Underwater Target Positioning with a Single Acoustic Sensor

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Abstract: The availability of reliable underwater positioning systems to localize one or more vehicles simultaneously based on information received on-board a support ship or an autonomous surface vessel is key to the operation of some classes of AUVs. Furthermore, there is considerable interest in reducing the number of sensors involved in acoustic navigation/positioning systems to reduce the costs involved and the time consumed in the deployment, callibration, and recovery phases. Motivated by these considerations, in this paper we address the problem of single underwater target positioning based on measurements of the ranges between the target and a moving sensor at the sea surface, obtained via acoustic ranging devices. In particular, and speaking in loose terms, we are interested in determining the optimal geometric trajectory of the surface sensor that will, in a well defined sense, maximize the range-related information available for underwater target positioning. To this effect, an appropriate Fisher Information Matrix is defined and its determinant is maximized to yield the sensor trajectory that maximizes the accuracy of the target position estimate that can possibly be obtained with any unbiased estimator. It is shown that the optimal trajectory depends on the relative velocity of the sensor, the sampling time between measurements, and the number of measurements acquired for the FIM computation. Simulation examples illustrate the key results derived.

Keywords: Autonomous Vehicles, Underwater Navigation, Optimal Sensor Placement, Positioning, Fisher Information Matrix, Cramer-Rao Bound

1. INTRODUCTION

One of the key issues in the operation of some classes of autonomous underwater vehicles (AUVs) is the availability of reliable underwater positioning systems capable of positioning one or more vehicles simultaneously, based on information received on-board a support ship or an autonomous surface vehicle. The info thus obtained can be used to follow the state of progress of a particular mission or, if reliable acoustic modems are available, to relay it as a navigation aid to the navigation systems existent on-board the AUV. Identical comments apply to a new generation of positioning systems to aid in the tracking of one or more human divers, as proposed in the context of the EC CO3AUVs project, Birk et al. [2011].

Interesting results in the area of sensor networks for positioning go back to the work of Abel [1990], where the Cramer-Rao bound is used as an indicator of the accuracy of source position estimation and a simple geometric interpretation of this bound is offered. Related work in this area can be found in Levanon [2000], Martinez & Bullo [2006], Zhang [1995], Neering et al. [2008], Jourdan & Roy [2008], Bishop et al. [2010]. In the marine systems field we can find the works Bahr et al. [2009], Moreno-Salinas et al. [2011] and Moreno-Salinas et al. [2013].

The body of work referred above exploits the geometric configuration of multiple acoustic sensors in order to define the position of a target based on range or bearings measurements. Because the latter are measured at different sensor locations, this makes it possible to determine the target position. However, in this paper an alternative approach is presented: a single sensor is used that exploits both spatial and temporal diversity in order to extract position information. In particular, we are interested in determining the optimal trajectory of a single sensor that will, in a well defined sense, maximize the range-related information available for underwater target positioning. We assume that the range measurements are corrupted by white Gaussian noise. The actual computation of the target position may be done by resorting to trilateration algorithms. See Alcocer [2009], Alcocer [2007], and the references therein for an introduction to this circle of ideas.

The challenge of reducing the number of sensors involved in underwater acoustic systems has been addressed previously in the literature in the different, yet related context of underwater navigation (in contrast with positioning, which is the core problem considered in this paper). This is patent in a vast number of publications that tackle the underwater navigation problem by assuming that only ranges from a moving vehicle to a single beacon/transponder installed at a known position are available. Early work along these lines is that of Larsen who pioneered the term Synthetic Long Baseline navigation, see Larsen [2001], and Larsen [2000]. Observability is the key issue in this class of problems. Recently, this problem has been addressed from diverse perspectives, some of them establishing connections with the multiple vehicle navigation problem.

