RSS BASED COOPERATIVE SENSOR NETWORK LOCALIZATION WITH UNKNOWN TRANSMIT POWER

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ABSTRACT

This work aims to estimate multiple node positions in the presence of unknown transmit powers within the context of cooperative sensor network localization. In the adopted scheme, each source can communicate with a set of anchors (probably not in sufficient numbers) and a set of other sources. Received Signal Strength (RSS) between them are measured. Since finding the Maximum Likelihood Estimates (MLE) of the positions and transmit powers given those measurements poses a difficult nonconvex optimization problem, it is approximated by a Nonlinear Least Squares problem. Then, the position and transmit power of multiple sources are estimated jointly by solving Euclidean Distance Matrix completion problem. Simulations show that the localization accuracy and the running time of the proposed method is better than the state of the art method and close to the Cramér-Rao Lower Bound for some scenarios.

Index Terms— Cooperative sensor network localization, unknown transmit power, received signal strength, Euclidean distance matrix, semidefinite programming.

1. INTRODUCTION

Many wireless sensor network (WSN) applications require the involved sensors to be localized [1]. Among the noisy measurements upon which localization could be based, Received Signal Strength (RSS) is an attractive method mainly because of its low complexity and cost [2]. Since the RSS measurement model is a function of the transmit power of the source node, which depends on its battery and antenna gain and might change with time, the anchor nodes are not able to find the location of a source node if its transmit power is not accounted for. Consequently, each source node has to report its transmit power to anchor nodes during RSS measurements which requires additional hardware and software in both anchor nodes and source nodes making the network more complex. Therefore, localization algorithms should tackle this issue.

In noncooperative sensor networks, source nodes can communicate only with anchor nodes [3]. The lack of accessible anchor nodes and also limited connectivity among anchor nodes and source nodes lead to the emergence of the cooperative localization paradigm, in which source nodes are able to communicate with both anchor nodes and other source nodes. Therefore, not only are RSSs between source nodes and anchor nodes measured, but also the source nodes themselves are involved and collect RSS measurements from each other. Thus, cooperative localization using RSS in the practical case where transmit powers are different and unknown are currently open problems.

One of the common solutions is to eliminate the dependency of the transmit power from the RSS measurement model by using the differential RSS between a source node and two anchor nodes [4] which enhances the noise and degrades the accuracy. Another very recent method is to estimate the transmit power of the source along with its location [5, 6] which uses Semidefinite Programming (SDP) similar to the proposed method with less accuracy and more computational complexity.

To find the Maximum Likelihood Estimator (MLE) for the sensor network localization problem with unknown transmit powers, it is necessary to solve a nonlinear and nonconvex optimization problem. To avoid this difficulty, the original MLE is transformed into an approximate Nonlinear Least Squares (NLS) problem. Then, relaxation techniques are applied to convert the NLS problem into a convex optimization problem by resorting to Euclidean Distance Matrix (EDM) completion (a type of SDP). Through this, the source transmit powers are considered as nuisance parameters and estimated jointly with the source locations. The advantage of an SDP is that its cost function does not have local minima and thus convergence to the global minimum is guaranteed [7]. The drawback is that the SDP technique is sub-optimal and cannot achieve the best possible performance under all conditions.

The remainder of the paper is organized as follows. Section 2 formalizes the problem and shows the EDM comple-

tion as an SDP method. Section 3 shows the derivation of Cramér-Rao Lower Bound (CRLB) for this problem. Simulations and computational complexity analysis are given in Section 4. Conclusions are drawn in Section 5.

2. PROBLEM FORMULATION

This section formulates the cooperative RSS based localization problem in which there are more than two source nodes with unknown locations, and moreover, source nodes can communicate not only with anchor nodes but also with each other. The power of the transmitted signal of each source can be measured at both anchor nodes and other source nodes. In other words, two sets of RSS measurements are available to the estimator: source-anchor and source-source measurements. Let $\mathbf{s}_j \in \mathbb{R}^2$, $j \in \mathcal{S} = \{1, \ldots, N\}$ and $\mathbf{a}_i \in \mathbb{R}^2$, $i \in \mathcal{A} = \{N+1, \ldots, N+M\}$ denote source and anchor locations[1], respectively. The following two sets are defined as

$$\mathcal{B}_j = \{\, i \mid \text{anchor } i \text{ can communicate with source } j \,\},$$
$$\mathcal{C}_j = \{\, i \mid \text{source } i \text{ can communicate with source } j, i>j \,\}.$$

