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Experimental validation of a nonlinear quadrotor controller with wind disturbance rejection

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Abstract— **This paper addresses the problem of designing and experimentally validating a controller for steering a quadrotor vehicle along a trajectory, while rejecting wind disturbances. The proposed solution consists of a nonlinear adaptive state feedback controller for thrust and torque actuation that asymptotically stabilizes the closed-loop system in the presence of constant force disturbances, used to model the wind action, and ensures that the actuation does not grow unbounded as a function of the position errors. A prototyping and testing architecture, developed to streamline the implementation and the tuning of the controller, is also described. Experimental results are presented to demonstrate the performance and robustness of the proposed controller.**

I. Introduction

Flight control of Unmanned Aerial Vehicles (UAV) is an active and challenging topic of research, with crucial importance to numerous civilian and military applications. Among UAVs, we highlight the quadrotor as an ideal platform for robotic systems, particularly suited for the development and test of new control strategies due to its simplicity, high maneuverability, and ability to hover.

Several linear and nonlinear approaches to the problem of quadrotor flight control have been proposed, namely PI and PID control [1], PID and LQ [2], backstepping [3], [4] and sliding mode control [5]. Linear methods have been applied to UAVs with success but are of limited applicability for extended flight envelope regions, i.e. aggressive maneuvers, where the linearity of the system breaks. Additionally, one can only guarantee stability of the closed loop system for small regions around the equilibrium point. The sliding mode approach relies on a feedback linearizion controller that is not globally stabilizing. There is a singularity in the control law for zero thrust force and the proposed controller does not actively avoid it.

Backstepping is a well known technique extensively used for control of nonlinear systems. For example, it has been applied to helicopter trajectory tracking [6] and [7], to control of a two tilt rotor aircraft [8] and also to quadrotor trajectory tracking [9] and tracking of parallel linear visual features [10]. Several methodologies can be combined with backstepping to attain desirable characteristics of a control law, such as robustness to external disturbances and actuation boundedness. The use of integral control to achieve zero steady-state error or equivalently rejection of constant disturbances in a closed-loop regulation system is standard in the control literature and can be combined with the backstepping technique as discussed in [11]. The control methodology known as adaptive backstepping [12] relies on an estimator to achieve the disturbance rejection effect of integral control. Similarly, other linear estimators have been proposed in connection with other nonlinear controllers for quadrotor vehicles, as in [13].

One problem with a straightforward application of linear estimators, as is the case of adaptive backstepping, is that the parameter estimate can grow, without an *a priori* bound, depending on the initial conditions of the system. The typical approach to this problem is to use a projection operator to constrain the parameter estimate to a given set [12]. The discontinuity of this projection method is a twofold problem. First, it leads to practical problems when applied to continuous systems. Second, the recursive application of the backstepping procedure is no longer possible, as Lipschitz continuity is violated and the usual theorems on the existence and uniqueness of differentiable equations can no longer be applied. To overcome both these problems, we employ the arbitrarily smooth projection operator proposed in [14], which generates parameter estimates with sufficient smoothness to complete the backstepping procedure.

In this work, we address the problem of trajectory tracking for quadrotors, using a backstepping procedure that builds on the dynamic augmentation principle presented in [15]. The desired trajectory is specified by a sufficiently smooth time-parameterized position vector. The desired attitude of the vehicle is not prescribed since attitude convergence (up to a rotation about the body z axis) is naturally accomplished by solving the position tracking problem. Robustness to external constant disturbances is accomplished through adaptive backstepping. These disturbances can be used to represent both exogenous inputs such as constant wind and model uncertainties such as quadrotor parameter mismatches.

This paper is structured as follows. Section II introduces the quadrotor model. The problem and control objectives are stated in Section III. Controller design is described in Section IV, including the necessary steps to ensure disturbance rejection. Experimental results illustrating the performance of the proposed control law are presented in Section V and Section VI summarizes the contents of the paper.

