CTMPmA* state as a function of time. A numerical method for solving this equation is discussed in [4].

5. Robotic Population Mission Control

In this section we will introduce a potential application of the theory developed in this paper to the mission control of a robotic population using stochastic signals. The mission scenario could be suitable for planetary system exploration or search and rescue missions.

This scenario assumes a robotic population of small robots, initially concentrated on a location over the unexplored terrain (e.g., the mission command station)

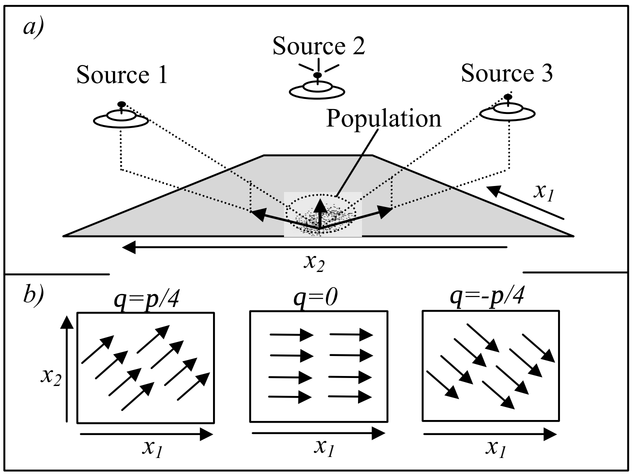

Fig. 3. *a*) A robotic population controlled by three aerial robots (sources); *b*) The vector fields created by control signal sources.

and controlled by the stochastic signals produced by aerial robots (Fig. 3*a*).

In this scenario, each robot in the population moves in the direction of the active signal source. Under the assumptions that aerial robots are far away from the population and that the robot velocity is constant *(v=1)*, the robotic motion model is

$$\begin{aligned}\dot{x}_1(t) &= \cos q(t) \\ \dot{x}_2(t) &= \sin q(t)\end{aligned} \qquad (32)$$

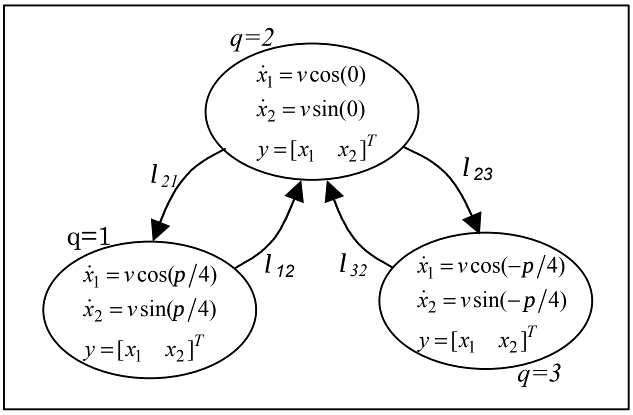

Fig. 4. The CTMPµA model of the population. Discrete states q=1,2,3 and motion between these states corresponds to the activation of signal sources 1,2,3, respectively, l_{ij} – transition rate from state *i* to state *j*.

The angles of the vector fields $f(x, i)=[v\cos q \quad v\sin q]^T$, i=1,2,3 in this example, are $q \in \{-\pi/4, 0, \pi/4\}$. A stochastic sequence of angles q acts as an input control signals for the population. If the stochastic sequence of angles (events) satisfies Equation (19), the population could be modeled as CTMPµA, depicted in Fig. 4 (in the example, no direct transitions between 1 and 3 exist). Using the transition rate matrix

$$L^T = \begin{bmatrix} -l_{12} & l_{21} & 0 \\ l_{12} & -l_{21} - l_{23} & l_{32} \\ 0 & l_{23} & -l_{32} \end{bmatrix} \qquad (33)$$

the evolution of the *CTMPmA* pdf is given by Equation (20). The solution contour plots of (20) for two different transition rates are presented in Figs.5 and 6. In both cases the initial pdf of the continuous state for discrete state $q=2$ is Gaussian with diagonal covariance matrix, and zero mean. For states $q = 1$ and 3 the pdf is zero. The figures present the pdf value of each *CTMPTmA* state, $r((x,q),t)$, and the pdf of the robot position $h(x,t)$ calculated as:

$$h(x,t)=r((x,1),t)+r((x,2),t)+r((x,3),t) \qquad (34)$$

we should notice that since $y=x$ the equation (34) is the pdf of the *CTMPTmA* output.

In case I most of the *mA* are in the state q=3 and overall population dominantly moves in the direction of the vector field $f(x,3)$. The vector field $f(x,2)$ has some initial influence on the population at the beginning of the observed time interval, but then this influence decreases. Since the contour shape of $r((x,1),t)$ has little influence on the shape of the contour plot of $h(x,t)$ we can conclude that influence of $f(x,1)$ is small. Both of these conclusions are reasonable and also come from the analysis of the transition rates values and initial probabilities of *CTMPTmA* discrete states $P_2(0)=1$ and $P_1(0)= P_3(0)=0$.