To some extent, the general problem that we address in this paper is the dual of the single beacon/transponder navigation problem referred to above, Here, we are interested in tracking (that is, estimating the successive position of a) target using a single acoustic device that measures the ranges from the target to a companion surface vessel. However, due to space limitations we restrict ourselves to the case where the position of the target is time-invariant. They key concept exploited is that, instead of a static surface sensor network, one envisions a surface vehicle that, by moving along convenient trajectories, exploits its spatial diversity while measuring ranges to the underwater platform in order to determine the position of the latter. An early reference to this problem can be found in Been et al. [1991] where target motion analysis (TMA) with respect to an unknown marine platform using sonar measurements is discussed, i.e., the estimation of the position and velocity of a target ship, given a sequence of bearings measurements, is studied; see also Song [1996] where observability requirements are obtained for three-dimensional maneuvering target tracking with bearings-only measurements. Other previous results in this challenging area go back to the work of Passerieux & Capel [1998] where optimal control theory is used to determine the course of a constant speed observer by minimization of a criterion based of an appropriately defined FIM with a mixed analytical and numerical procedure. In Oshman & Davidson [1999] a fixed target location is estimated from a sequence of noisy bearings measurements, and the optimal trajectories for bearings-only target localization are based on the maximization of the determinant of a FIM subject to some constraints. Other interesting references on the subject are Fallon et al. [2010] that describes the experimental implementation of an online algorithm for cooperative localization of AUVs supported by an autonomous surface craft, or Arrichiello et al. [2011] where the problem of observability of the relative motion of two AUVs equipped with velocity and depth sensors, and inter-vehicle ranging devices, is studied. Finally, in Scherbatyuk & Dubrovin [2012] a number of algorithms to position an AUV based on range measurements obtained with a single acoustic sensor at the ocean surface are described. The single moving beacon uses an acoustic positioning system with long base line. The AUV position is computed using a Kalman Filter, and the algorithm for mobile beacon trajectory that minimizes the AUV positioning error is presented. Motivated by this circle of ideas, in this paper we seek to characterize the optimal trajectories that a single sensor must execute, in order to maximize the accuracy with which a target can be localized. From a practical standpoint, this will provide guidelines as to how one should operate in practical scenarios.

The key contributions of the present paper are twofold: i) two different approaches to determine the optimal sensor trajectories are studied; the first approach computes the next desired way point for the moving sensor to move to, the computation being repeated as the mission unfolds, while the second approach computes a complete trajectory off-line, ii) a general solution is obtained analytically and numerically for positioning of a static underwater target with the above approaches.

The paper is organized as follows. In Section 2 the optimal sensor trajectory problem is formulated and the assumptions underlying the computation of the optimal trajectories are established. Section 3 contains the derivation of optimal sensor trajectories for the two above different approaches. Simulation examples are included. Finally, Section 4 contains the conclusions.

2. PROBLEM FORMULATION

Consider the problem of estimating the position of an underwater target given a series of measurements of its range to a moving sensor, the position of which is known with good accuracy (target positioning problem). In an estimation theoretic context, the optimal sensor trajectory can be determined by examining an appropriately defined Cramer-Rao Lower Bound (CRLB) or Fisher Information Matrix (FIM), see Van Trees [2001]. Stated in simple terms, the FIM captures the amount of information that measured data provide about an unknown parameter (or vector of parameters) to be estimated. Under well known assumptions, the FIM is the inverse of the CRLB, which provides a lower bound for the covariance of the estimation error that can possibly be obtained with any unbiased estimator. Thus, "minimizing the CRLB" may yield (by proper estimator selection) a decrease of uncertainty in the parameter estimation process. In particular, the FIM determinant is used as an indicator of the performance that is achievable with a given sensor trajectory. Maximizing this quantity yields the most appropriate sensor movements.

Given a target positioning problem, the optimal sensor trajectory depends strongly on the constraints imposed by the task itself (e.g. maximum number of measurements used for the computation of the FIM and the type of sensor that can be used) and the environment (e.g. ambient noise). In what follows, we assume that the acoustic sensor used is installed on board an unmanned surface vessel (USV). As is well known, an inadequate sensor trajectory may yield large positioning errors. It is therefore of the utmost importance to define the constraints and assumptions considered for the problem at hand, stated next (see also Fig. 1).