The cooperative RSS measurement model is expressed as

$$P_{ij} = P_j - 10\beta \log_{10} d_{ij} + n_{ij}, \quad j \in \mathcal{S}, \ i \in \mathcal{B}_j \cup \mathcal{C}_j, \quad (1)$$

where P_j is the reference power at a reference distance (1m) from the jth source, $d_{ij} = \|\mathbf{s}_i - \mathbf{s}_j\|$, $i \in \mathcal{C}_j$ and $d_{ij} = \|\mathbf{a}_i - \mathbf{s}_j\|$ $i \in \mathcal{B}_j$. In addition, n_{ij} are the log normal shadowing terms which are modeled as independent and identically distributed (i.i.d.) zero mean Gaussian random variables with standard deviation σ_{ij}. Consequently, there are in total 2N+N elements that should be estimated including the source node locations and the transmit powers of the source nodes defined as $\mathbf{S} = [\mathbf{s}_1, \ldots, \mathbf{s}_N] \in \mathbb{R}^{2 \times N}$ and $\mathbf{P} = [P_1, \ldots, P_N]^T \in \mathbb{R}^N$ respectively.

2.1. EDM Formulation

By rearranging the logarithmic term and dividing both sides by 5β, (1) can be reformulated [6] as

$$d_{ij}^2 \lambda_{ij} = \alpha_j 10^{\frac{n_{ij}}{5\beta}}, \quad (2)$$

where $\lambda_{ij} = 10^{\frac{P_{ij}}{5\beta}}$ and $\alpha_j = 10^{\frac{P_j}{5\beta}}$. For sufficiently small noise, the right hand side (RHS) of (2) can be approximated using the first order Taylor series expansion as

$$d_{ij}^2 \lambda_{ij} = \alpha_j (1 + \frac{\ln 10}{5\beta} n_{ij}), \quad (3)$$

and this can be rewritten as

$$d_{ij}^2 \lambda_{ij} = \alpha_j + \epsilon_{ij}, \quad (4)$$

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[1] The generalization to three-dimensional space is straightforward.

where ϵ_{ij} is a zero mean Gaussian random variable with standard deviation $\alpha_j \frac{\ln 10}{5\beta} \sigma_{ij}$. The corresponding NLS estimator of the unknown parameters $\mathbf{S}$ and α is

$$\underset{\mathbf{S},\alpha}{\text{minimize}} \quad \sum_{j \in \mathcal{S}} \sum_{i \in \mathcal{B}_j \cup \mathcal{C}_j} (d_{ij}^2 \lambda_{ij} - \alpha_j)^2. \quad (5)$$

The unknown squared distances can be arranged into a single symmetric EDM matrix of size $(N+M) \times (N+M)$, with elements $E_{ij} = d_{ij}^2$, and satisfying the properties of the EDM cone $\mathcal{E}$ [7, 8]

$$E_{ii} = 0, \qquad E_{ij} \geq 0, \qquad -\mathbf{JEJ} \succeq 0, \quad (6)$$

where $\mathbf{J} = \left(\mathbf{I}_\rho - \frac{1}{\rho}\mathbf{1}_\rho \mathbf{1}_\rho^T\right)$, $\rho = N + M$, is a centering operator which subtracts the mean of a vector from each of its components and $\mathbf{I}_\rho$ is $\rho \times \rho$ identity matrix. Therefore, the nearest EDM problem is formulated as

$$\begin{aligned} \underset{\mathbf{E},\alpha}{\text{minimize}} \quad & \sum_{j \in \mathcal{S}} \sum_{i \in \mathcal{B}_j \cup \mathcal{C}_j} (E_{ij} \lambda_{ij} - \alpha_j)^2 \\ \text{subject to} \quad & \mathbf{E} \in \mathcal{E}, \ \mathbf{E}(\mathcal{A}) = \mathbf{A} \\ & \text{rank}(\mathbf{JEJ}) = 2, \end{aligned} \quad (7)$$

The constraint $\mathbf{E}(\mathcal{A}) = \mathbf{A}$ enforces the known *a priori* spatial information related with anchors in the appropriate EDM submatrix. The rank constraint in (7) ensures that the solution is compatible with a constellation of source/anchor points in $\mathbb{R}^2$. Dropping the rank constraint, a compact relaxed SDP formulation is obtained.