II. Quadrotor Model

The quadrotor vehicle, depicted in Figure 1(a), is modeled as a rigid body able to generate a thrust force along the body z axis and is controlled in attitude through angular

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velocity inputs. A sketch of the quadrotor setup is presented in Figure 1(b), together with illustrations of reference frames, the force generated by each motor F_i and the direction of rotation for each propeller. Consider a fixed inertial frame $\{I\}$ and a frame $\{B\}$ attached to the vehicle's center of mass. The configuration of the body frame $\{B\}$ with respect to $\{I\}$ can be viewed as an element of the Special Euclidean group, $(R, \mathbf{p}) = ({}^{I}_{B}R\,, {}^{I}\mathbf{p}_{B}) \in \mathrm{SE}(3)$, where $\mathbf{p} \in \mathbb{R}^3$ is the position and $R \in \mathrm{SO}(3)$ the rotation matrix. The kinematic and dynamic equations of motion for the rigid body can be written as

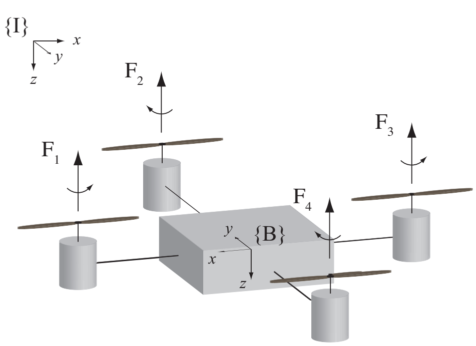
$$\dot{\mathbf{p}} = R\mathbf{v}, \tag{1}$$

$$\dot{\mathbf{v}} = -S(\boldsymbol{\omega})\mathbf{v} + \frac{1}{m}\mathbf{f} + R^T\mathbf{b}, \tag{2}$$

$$\dot{R} = RS(\boldsymbol{\omega}), \tag{3}$$

where the angular velocity $\boldsymbol{\omega} \in \mathbb{R}^3$, linear velocity $\mathbf{v} \in \mathbb{R}^3$ and the external force $\mathbf{f}$ are expressed in the body frame $\{B\}$ and the map $S(.) : \mathbb{R}^3 \mapsto \mathbb{R}^{3\times 3}$ yields a skew symmetric matrix that verifies $S(\mathbf{x})\mathbf{y} = \mathbf{x} \times \mathbf{y}$, for $\mathbf{x}$ and $\mathbf{y} \in \mathbb{R}^3$. The unknown external disturbance $\mathbf{b} \in \mathbb{R}^3$ is constant and expressed in the inertial frame $\{I\}$ and the scalar m represents the quadrotor's mass. The force disturbance can model exogenous inputs, such as constant wind, and also model uncertainties, such as variations in the vehicle mass or in relation between the thrust input and the thrust force output.

(a) Quadrotor platform



(b) Quadrotor setup

Fig. 1. Quadrotor experimental platform and diagram.

The quadrotor platform used in this work is equipped with an inner-loop control circuit, responsible for generating the forces F_i applied to each motor, so that the total thrust force T and the angular velocity $\boldsymbol{\omega}$ can be considered as inputs for the trajectory tracking controller design. The external force in body coordinates is given by

$$\mathbf{f} = -T\mathbf{e}_3 + mgR^T\mathbf{e}_3 \tag{4}$$

where $\mathbf{e}_3 = [0\ 0\ 1]^T$ and g is the gravitational acceleration.

III. Problem Statement

Let the desired trajectory $\mathbf{p}_d(t) \in \mathbb{R}^3$ be a curve of class at least C^3. The control objective consists of designing a control law for the quadrotor actuations $T(t)$ and $\boldsymbol{\omega}(t)$ that ensures convergence of the vehicle's position $\mathbf{p}(t)$ to the trajectory $\mathbf{p}_d(t)$ with the largest possible basin of attraction. Throughout the remainder of the paper, the time dependence of variables is often omitted to lighten notation.

Due to the underactuated nature of the vehicle, the desired attitude cannot be arbitrarily selected. From (2) and (4), it is easy to observe that the equilibrium for trajectory tracking satisfies

$$T_d R_d \mathbf{e}_3 = mg\mathbf{e}_3 - m\ddot{\mathbf{p}}_d + m\mathbf{b}.$$

Consequently, the desired rotation matrix R_d is automatically prescribed up to a rotation about the body z axis ($T_d R_d R_z(\psi)\mathbf{e}_3 = mg\mathbf{e}_3 - m\ddot{\mathbf{p}}_d + m\mathbf{b}$, with $\psi \in \mathbb{R}$), which we take as a degree of freedom.