- The variance of the range measurement error ω is constant and equal to σ.
- The USV must localize a static target whose position is estimated with a fixed number of measurements.
- The initial USV position is arbitrary because it should not condition the final optimal solution.
- For ranging purposes, the acoustic signals are emitted at constant intervals of time Δt and there exists a delay between the emission by the pinger on board the USV and the reply from the target.
- The sensor moves with constant speed $V(t) = V$.

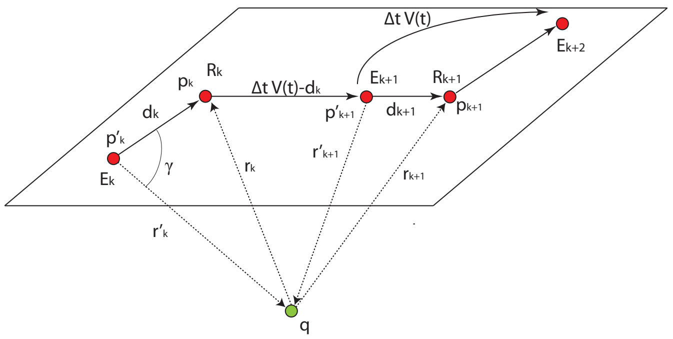

Fig. 1. Problem Setup: the acoustic signal is emitted at points E_k and received by the sensor at points R_k.

Notice how the sensor (red) emits the acoustic signal at time E_k and the reply from the target (green) is received by the sensor at time R_k, with d_k being the distance between the two above points. Clearly, d_k depends on the velocity of sound in water, the sensor speed V, and the range distances r'_k and r_k associated with the "interrogate and reply" time travel of the acoustic signal. The emission point E_k defines the point p'_k, the reception point R_k defines the $k-th$ measurement point p_k, and the range distance measured for the FIM computation is considered to be r_k, i.e., the distance between the target position q and the point p_k, at the moment of the reception of the acoustic signal. In this theoretical framework it is considered that r'_k and r_k, and therefore p'_k and p_k, are known, so we can define analytically the distance d_k that separates the emission and reception points. Let c_s be the speed of sound in the water. Then,

$$d_k/V = r'_k/c_s + r_k/c_s \tag{1}$$

Moreover, if γ is the angle defined by r'_k and d_k, from the theorem of the cosines it follows that, $r_k^2 = {r'}_k^2 + d_k^2 + 2d_k r'_k \cos(\gamma)$ with

$$\gamma = \arccos\left(\frac{\langle (q-p'_k)\,(p'_k - p_{k-1})\rangle}{r'_k \cdot (\Delta t V(t) - d_{k-1})}\right) \tag{2}$$

where $<>$ denotes the inner product between its operands, see Fig. 1. It follows from (1) that

$$d_k/V - r'_k/c_s = \sqrt{{r'}_k^2 + d_k^2 + 2d_k r'_k \cos(\gamma)}/c_s \tag{3}$$

Now taking the square of both sides and rewriting the equation yields

$$d_k = \frac{2r'_k}{c_s}\left(\frac{\cos(\gamma)}{c_s} - \frac{1}{V}\right)\left(\frac{1}{c_s^2} - \frac{1}{V^2}\right)^{-1} \tag{4}$$

so that the measurement points may be explicitly defined considering only the orientation angles α_i that the surface sensor takes at the R_k points, and the past known trajectory information.

As is well known, the FIM associated with a classical estimation problem is defined as the expected value of the logarithm of the derivative of the maximum likelihood function, see Van Trees [2001]. In the present case, straightforward computations yield

$$FIM = \frac{1}{\sigma^2}\sum_{i=1}^{n}\begin{pmatrix} (u_{ix})^2 & (u_{iy})\,(u_{ix}) & (u_{iz})\,(u_{ix}) \\ (u_{ix})\,(u_{iy}) & (u_{iy})^2 & (u_{iz})\,(u_{iy}) \\ (u_{ix})\,(u_{iz}) & (u_{iy})\,(u_{iz}) & (u_{iz})^2 \end{pmatrix} \tag{5}$$