Note that the solution of (7) is a distance matrix $\mathbf{E}$. Detailed explanations of how to estimate the spatial coordinates of the sources from EDM and the usage of anchors are given in [9]. The basic idea is to use a linear transformation to obtain the Gram matrix $(\mathbf{ZJ})^T \mathbf{ZJ} = -\frac{1}{2}\mathbf{JEJ}$, from which spatial coordinates $\mathbf{Z} = [\mathbf{s}_1, \ldots, \mathbf{s}_N, \mathbf{a}_{N+1}, \ldots, \mathbf{a}_{M+N}]$ are extracted by the singular value decomposition up to a unitary matrix. The anchors are then used to estimate the residual unitary matrix by solving a Procrustes problem [9].

3. DERIVATION OF CRLB

The log of the joint conditional pdf for the RSS based cooperative localization problem with unknown transmit power is (up to an additive constant) [2]

$$\log f(\mathbf{p}|\mathbf{S},\mathbf{P}) = -\frac{1}{2\sigma_{ij}^2} \left\{ \sum_{j \in \mathcal{S}, i \in \mathcal{B}_j} (P_{ij} - P_j - 10\beta \log_{10}(\|\mathbf{a_i} - \mathbf{s_j}\|)) \right.$$

$$\left. + \sum_{j \in \mathcal{S}, i \in \mathcal{C}_j} (P_{ij} - P_j - 10\beta \log_{10}(\|\mathbf{s_i} - \mathbf{s_j}\|)) \right.$$

$$(8)$$

The Fisher information matrix, $\mathbf{F}$, is obtained by taking the negative expected value of the second derivative of (8) with respect to $\mathbf{S}$ and $\mathbf{P}$ as [10]

$$\mathbf{F} = \begin{bmatrix} F_{xx} & F_{yx} & F_{px} \\ F_{xy} & F_{yy} & F_{py} \\ F_{xp} & F_{yp} & F_{pp} \end{bmatrix} \tag{9}$$

where

$$F_{xx} = \begin{cases} \gamma\{\sum_{k\in\mathcal{B}_j} \frac{(a_{kx}-s_{jx})^2}{\|\mathbf{a}_k-\mathbf{s}_j\|^4} + \sum_{k\in\mathcal{C}_j} \frac{(s_{kx}-s_{jx})^2}{\|\mathbf{s}_k-\mathbf{s}_j\|^4}\} & , i=j \\ -\gamma\frac{(s_{jx}-s_{ix})^2}{\|\mathbf{s}_j-\mathbf{s}_i\|^4} & , i\neq j \end{cases}$$

$$F_{yy} = \begin{cases} \gamma\{\sum_{k\in\mathcal{B}_j} \frac{(a_{ky}-s_{jy})^2}{\|\mathbf{a}_k-\mathbf{s}_j\|^4} + \sum_{k\in\mathcal{C}_j} \frac{(s_{ky}-s_{jy})^2}{\|\mathbf{s}_k-\mathbf{s}_j\|^4}\} & , i=j \\ -\gamma\frac{(s_{jy}-s_{iy})^2}{\|\mathbf{s}_j-\mathbf{s}_i\|^4} & , i\neq j \end{cases}$$

$$F_{yx} = \begin{cases} \gamma\{\sum_{k\in\mathcal{B}_j} \frac{(a_{ky}-s_{jy})(a_{kx}-s_{jx})}{\|\mathbf{a}_k-\mathbf{s}_j\|^4} + \sum_{k\in\mathcal{C}_j} \frac{(s_{ky}-s_{jy})(s_{ky}-s_{jx})}{\|\mathbf{s}_k-\mathbf{s}_j\|^4}\} & , i=j \\ -\gamma\frac{(s_{jy}-s_{iy})(s_{jx}-s_{ix})}{\|\mathbf{s}_j-\mathbf{s}_i\|^4} & , i\neq j \end{cases}$$