We consider the full state of the vehicle to be available for feedback. In our setup, the state measurements are obtained through a high speed motion tracking system, based on external cameras tracking reflective markers on the vehicle, as described in section V. An estimate of the external disturbance, $\hat{\mathbf{b}}$ is obtained by adaptive backstepping and used for feedback control. We write the estimation error as $\tilde{\mathbf{b}} = \hat{\mathbf{b}} - \mathbf{b}$. Stability and convergence of the estimation error to zero is guaranteed by Lyapunov-like methods. The external force disturbance $\mathbf{b}$ is assumed to be bounded in norm, so that the quadrotor can perform a trajectory tracking manoeuvre with bounded thrust input.

Assumption 1. *The force disturbance* $\mathbf{b}$ *is bounded in norm by* $\|\mathbf{b}\| \leq B$ *with* $B > 0$.

Even though the disturbance is bounded, straightforward or naive implementations of estimators can lead to wind-up phenomena and result in unbounded growth of the estimate. To avoid a wind-up effect on the disturbance estimator, and keep the estimate bounded, we employ a projection operator when designing the estimator. This procedure is detailed in the next section, together with the design of the controller.

IV. Controller Design

The translational subsystem of the quadrotor, viewed in the inertial frame, can be regarded as a vectorial double integrator, driven by

$$\ddot{\mathbf{p}} = -\frac{T}{m}R\mathbf{e}_3 + g\mathbf{e}_3 + \mathbf{b}.$$

We start the design process by considering a virtual controller for the translational subsystem, which is backstepped through the angular subsystem to obtain the final implementable controller. In order to define the virtual controller consider the following error states for the double integrator

$$\mathbf{z}_1 = \mathbf{p} - \mathbf{p}_d, \tag{5}$$

$$\mathbf{z}_2 = \dot{\mathbf{z}}_1 + \sigma(\mathbf{z}_1), \tag{6}$$

where σ is a sigmoidal saturation function, and

$$\ddot{\mathbf{z}}_1 = \mathbf{u} = -\frac{T}{m}R\mathbf{e}_3 + g\mathbf{e}_3 + \mathbf{b} - \ddot{\mathbf{p}}_d.$$

A tentative Lyapunov function is devised as

$$V_{DI} = \phi(\mathbf{z}_1)^T \mathbf{1} + \frac{1}{2}\big(\Omega(\mathbf{z}_2 - \sigma(\mathbf{z}_1)) + \Omega(\sigma(\mathbf{z}_1))\big)^T \big(\Omega(\mathbf{z}_2 - \sigma(\mathbf{z}_1)) + \Omega(\sigma(\mathbf{z}_1))\big)$$

with $\phi(s) = \int_0^s \sigma(t)\mathrm{d}t$. For a fully-actuated vehicle, the control law

$$\mathbf{u}^\star = \begin{bmatrix} u(z_{11}, \mathbf{z}_{21} - \sigma(z_{11})) \\ u(z_{12}, \mathbf{z}_{22} - \sigma(z_{12})) \\ u(z_{13}, \mathbf{z}_{23} - \sigma(z_{13})) \end{bmatrix}$$

where

$$u(x_1, x_2) = -\frac{(x_2 + \sigma(x_1))(\rho(x_2 + \sigma(x_1)) + \sigma(x_1))}{\Omega'(x_2)\big(\Omega(x_2) + \Omega(\sigma(x_1))\big)} + \frac{\sigma'(x_1)x_2}{\Omega'(x_2)}$$

and ρ and σ are saturation functions, globally asymptotically stabilizes the system and renders the Lyapunov function derivative negative definite

$$\dot{V}_{DI} = -\sigma(\mathbf{z}_1)^T\sigma(\mathbf{z}_1) - k_2\mathbf{z}_2^T\rho(\mathbf{z}_2) = -W_2(\mathbf{z}_1, \mathbf{z}_2).$$

In the next step, we consider the real vehicle and the errors introduced by the underactuation. Furthermore, a term is added to the Lyapunov function to allow for disturbance rejection. The new tentative Lyapunov function is

$$V_2 = V_{DI} + \frac{1}{2k_b}\tilde{\mathbf{b}}^T\tilde{\mathbf{b}}$$

and has the following time derivative

$$\dot{V}_2 = -W_2(\mathbf{z}_1, \mathbf{z}_2) + \frac{\partial V_2}{\partial \mathbf{z}_2}(\hat{\mathbf{u}} - \mathbf{u}^\star) + \tilde{\mathbf{b}}^T\Big(\frac{\partial V_2}{\partial \mathbf{z}_2}^T - \frac{1}{k_b}\dot{\hat{\mathbf{b}}}\Big),$$

where the real control input, computed using the estimated disturbance, is

$$\hat{\mathbf{u}} = -\frac{T}{m}R\mathbf{e}_3 + g\mathbf{e}_3 + \hat{\mathbf{b}} - \ddot{\mathbf{p}}_d.$$