where $u_{ij} = \frac{\partial \|q_i - p_i\|}{\partial q_{i,j}}$, for $i \in \{1,...,n\}$ and $j \in \{x,y,z\}$, $p_i = R_i$, and q_i corresponds to the target position at the moment at which the measurement i is taken (for this particular case of study, the origin of the inertial coordinate frame). In this work the simpler situation where the position of the target is known is studied to characterize the types of optimal solutions. In a practical situation, the position of the target is only known with uncertainty. One can think of an iterative cycle where an initial estimate of the target position is used to compute the corresponding optimal trajectory. Once the mission unfolds the information acquired by the sensor can be used to refine the underwater target position, after which the cycle repeats itself. Clearly, having the means to generate, for an assumed position of the target, the corresponding trajectory is also advantageous in this case. See Moreno-Salinas et al. [2013] for a discussion of this circle of ideas in the case of positioning with multiple sensor.

3. OPTIMAL TRAJECTORY COMPUTATION

In this section we define the trajectory that a moving surface sensor must follow in order to maximize the accuracy with which a static underwater target can be localized by resorting to any unbiased estimator. The computation of the optimal trajectory is done using two different approaches: i) iteratively, by computing the immediate best next measurement point in the current sensor trajectory (to update the FIM), after eliminating the oldest one, and ii) using batch optimization to compute a complete trajectory, with a number of points equal to the number of range measurements available.

3.1 Next optimal range measurement algorithm

Once the mission is running and an initial estimation of the target position is available, possibly with a large error, it is necessary to determine the next measurement point, i. e., the direction in which the single tracker must move in order to maximize the FIM determinant. Suppose a given number of measurements have been taken and one wishes to determine the next point at which a new measurement should be made. For given values of sensor speed and sampling time, it is easy to derive the analytical expression that provides the next optimal point because the new FIM determinant will only have one unknown parameter, the new angle α_{k+1}, that defines the sensor movement direction. As mentioned above, the single tracker computes the FIM with a given number of measurements, therefore it is necessary to delete the oldest one in order to update the FIM. Since the sensor speed V and the sampling time Δt are known, the new measurement point yields $p_{k+1} = p_k + [\xi \cos(\alpha_{k+1}); \xi \sin(\alpha_{k+1}),\ q_z]$, with $\xi = (V\Delta t - d_k + d_{k+1})$; and d_k, d_{k+1} defined as in Section 2.

The derivative of the FIM determinant with respect to the new direction angle α_{k+1} can now be computed. We consider that FIM_k^* is the FIM computed with the current n known range measurements except the oldest one, and FIM_{k+1} is the updated FIM that has been computed with the new range measurement obtained from a point to be defined. From the above, the new (unknown) FIM is given by $FIM_{k+1} = FIM_k^* + FIM'_{k+1}$, where FIM'_{k+1} is a 3x3 symmetrical matrix with elements $FIM'_{k+1}(i,j) = \frac{p_{k+1,i} \cdot p_{k+1,j}}{r^2_{k+1}}$, with $i,j = x,y,z$. Thus, the problem to solve can be defined as that of finding

$$\alpha^*_{k+1} = \arg\max_{\alpha_{k+1}} |FIM_{k+1}| \tag{6}$$

The derivative of (6) with respect to α_{k+1} can now be computed by decomposing the above determinant in terms of its adjoints, that is,

$$\frac{\partial |FIM_{k+1}|}{\partial \alpha_{k+1}} = \sum_{i,j}^{n} (-1)^{i+j} |Adj_{i,j}(FIM_{k+1})| \cdot \Theta(i,j) \tag{7}$$

where $\Theta(i,j) = \frac{\partial FIM_{k+1}(i,j)}{\partial \alpha_{k+1}}$ and $|Adj_{i,j}(FIM_{k+1})|$ is the determinant of the adjoint matrix of FIM_{k+1} with respect to the element (i,j). The derivatives of each FIM_{k+1} element with respect to α_{k+1} are actually the derivatives of each element of the matrix FIM'_{k+1} with respect to α_{k+1}. The details are omitted due to space limitations.