$$F_{xp} = \begin{cases} \rho\{\sum_{k\in\mathcal{B}_j} \frac{a_{kx}-s_{jx}}{\|\mathbf{a}_k-\mathbf{s}_j\|^2} + \sum_{k\in\mathcal{C}_j,j<k} \frac{s_{kx}-s_{jx}}{\|\mathbf{s}_k-\mathbf{s}_j\|^2}\} & , i=j \\ -\rho\frac{s_{jx}-s_{ix}}{\|\mathbf{s}_j-\mathbf{s}_i\|^2} & , i<j \end{cases}$$

$$F_{yp} = \begin{cases} \rho\{\sum_{k\in\mathcal{B}_j} \frac{a_{ky}-s_{jy}}{\|\mathbf{a}_k-\mathbf{s}_j\|^2} + \sum_{k\in\mathcal{C}_j,j<k} \frac{s_{ky}-s_{jy}}{\|\mathbf{s}_k-\mathbf{s}_j\|^2}\} & , i=j \\ -\rho\frac{s_{jy}-s_{iy}}{\|\mathbf{s}_j-\mathbf{s}_i\|^2} & , i<j \end{cases}$$

$$F_{pp} = \begin{cases} \frac{1}{\sigma_{ij}^2}\{\sum_{k\in\mathcal{B}_j} 1 + \sum_{k\in\mathcal{C}_j,j<k} 1\} & , i=j \\ 0 & , j\neq i \end{cases}$$

and $\gamma = \frac{(10\beta)^2}{(\ln 10\sigma_{ij})^2}$, $\rho = \frac{10\beta}{\ln 10\sigma_{ij}^2}$. The CRLB for the RSS based cooperative localization with unknown transmit power is taken as the inverse of $\mathbf{F}$.

4. SIMULATIONS

In this section, computer simulations are performed to evaluate the performance of the proposed algorithm which will be called "EDM" in the figures. The comparison metric is the total root mean-square error (RMSE) defined as

$$\text{RMSE} = \sqrt{\frac{1}{L}\frac{1}{N}\sum_{k=1}^{L}\sum_{i=1}^{N}\|\mathbf{s}_i - \hat{\mathbf{s}}_i^k\|^2}, \tag{10}$$

where $\hat{\mathbf{s}}_i^k$ denotes the i-th estimated source position in the k-th Monte Carlo run ($L = 1000$) for the specific noise realization. To assess the fundamental hardness of the position estimation, error plots also show the total CRLB with known ("CRLB") and unknown transmit power ("CRLB-Unknown-P") for each noise variance. Through out the simulations the value of the path loss exponent β was known and set to 4. The standard deviation of the shadowing is $\sigma_{ij} = \sigma \in [1\,8]$.

To compare the proposed algorithm with MLE, Matlab's function *lsqnonlin* is initialized with true values of the positions and transmit power of sources, denoted below as "MLE". Additionally, the results will be benchmarked with a recently published method "SDP-URSS" [6] which resorts
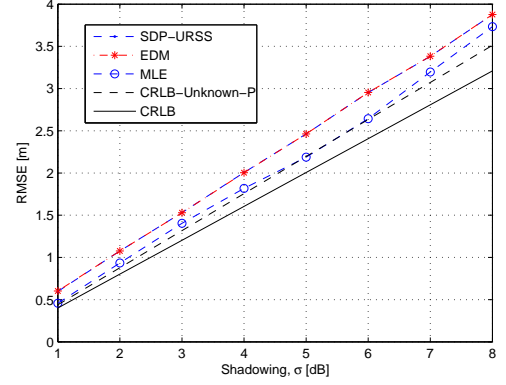

Fig. 1. RMSE comparisons for the first scenario where the sources are inside the convex hull of the anchors.

to similar formulations but uses different semidefinite relaxations.

Experiment 1: In the first scenario, five anchor nodes were placed regularly on the corners and in the center of a square 20 m x 20 m and ten source nodes were distributed in a square area 19 m x 19 m inside the convex hull of the anchor nodes, i.e, $\mathbf{S} = [2\,19;\ 4\,3;\ 6\,4;\ 6\,10;\ 12\,2;\ 14\,4;\ 15\,7;\ 15\,16;\ 16\,3;\ 18\,18]$. The corresponding reference powers are $\mathbf{P} = -[3.92;\,11.55;\,9.48;\,19.47;\,5.11;\,19.63;\,12.42;\,14.65;\,2.51;\,10]$. Full connectivity was assumed, meaning that each source node was connected to all anchor nodes and also to all other source nodes. Fig. 1 shows that the RMSE of EDM and SDP-URSS are almost the same and they are close to MLE and CRLB.