The term $\hat{\mathbf{u}} - \mathbf{u}^\star$ can be regarded as an actuation error due to the fact that the quadrotor is an underactuated vehicle and that thrust must be align with the z body axis. Applying with the backstepping procedure, we define the new backstepping error

$$\mathbf{z}_3 = \hat{\mathbf{u}} - \mathbf{u}^\star \tag{7}$$

and the new Lyapunov function

$$V_3 = V_2 + \frac{1}{2}\mathbf{z}_3^T\mathbf{z}_3,$$

with

$$\dot{V}_3 = -W_3(\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3) + \mathbf{z}_3^T\Big(k_3\mathbf{z}_3 + \frac{\partial V_2}{\partial \mathbf{z}_2} + \dot{\mathbf{z}}_3\Big) + \tilde{\mathbf{b}}^T\Big(\frac{\partial V_2}{\partial \mathbf{z}_2}^T - \frac{1}{k_b}\dot{\hat{\mathbf{b}}}\Big). \tag{8}$$

The time derivative of the error state $\mathbf{z}_3$ is

$$\dot{\mathbf{z}}_3 = -\frac{\dot{T}}{m}R\mathbf{e}_3 - \frac{T}{m}RS(\boldsymbol{\omega})\mathbf{e}_3 + \dot{\hat{\mathbf{b}}} - \mathbf{p}_d^{(3)} - \hat{\dot{\mathbf{u}}}^\star - \frac{\partial \mathbf{u}^\star}{\partial \mathbf{z}_2}\tilde{\mathbf{b}}$$

where $\hat{\dot{\mathbf{u}}}^\star$ denotes the estimation of the time derivative of $\dot{\mathbf{u}}^\star$ obtained using $\hat{\mathbf{b}}$ instead of the unknown $\mathbf{b}$. The error when performing such estimation is given by

$$\dot{\mathbf{u}}^\star - \hat{\dot{\mathbf{u}}}^\star = \frac{\partial \mathbf{u}^\star}{\partial \mathbf{z}_2}\tilde{\mathbf{b}}$$

Substituting $\dot{\mathbf{z}}_3$ in (8) and defining the input vector $\boldsymbol{\mu} = \begin{bmatrix}\dot{T} & \omega_1 & \omega_2\end{bmatrix}^T$ and the matrix

$$M(T) = \begin{bmatrix} 0 & 0 & -1 \\ 0 & T & 0 \\ -T & 0 & 0 \end{bmatrix},$$

we get an expression for the Lyapunov function time derivative,

$$\dot{V}_3 = -W_3(\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3) + \mathbf{z}_3^T\Big(\frac{1}{m}RM(T)\boldsymbol{\mu} + k_3\mathbf{z}_3 + \frac{\partial V_2}{\partial \mathbf{z}_2} + \dot{\hat{\mathbf{b}}} - \mathbf{p}_d^{(3)} - \hat{\dot{\mathbf{u}}}^\star\Big) + \tilde{\mathbf{b}}^T\Big(\frac{\partial V_2}{\partial \mathbf{z}_2}^T - \frac{\partial \mathbf{u}^\star}{\partial \mathbf{z}_2}^T\mathbf{z}_3 - \frac{1}{k_b}\dot{\hat{\mathbf{b}}}\Big),$$

that can be rendered negative semi-definite to achieve convergence of the trajectory tracking error to zero, with the appropriate control inputs, $\dot{T}$ and $\boldsymbol{\omega}$, and estimator law for $\dot{\hat{\mathbf{b}}}$.

To tackle the estimation of the disturbance we use a projection operator that keeps the estimate $\hat{\mathbf{b}}$ within some *a priori* defined set and verifies the smoothness properties required for the iterative application of the backstepping procedure. Consider the estimate control law,

$$\begin{aligned}\dot{\hat{\mathbf{b}}} &= k_b \operatorname{Proj}(\xi, \hat{\mathbf{b}}) \\ &= k_b\left(\xi - \frac{\eta_1\eta_2}{2(\epsilon^2 - 2\epsilon B)^{n+1}B^2}\hat{\mathbf{b}}\right)\end{aligned} \tag{9}$$

