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DECENTRALIZING FORMATIONS OF MULTI-AGENT SYSTEMS

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Abstract

Formation control and coordination of a group of agents is now an active area of research. While most researchers address the problem of controlling a set of agents to a given formation, no satisfactory results exist regarding the interplay between the formation configuration, its implementation at the level of feedback controllers and the required exchange of information between the agents. We present a methodology that allows to decide when a given formation specification is implementable by local controllers, that is, controllers using only partial state information. The existence of such decentralized implementation is clearly useful as it simplifies control design and reduces communication needs between the agents.

1 Introduction

Control and coordination of multi-agent and unmanned systems is a rapidly growing area of research. The thrust comes from potential applications (see for example [9] and the references therein) as well as the need to perform complex missions with simple and inexpensive systems. One of the important problems if this area is that of controlling the individual agents to a specific formation. Many different approaches have been reported for this problem [2, 4, 8, 10, 3, 7, 6] but in all of them it is assumed that the formation is given. However there is a strong coupling between the formation topology, its implementability at the level of feedback controllers and the necessary information exchange. Attempts to understand these interactions have not yet appeared in the literature, except for [5], where it is shown that for autonomous formation flight, redundancy requirements in communication channels, impose constraints at the level of the formation topology.

As the number of agents increase, the interaction between inter-agent communication, formation topology and its feedback implementation also increase in complexity. It is therefore necessary to develop a systematic way of determining when can the constrains be implementable by feedback control laws requiring only local information. This leads to simpler control laws and to a reduction in the communication flow between the agents. Furthermore, it is also important to determine if the agents need to coordinate their actions to enforce the formation constraints or if each agents can become responsible for enforcing a constraint, independently of the other agents actions. These two problems are formally addressed in this paper and computationally simple solutions are proposed. We also model the sensing capabilities of each agent explicitly and determine when the previous problem has a solution under the agents sensing constraints.

2 Formation Graphs

In this section we introduce the notion of formation graphs. We assume that the reader is familiar with several differential geometric concepts at the level of [1].

Given n heterogeneous agents, we denote the state of agent i by x_i ranging in manifold M_i. Each agent is modeled by a control system describing its kinematics:

$$\dot{x}_i = \sum_l X_l(x_i) u_l \tag{2.1}$$

where u_l are control inputs and X_l vector fields in TM_i. Note that agent i is allowed to move in all directions that can be written as linear combinations (with smooth real valued maps as scalars) of the vector fields X_l, therefore all possible directions of motion are captured by the distribution $\mathcal{D}_i \subset TM_i$ given by the span of all vector fields X_l. Distribution $\mathcal{D}_i$ can also be modeled by a dual object, namely $\mathcal{K}_i$, the annihilating codistribution of $\mathcal{D}_i$ defined as:

$$\mathcal{K}_i = \{\alpha \in T^*M \ : \ \alpha(X) = 0 \ \text{ for every } X \in \mathcal{D}_i\} \tag{2.2}$$

We now combine each agent kinematics with inter-agent constraints in a formation graph:

Definition 2.1 (Formation Graph) *A formation graph $F = (V, E, C)$ consists of:*

- *A finite set V of n vertices, where n is the number of agents in the formation. Each vertex v_i is a codistribution modeling agent i kinematics, as described in (2.2).*
- *A binary relation $E \subseteq V \times V$ representing a bond or link between the agents.*
- *A family of constraints C indexed by the set E, $C = \{c_e\}_{e \in E}$. For each edge $e = (v_i, v_j)$, c_e is a function $c(x_i, x_j)$ defining the formation constraints between agents i and j. The constraint is enforced when $c(x_i, x_j) = 0$.*

We shall consider directed graphs where an edge $e = (v_i, v_j)$ represents an edge from v_i to v_j. Such an edge, denotes a constraint between agent i and j that must be enforced by agent i, independently of agent j actions. Clearly, it is not possible to decide a priori, how to direct the arrows in a formation. The initial specification will be given by two arrows with opposite direction between agents, symbolizing the fact that the constraint responsibilities have not been assigned. These arrows represent an undirected edge which will be represented graphically by a straight line.