Experiment 2: In the second scenario, the location of the source nodes is the same as in experiment 1, but the anchor nodes placed irregularly as $\mathbf{A} = [2\,2;\ 4\,16;\ 10\,10;\ 12\,14;\ 17\,5]$. For this irregular scenario the performance of EDM is superior than SDP-URSS as shown in Fig. 2. However, the performance gap between MLE and the algorithms are higher than the previous scenario. Moreover, MLE attains the CRLB only at small noise levels.

A Note on Practical Computational Complexity: The worst case computational complexity of SDP based algorithms for sensor network localization is bounded by $O((N + M)^6)$ [11]. In detail, without imposing any structure on matrix variables [12], $\mathcal{O}_{EDM} = ((N+M)^2 + N)^2(N+M)^2)$ and $\mathcal{O}_{SDP-URSS} = (3N + N^2 + L)^2 L^2)$, where L is the total number of connections. For full connectivity $L = N(M + (N-1)/2)$.

For the proposed algorithm, CPU time empirically increases with $(N + M)^{4.5}$. The experiments were conducted on a laptop with Intel Core i5-2430M 2.4 GHz CPU and 4 GB of RAM, using MATLAB 7.11, CVX 1.22 and SeDuMi as a general purpose SDP solver. The CPU time used by EDM and SDP-URSS is about 0.3 and 0.7 seconds, respectively for this network.
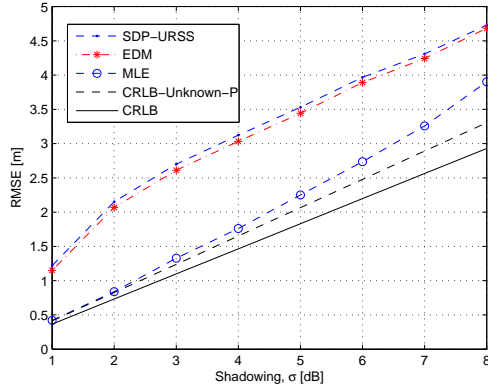
Fig. 2. RMSE comparisons for the second scenario where the sources are not inside the convex hull of the anchors

Experiment 3: The sample mean and the uncertainty ellipsoids of EDM are given in Fig. 3 when $\sigma = 4$ for the first scenario. Two of the sources ($[\mathbf{s}_1\,\mathbf{s}_{10}] = [2\,19; 18\,18]$) are only connected to two anchors and all others communicate with five anchors. With the limited connectivity to anchors the localization problem becomes harder, similarly to what is known to occur even with full connectivity when some of the sources lie outside the convex hull of the set of anchors. Moreover, although two anchors are not enough for those sources to be localized in 2D, all positions are eventually determined with good accuracy through cooperation, as the remaining sources are within range of a sufficient number of anchors.
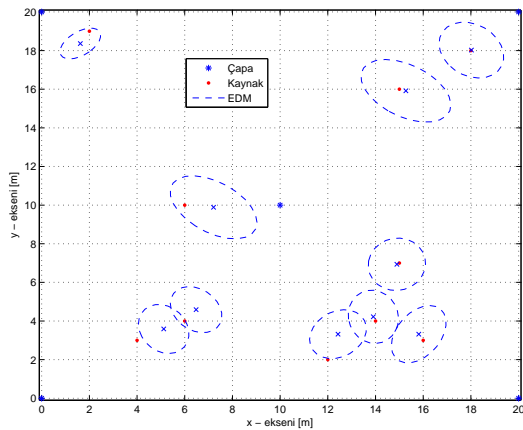

Fig. 3. Sample mean (shown as crosses) and uncertainty ellipsoids of EDM when sources $[\mathbf{s}_1\,\mathbf{s}_{10}] = [2\,19; 18\,18]$ are connected to only two anchors.

5. CONCLUSION

EDM completion, a type of SDP technique with reasonable computational cost, is proposed to localize multiple sources when transmit powers are not known. It is shown that cooperation among sources provides accurate localization even if some sources are connected to few anchors. Additionally, EDM is better than the recently published method both in accuracy and computational complexity. Its performance is close to CRLB at some scenarios.

6. REFERENCES

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