with

$$\xi = \frac{\partial V_2}{\partial \mathbf{z}_2}^T - \frac{\partial \mathbf{u}^\star}{\partial \mathbf{z}_2}^T\mathbf{z}_3,$$

$$\eta_1 = \begin{cases} (\hat{\mathbf{b}}^T\hat{\mathbf{b}} - B^2)^{n+1}, & \text{if } (\hat{\mathbf{b}}^T\hat{\mathbf{b}} - B^2) > 0 \\ 0, & \text{otherwise} \end{cases},$$

$$\eta_2 = \hat{\mathbf{b}}^T\xi - \sqrt{(\hat{\mathbf{b}}^T\xi)^2 + \delta^2},$$

where $\epsilon > 0$ and $\delta > 0$ are arbitrary parameters and B is the bound on the norm of the unknown parameter. The smooth projection operator is taken from [14] and has the following properties,

P1 – $\left\|\hat{\mathbf{b}}(t)\right\| \leq B + \epsilon, \forall t \geq 0$;

P2 – $\tilde{\mathbf{b}}^T \operatorname{Proj}(\xi, \hat{\mathbf{b}}) \geq \tilde{\mathbf{b}}^T\xi$;

P3 – $\left\|\operatorname{Proj}(\xi, \hat{\mathbf{b}})\right\| \leq \|\xi\| \left[1 + ((B + \epsilon)/B)\right]^2 + ((B + \epsilon)/(2B^2))\delta$;

P4 – $\operatorname{Proj}(\xi, \hat{\mathbf{b}})$ is C^n.

From the estimator control law (9) and property P2 we derive the lower bound for the Lyapunov function derivative

$$\dot{V}_3 \leq -W_3(\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3) + \mathbf{z}_3^T\Big(\frac{1}{m}RM(T)\boldsymbol{\mu} + k_3\mathbf{z}_3 + \frac{\partial V_2}{\partial \mathbf{z}_2} + \dot{\hat{\mathbf{b}}} - \mathbf{p}_d^{(3)} - \hat{\dot{\mathbf{u}}}^\star\Big).$$

Moreover, property P4 ensures that the derivatives of the estimate are continuous up to $\hat{\mathbf{b}}^{(n+1)}$. We are now able to state Theorem 3, regarding the design of a stabilizing control law. The Theorem requires the following assumption, which is met under the appropriate conditions, that are to be detailed in the sequel.

Assumption 2. *The thrust actuation verifies $T(t) \geq \epsilon > 0$ for all time $t > 0$.*

Theorem 3. *Let the quadrotor kinematics and dynamics be described by* (1)-(3)*, let $\mathbf{p}_d(t) \in C^3$ be the desired trajectory, and consider the transformation to error coordinates $\mathbf{z}_1$, $\mathbf{z}_2$, $\mathbf{z}_3$ given by* (5)*,* (6)*,* (7)*, respectively. For any bounded $\omega_3(t) \in C^1$, the closed-loop system that results from applying the control law*

$$\boldsymbol{\mu} = -mM^{-1}(T)R^T\left(k_3\mathbf{z}_3 + \frac{\partial V_2}{\partial \mathbf{z}_2} + \dot{\hat{\mathbf{b}}} - \mathbf{p}_d^{(3)} - \hat{\mathbf{u}}^\star\right) \tag{10}$$

and the estimator law (9) *achieves global trajectory tracking by guaranteeing that the errors $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ converge to zero for any initial condition.*

Proof. The proposed control law (10) is well defined, in the conditions of Assumption 2. Starting with the positive definite Lyapunov function

$$V = \phi(\mathbf{z}_1)^T\mathbf{1} + \frac{1}{2}\big(\Omega(\mathbf{z}_2 - \sigma(\mathbf{z}_1)) + \Omega(\sigma(\mathbf{z}_1))\big)^T \big(\Omega(\mathbf{z}_2 - \sigma(\mathbf{z}_1)) + \Omega(\sigma(\mathbf{z}_1))\big) + \tfrac{1}{2}\mathbf{z}_3^T\mathbf{z}_3 + \tfrac{1}{2k_b}\tilde{\mathbf{b}}^T\tilde{\mathbf{b}},$$

and computing its time derivative, in closed-loop, we have that

$$\dot{V} \leq -\sigma(\mathbf{z}_1)^T\sigma(\mathbf{z}_1) - k_2\mathbf{z}_2^T\rho(\mathbf{z}_2) - k_3\mathbf{z}_3^T\mathbf{z}_3,$$