Equating (7) to 0 yields the angle that maximizes the determinant of FIM_{k+1}. It can be easily seen that even though (7) depends only on α_{k+1} and an analytical solution may be defined from this equation, the computation of the optimal solution is not immediate. In a practical situation, the optimal value of α_{k+1} can be obtained by using the gradient of the FIM determinant, i.e., by using (7) at the current sensor position to define the desired sensors direction of motion. In fact, the solution that the gradient provides is very close to the analytical one.

3.2 Optimal trajectory algorithm

In this approach, we now determine the optimal trajectory to be followed by the sensor so that the next n range measurements maximize the positioning accuracy of the underwater target. Therefore, in contrast to the previous approach, the whole trajectory of n points is optimized and a new target position estimate is obtained at each $n \cdot \Delta t$ seconds. The same assumptions about the sensor speed V, sampling time Δt, target q, and noise ω still hold for the scenario at hand, the only difference being that the optimization procedure deals with n range measurements to be computed, not just one.

The solution may be computed analytically from the derivatives of the FIM determinant with respect to the angles α_i, $i = 2, \cdots, n$, that determine the distance and relative orientation of two consecutive measurements. It is clear, by considering that the initial sensor position is known, that we have $n-1$ variables α_i , $i = 2, \cdots, n$, and $n-1$ derivatives with respect to these angles α_i, so that a system of equations with the same number of equations and unknowns is obtained. The complexity of this approach resides in the fact that the process to obtain the solution of this system of equations is complex and tedious. Moreover, we must resort to numerical methods to solve it. Therefore, these derivatives are used in a gradient optimization algorithm. Following the methodology adopted in the previous section, the derivatives can be computed as follows:

$$\frac{\partial |FIM|}{\partial \alpha_i} = \sum_{j,k}^{3} (-1)^{j+k} |Adj_{j,k}(FIM)| \cdot \frac{\partial FIM(j,k)}{\partial \alpha_i} \tag{8}$$

where $|Adj_{j,k}(FIM)|$ is the determinant of the adjoint matrix of the FIM with respect to the element (j,k). The details are omitted. The optimal solutions are obtained using a gradient optimization algorithm with the Armijo rule. As will become clear in the forthcoming examples, it is interesting to notice that this approach provides optimal trajectories very similar to those obtained with the algorithm of Section 3.1. The difference lies in that, for the approach at hand, the optimal trajectories are defined with a far less number of iterations of the algorithm, and therefore, in a practical situation, the optimal trajectory will be reached faster. At this point it is interesting to comment that if the values of V, Δt, and n are the optimal ones for the target depth so that the maximum theoretical FIM determinant can be obtained, the solution defined in Moreno-Salinas et al. [2011] for surface sensor networks is recovered.

3.3 Simulation Examples

We now present some examples of optimal sensor trajectory computations. For comparison purposes, the two algorithms described will be computed for each example. We consider that the optimal trajectories are computed for $n = 5$ range measurements but the procedure is similar for any number of measurements. The initial sensor position is $p_1 = [170, 170, 200]^T$ m and the target is placed at the origin of the inertial coordinate frame. For the *next optimal measurement* algorithm the simulation is run until the optimal trajectory is reached. The algorithm *optimal trajectory* is recursively executed 30 times, therefore 120 points are computed. This algorithm needs less iterations to compute the optimal trajectory, and the latter is reached with less measurement points; however, the computation of the solution is more complex. For each iteration of the latter algorithm, the first point in the new trajectory is the last in the old one.