In case II the distribution of the CTMC*mA* states over the discrete space Q is more uniform. This explains $h(x,t)$ more symmetric shape than in case I. However, it could be seen from the density of the contours that the probability of the *CTMCmA* staying in state $q=1$ is slightly bigger than the others. Analysis of the transition rate values shows that it is due to the probability $P_1(t)$ being slightly larger than $P_2(t)$ and $P_3(t)$

From the previous results we may conclude that, in these examples, the dominant direction of the population motion is directed by the vector field $f(x,q)$ of the most

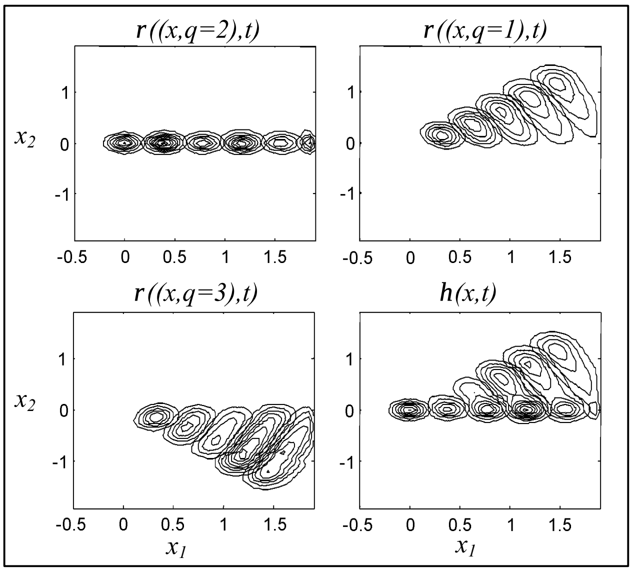

Fig. 5. The pdf of the robots population states $r((x,q),t)$, the pdf of the robots position $h(x,t)$. Case I: λ_{12}=0.5, λ_{21}=0.1, λ_{23}=0.9, λ_{32}=0.1, plots shown at time instants t= 0, 0.39, 0.79,1.18,1.57, 1.96 (from left to the right in picture)

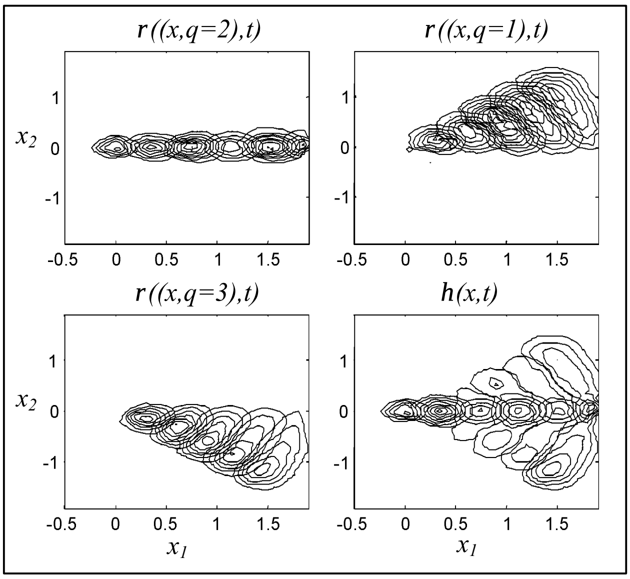

Fig. 6. The pdf of the robotic population states *r((x,q),t)*, the pdf of the robots position *h(x,t)*. Case II: λ_{12}=0.1, λ_{21}=0.5, λ_{23}=0.5, λ_{32}=0.4, plots shown at time instants at time instants *t*= 0, 0.39, 0.79,1.18,1.57, 1.96 (from left to the right in picture)

probably discrete state *q*. However, the population spreading shape depends in general on the initial pdf and transition matrix *L.* The latter should be seen as a design parameter for the population mission control.

6. Conclusion

In this paper a biologically inspired approach to the study of a robotic population is discussed. The robotic individuals were described by a deterministic Micro Agent model, which is defined within the Hybrid Automata framework. Under a stochastic assumption about the Micro Agent input event sequence the Stochastic Micro Agent model of robotic population is introduced. The relation between the deterministic model Continuous Markov Chain stochastic event sequence and the pdf of the Stochastic Micro Agent state is derived. Using this analytical relation an example of using the stochastic control signals to control a robotic population was presented.

Potential future work along this research line includes the design of the multivariable feedback mission controller of the population spreading shape. This also includes problem of trajectory control of the population center of the mass or maximum likelihood position. An interesting research topic will be the design of a mission controller for the population splitting and independent control of the resulting subpopulations. Direct applications to biological experiments are also currently underway.

Acknowledgment

This work was supported by grant SHFR/2960/2000 from the Portuguese *Fundação para a Ciência e a Tecnologia*.

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