which is a negative semidefinite function. Since the quadrotor error dynamics are non-autonomous, we resort to Barbalat's Lemma to prove convergence of $\dot{V}$ to zero. From the unboundedness of V with respect to $\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3$ and $\tilde{\mathbf{b}}$, and observing that $\dot{V}$ is negative semi-definite, we conclude that the states $\mathbf{z}_1$, $\mathbf{z}_2, \mathbf{z}_3$ and $\tilde{\mathbf{b}}$ are bounded. The external input $\mathbf{p}_d^{(3)}$ is bounded by assumption and $\dot{\hat{\mathbf{b}}}$ is bounded from property P4 of the projection operator. We have thus that $\ddot{V}$ is bounded and, consequently, $\dot{V}$ is uniformly continuous. We can therefore apply Barbalat's Lemma to prove convergence of $\dot{V}$ to zero and, consequently, of the states $\mathbf{z}_1$, $\mathbf{z}_2$ and $\mathbf{z}_3$ to the origin. □

The proposed control law (10) can only be applied if Assumption 2 holds. A conservative estimate of the initial states for which $T(t) \geq \epsilon$ is guaranteed for all $t > 0$ can be obtained using (7) together with the bounds for the errors and estimation derived from the Lyapunov function and its derivative. Using $\|\cdot\|_\infty$ to denote the maximum norm of a function, we obtain the following lower bound for the thrust, for all $t > 0$,

$$|T(t)| \geq mg - \left\|\hat{\mathbf{b}}\right\| - \|\ddot{\mathbf{p}}_d(t)\|_\infty - \|\mathbf{u}^\star\|_\infty - \sqrt{2V(0)}.$$

If the initial conditions and desired trajectories are such that the lower bound for $|T(t)|$ is positive, then the thrust $T(t)$ that results from applying the proposed control law is guaranteed to take only positive or only negative values.

V. Experimental Results

In order to experimentally validate the proposed control algorithms we developed a rapid prototyping and testing architecture using a Matlab/Simulink environment to seamlessly integrate the sensors, the control algorithm and the communication with the vehicles. The vehicle used for the experiments is a radio controlled Blade mQX quadrotor [16], depicted in Figure 1(a). This aerial vehicle is very agile and maneuverable, readily available and inexpensive, making it the ideal platform for the present work. The quadrotor weighs 80 g with battery included and the arm length from the center of mass to each motor is 11 cm. The available commands are thrust force and angular velocity. The maximum thrust generated by the propellers is approximately 1.37 N (equivalent to 140 g) and varies slightly with the battery charge. The maximum angular velocity that can be commanded is 200 deg/s for the x and y axes and 300 deg/s for the z axis. However, the commands issued to the quadrotor are not instantaneously followed. This delay nonlinearity can be well approximated by considering the motors as first order dynamic systems with a pole at 1.5 Hz.

Due to the lack of support for on-board sensors, the state of the vehicle must be estimated through external sensors. In our setup we use a VICON Bonita motion capture system [17], comprising 12 cameras, together with markers attached to the quadrotor. The motion capture system is able to accurately locate and estimate the positions of the markers, from which it obtains position and orientation measurements for the aircraft. VICON Bonita is a high performance system, able to operate with sub-millimeter precision at up to 120 Hz. The performance of the motion capture system is such that the linear velocity can be well estimated from the position measurements by a simple backwards Euler difference, with relatively low noise level. For the experimental setup, the state measurements from the motion capture system are obtained at 50 Hz, allowing for improved accuracy without compromising the overall stability due to delays.

The commercial off-the-shelf quadrotor vehicles are designed to be human piloted with remote controls but not directly from a computer. In order to be able to send commands to a quadrotor from a computer we identified the radio chip inside the remote control and connected the serial interface of the RF module to a computer serial port. To maintain the radio link, the radio transmitters must receive the control signals via serial port and send them to the vehicle once every 22.5 ms.

A graphical representation of the overall architecture is presented in Figure 2. We use two computer systems, one running the VICON motion tracking software and the Simulink model which generates the command signals sent to the other computer through Ethernet; and a second one that receives the command signals and sends them through serial port to the RF module at intervals of 22.5 ms. The decision to separate control and communications was made to avoid jitter in the transmission of the serial-port signals to the RF module, which occurred when running all the systems in the same computer, and lead to erratic communication with the vehicle.