Example 1: In this example, $V = 3$ m/s and $\Delta t = 3$ s. In Figure 2(a) the trajectory followed by the sensor is shown for the *next optimal measurement* algorithm. The sensor describes a spiral while it approaches the target position. In the lower right corner of Figure 2(a) the last 100 points of the simulation that correspond to a limit optimal trajectory are shown, i.e., the trajectory that the sensor will keep repeating if the simulation continues, because this trajectory provides the largest accuracy possible for the approach adopted. The values of V and Δt determine the number of points (or equivalently, time) needed to reach the optimal trajectory, as will be seen in the next example. The optimal trajectory is a circumference around the target projection in the horizontal plane, whose radius depends directly on the sensor speed V, the sampling time Δt, and the number of points n used for the computation of the FIM. In fact, although this is not shown in this work due to space limitations, the radius of the optimal trajectory grows with the number of measurements used for the computation of the FIM. The FIM determinant computed at each iteration of the algorithm grows from an initial value around $20m^{-6}$ until the final constant value $220m^{-6}$, because the sensor describes trajectories closer to the optimal one. After 7000 iterations the FIM determinant has a constant value meaning that the sensor has converged to the optimal trajectory.

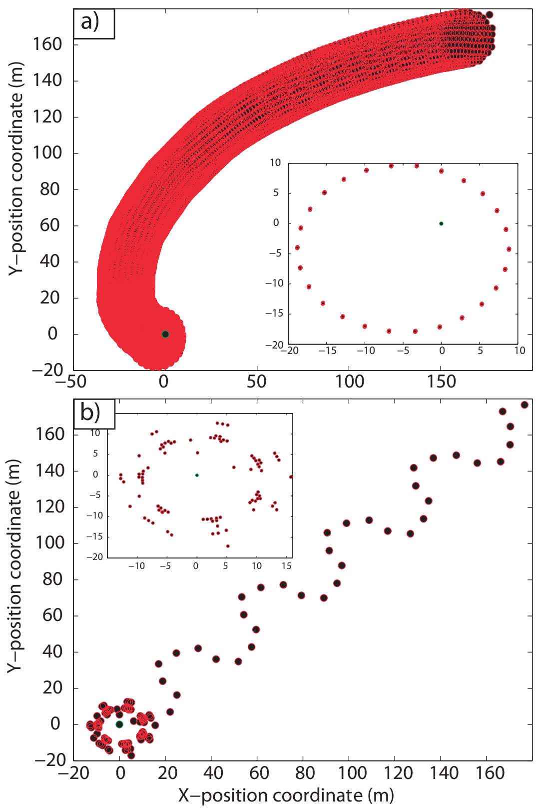

Fig. 2. Limit optimal trajectories for $V = 3m/s$ and $\Delta t = 3s$; (a) *next best measurement* algorithm, (b) *optimal trajectory* algorithm.

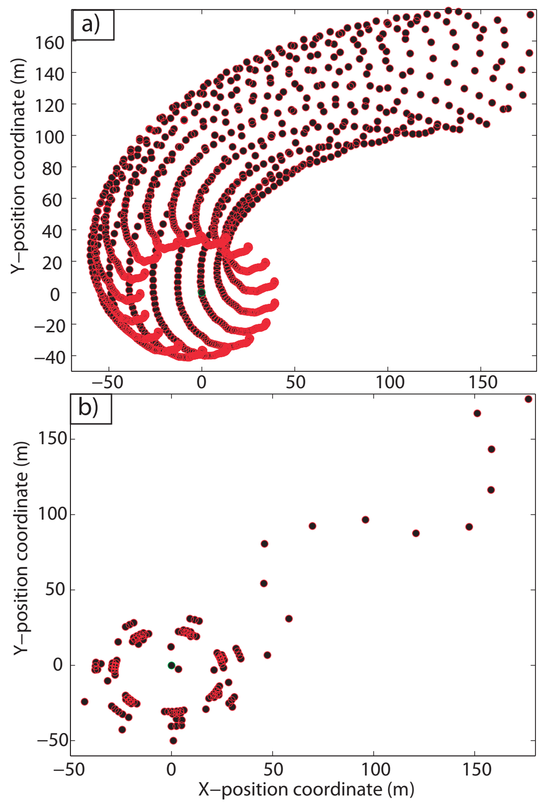

Fig. 3. Limit optimal trajectories for $V = 5m/s$ and $\Delta t = 5s$; (a) *next best measurement* algorithm, (b) *optimal trajectory* algorithm.