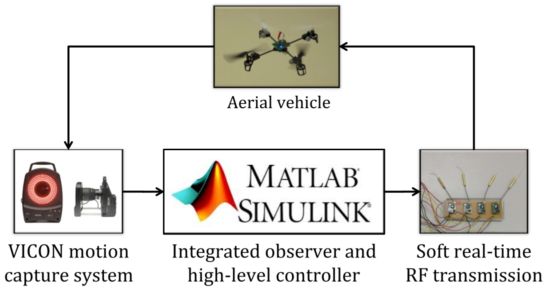


Fig. 2. Quadrotor control architecture.

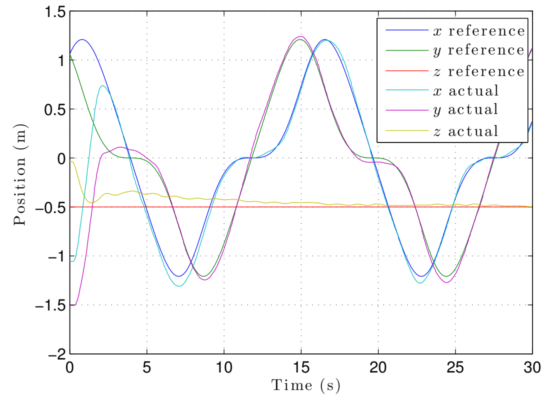


Fig. 3. Time evolution of the position and reference signals.

Trajectory tracking

For the first experimental evaluation of the proposed controller we selected for the desired trajectory a lemniscate (figure eight) parameterized as

$$\mathbf{p}_d(t) = \begin{bmatrix} \cos(\pi/4) & \sin(\pi/4) & 0 \\ -\sin(\pi/4) & \cos(\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sin(\phi(t))\cos(\phi(t))}{\sin(\phi(t))^2+1} \\ \frac{\cos(\phi(t))}{\sin(\phi(t))^2+1} \\ 1 \end{bmatrix},$$

where $\phi(t)$ obeys

$$\dot{\phi}(t) = V\sqrt{1+\sin^2 t}.$$

This parametrization results in a trajectory with unitary norm time derivative and constant desired speed for the quadrotor of V m/s, chosen for this experiment to be $V = 0.5$. The control law coefficients are $k_2 = 2$, $k_3 = 4$, $k_b = 1$ and the initial estimate $\hat{\mathbf{b}}(0)$ is set to zero. For the sigmoid saturation function we use $\sigma(s) = M\,r\,s/\sqrt{1+r^2s^2}$, which has the bound $|\sigma(s)| \leq M$ and derivative at the origin $\sigma'(s) = r$, with $M = 1.5$ and $r = 3$.

The time evolution of the actual quadrotor position and the reference is shown in Figure 3. The quadrotor follows closely the desired path, with neglectible error in steady state. The RMS trajectory tracking error in steady state is 4.61 cm and the maximum error is 7.6 cm. The position error in steady state can be attributed to unmodeled dynamics of the plant and to the fact that the issued commands are not perfectly followed by the aircraft. The main contributions to the unmodeled dynamics are threefold: *i)* there exist unmodeled cross-couplings between the angular velocity commands and lateral forces acting on the quadrotor, due to an uneven and not perfectly symmetric mass distribution of the vehicle; *ii)* the issued thrust and angular velocity commands are not perfectly followed due to motor inertia and incorrect thrust command to thrust force identification; *iii)* there exists a non-constant wind disturbance affecting the vehicle. Notice that the vehicle has a large initial position error, leading to the saturation function having a preponderant role in the control signal. Despite this unfavorable initial configuration, the actuation commands are kept within their performance limits (see Figure 5) and convergence to the reference trajectory occurs in just 5 seconds time, after which only small corrections are performed to the quadrotor trajectory.

Although the trajectory tracking experiment is performed on a closed division, with wind disturbances arising only from an air conditioning system, the effect of the integral action is evidenced on the vertical axis. After the initial transient, where the vertical error decreases rapidly, there is a slower approximation of the altitude to the desired one, until they match in steady state. This slower convergence is the result of the imposed integral action, through the disturbance estimator, which enables perfect theoretical tracking, even though the thrust command to thrust force relation is not perfectly known. The time evolution of the disturbance estimate is presented in Figure 4 where a convergence of the estimated can be seen, although they do not converge perfectly due to the presence of unmodelled dynamics.