In Figure 2(b) the trajectory followed by the sensor is shown for the *optimal trajectory* algorithm. In this case the final trajectory is reached faster compared with the previous case. Notice how the optimal trajectory is not exactly a circumference, the optimal measurement points are concentrated in two concentric circumferences around the target projection, and the sensor moves between them in the optimal trajectory. However, the size of them is very close to the size of the circumference of Figure 2(a). In the left upper corner of Figure 2(b) the optimal trajectory is shown for the last 80 measurement points. The FIM determinant computed for the 30 iterations of the algorithm grows from $20m^{-6}$ until $220m^{-6}$. It can be seen that the maximum FIM determinant obtained is the same, but it is obtained with less iterations, and in a practical situation, the optimal trajectory will be reached faster.

Example 2: In this example, $V = 5\ m/s$ and $\Delta t = 5\ s$. The values of V and Δt are larger, and therefore, the optimal trajectories are reached in less iterations than in the previous example. Figure 3(a) shows the trajectory followed by the sensor for the *next optimal measurement* algorithm. In this case, the optimal trajectory is reached in less iterations than in the case shown in Fig. 2(a), that is, in approximately 1000 iterations. Again, notice how this final trajectory is a circumference around the target projection in the horizontal plane, but the circumference has now a radius of approximately 35 meters, in contrast to 15 meters in the previous example. Thus, with larger values of V and Δt the final trajectory follows a larger circumference and is computed with less iterations of the algorithm. The FIM determinant computed during the simulation grows because the sensor describes trajectories closer to the optimal one. In this example, the FIM determinant obtained is $10,000m^{-6}$, that is, approximately 50 times larger than in the previous example, thus showing that it is adequate that V and Δt be large enough so that the optimal trajectory can be reached with a lower number of iterations (for the algorithm considered) while yielding better positioning accuracy. Of course, the selection of V, Δt and n will be mission dependent.

Figure 3(b) shows the trajectory followed by the sensor for the *optimal trajectory* algorithm. Notice that the final trajectory is not a circumference, and the sensor moves between 2 circumferences. This optimal trajectory is computed in less iterations than in the example of Figure 3(a), so in a practical scenario the maximum accuracy would be obtained faster and with less iterations of the optimization algorithm. Compared to the case shown in Fig. 2(b), the final trajectory defines arcs of circumferences of larger radius, similarly to what happened with the Fig. 3(a) case and its dual case in Fig. 2(a). The optimal FIM determinant is also larger, $10,500m^{-6}$, therefore for larger values of V and Δt the determinant of the FIM grows as well. Moreover, the maximum FIM determinant is obtained in less iterations of the algorithm. Again the accuracy is similar to that obtained in the case of Figure 3(a), but the optimal trajectory is computed with a very significant less number of iterations and measurement points.

Therefore, for a static target, although both approaches provide the same maximum FIM determinant and, thus, the same positioning accuracy for similar mission constraints, the latter approach allows for the computation of the optimal trajectory in less iterations. As a consequence, the optimal trajectory is reached with few sensor movements. Still, this algorithm is more complex to implement and the computation of the optimal solution may take more time than in the first approach, whose implementation is quite easier and faster.

4. CONCLUSIONS

The paper addressed the problem of a single static underwater target positioning using a single acoustic range measuring device at the surface. The analysis of optimal sensor trajectories exploited the spatial and temporal diversity of the measurements taken by the surface sensor. Two different approaches for the computation of the optimal trajectories were studied considering a fixed number n of range measurements. The first approach involves the computation of the next measurement point that maximizes the current FIM determinant. The second approach optimizes the complete trajectory for the number of range measurements considered. Thus, in contrast with the previous approach, the sensor trajectory is computed each time n measurements have been accumulated, instead of recomputing it after a new range measurement has been acquired. The examples showed that for a static target both approaches provide similar accuracies and optimal trajectories. Future work will address the challenging problem of positioning single and multiple moving underwater targets, as well as implementing and testing the efficacy of the algorithms with real vehicles at sea.

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