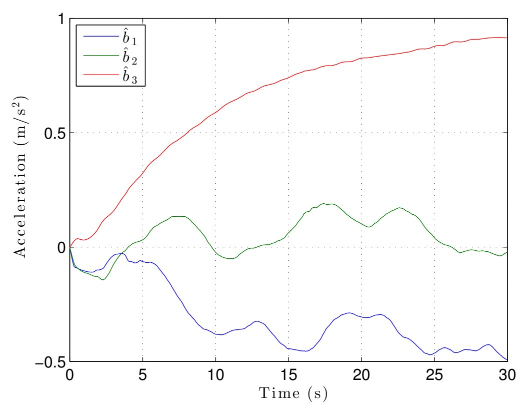


Fig. 4. Force disturbance estimate.

The quadrotor actuation signals are depicted in Figure 5. The initial transient starts with a high thrust, to take the quadrotor to the desired height, and large angular velocity commands, to turn the quadrotor to the desired direction to minimize the errors. Once in steady state, the actuation signals are primarily the ones necessary to drive the controller through the reference trajectory, with only small corrections being performed according to the control law, without large variations. The effect of the integral action can also be seen in the thrust actuation, as it slowly increases with time, compensating for imprecisions in the conversion between commanded and actual thrust due to its variation with battery charge. Moreover, the thrust actuation is always well above zero and Assumption 2 holds for all the trajectory.

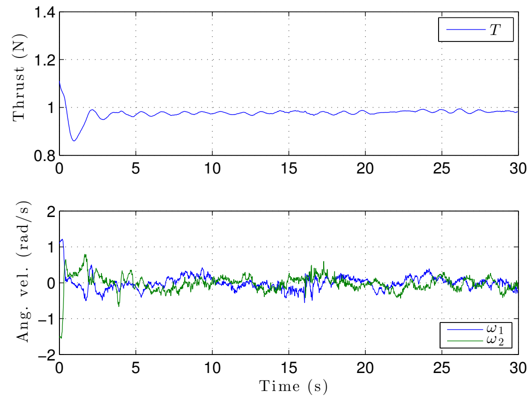


Fig. 5. Time evolution of the actuation signals.

Finally, the time evolution of the backstepping errors is shown in Figure 6. A video of the quadrotor takeoff and trajectory tracking is available at [18].

VI. Conclusions

This paper presented a state feedback solution to the problem of stabilizing an underactuated quadrotor vehicle along a predefined trajectory in the presence of constant force disturbances. A Lyapunov function for the system was derived using adaptive backstepping techniques, where an adaptive estimator was introduced so as to compensate for the force disturbance and add integral action to the system. A rapid prototyping and testing architecture was developed to expedite the development process by creating an abstraction layer that integrates the sensors, controller, and communication with the vehicle. Experimental data for trajectory tracking applied to a small-scale quadrotor vehicle was presented which evidenced the effects of the adaptive action and demonstrated the robustness and performance of the proposed control law.

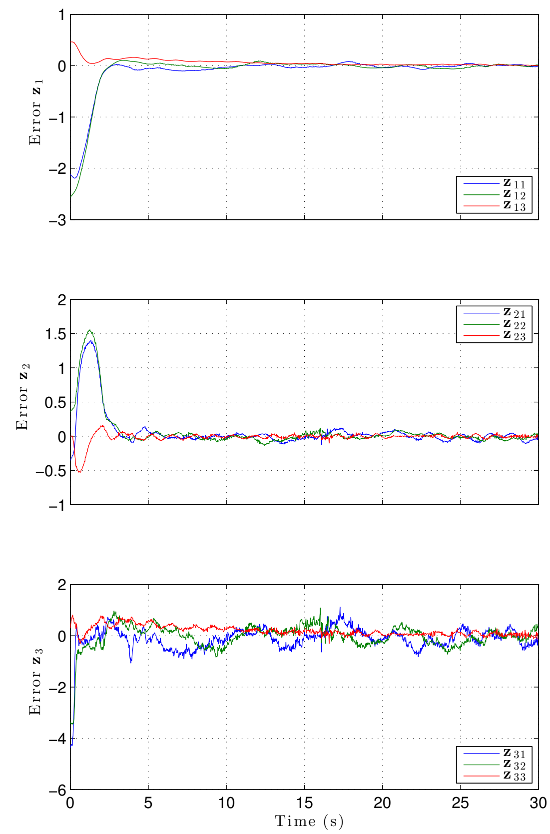


Fig. 6. Time evolution of the error signals.

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