In this paper we will solve the problem of determining if a given constraint between two agents can be realized by a directed edge, thereby decentralizing the feedback implementation of that constraint. More specifically we will solve:

Problem 2.2 *Given a formation graph F, determine if it is possible to replace a pair of edges (v_i, v_j) and (v_j, v_i) by a single one.*

Once we can determine if a given constraint is decentralizable, we can analyze all the formation by recursively applying the solution of Problem 2.2. When each agent has limited sensory information, there are additional constraints that must be taken into account. We thus solve the following version of Problem 2.2:

Problem 2.3 *Given a formation graph F, determine if it is possible to replace an undirected edges (v_i, v_j) by a directed one, while respecting the observation capabilities of agents i and j.*

3 Decentralization with full observations

To simplify the exposition we start by considering a formation with two agents as displayed in Figure 1. Our goal is to determine if it is possible for agent i to be the single responsible for constraint c_e. To formalize this problem, we denote by $\mathcal{D}_{ij}^{c_e}$ the set of vector fields in $TM_i \times TM_j$ representing directions of motion satisfying the kinematics of each agent, as well as the formation constraint

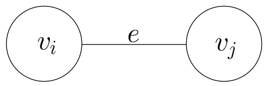

Figure 1: Formation constituted by two agents and a single edge.

c_e. We also introduce the set $0 \times TM_j$, representing the tangent space of M_j, but regarded as a subspace of $TM_i \times TM_j$, that is, a vector field X in $0 \times TM_j$ has coordinates $X = [0 \;\; X_j]^T$ for $X_j \in TM_j$. The kinematics of agent j, defined by the distribution $\mathcal{D}_j$ is also regarded as a subspace of $TM_i \times TM_j$ and is denoted by $0 \times \mathcal{D}_j$. The question we are trying to solve can now be put in the following terms: is the set $\mathcal{D}_{ij}^{c_e}$ such that its projection on $0 \times TM_j$ is $0 \times \mathcal{D}_j$? If this is the case, then agent i can be assigned the responsibility of the constraint, as for any direction of motion $X_j \in \mathcal{D}_j$ for agent j, there exists a direction of motion $[X_i \;\; X_j]^T \in \mathcal{D}_{ij}^{c_e}$ projecting on X_j. The vector field $[X_i \;\; X_j]^T$ provides direction of motion X_i for agent i allowing him to fulfill the constraint imposed by c_e. Unfortunately, the computation of the set $\mathcal{D}_{ij}^{c_e}$ is something too expensive to be performed for each edge. We thus seek a simple test to the above question. For this we review from [11] how to model all formation constraints in a single object Ω. Since $\mathcal{K}_i$ and $\mathcal{K}_j$ model the kinematics of agents i and j, on the product of the state spaces $M_i \times M_j$ we can construct another codistribution $\mathcal{K}_{ij}$ capturing both $\mathcal{K}_i$ and $\mathcal{K}_j$. We denote the natural projections from $M_i \times M_j$ to M_i and M_j by π_i and π_j, respectively, and define:

$$\mathcal{K}_{ij} = \pi_i^* \mathcal{K}_i + \pi_j^* \mathcal{K}_j$$

If we regard $\mathcal{K}_i$ and $\mathcal{K}_j$ as matrices, where each line represents the coefficients of one-forms in $\mathcal{K}_i$ and $\mathcal{K}_j$, respectively, the construction of $\mathcal{K}_{ij}$ simply amounts to constructing the matrix:

$$\begin{bmatrix} \mathcal{K}_i & \mathbf{0} \\ \mathbf{0} & \mathcal{K}_j \end{bmatrix}$$

where $\mathbf{0}$ denotes a matrix of zeros of appropriate dimension. To obtain Ω we need to capture $\mathcal{K}_{ij}$ and also the constraints defined by c_e. A direction of motion for agents i and j, represented by vector field $Y \in \mathcal{K}_{ij}$, satisfies the formation constraints c_e if the trajectories of the agents satisfy:

$$c_e(x_i(t), x_j(t)) = 0$$

for all t. Time differentiation of the previous expression provides $L_Y c_e = 0 = \mathrm{d}c_e(Y)$, where $L_Y c_e$ is the Lie derivative of c_e along Y and d is the exterior derivative[1] of c_e (which is simply the gradient regarded as a row vector). We thus see that the formation constraints can be modeled by the one-form $\mathrm{d}c_e$ which combined with $\mathcal{K}_{ij}$ will give:

$$\Omega = \mathcal{K}_{ij} + \mathrm{d}c_e$$

In coordinates, Ω is the matrix obtained from the matrix of $\mathcal{K}_{ij}$ by adding the additional lines representing $\mathrm{d}c_e$.

We now return to the solution of the decentralization problem with a preparatory lemma:

Lemma 3.1 *Let F be a formation graph modeling a formation with two agents (i and j) and a single constraint c_e associated with the undirected edge $e = (v_i, v_j)$. Then $\pi_{0 \times TM_j}(\mathcal{D}_{ij}^{c_e}) = 0 \times \mathcal{D}_j$ iff the exists a map $f : 0 \times TM_j \longrightarrow TM_i \times TM_j$ such that $\pi_{0 \times TM_j} \circ f = id_{0 \times TM_j}$ and $f^* \Omega \subseteq \mathcal{K}_j$, where $\pi_{0 \times TM_j}$ the natural projection from $TM_i \times TM_j$ to $0 \times TM_j$.*

Proof: Assume that $\pi_{0 \times TM_j}(\mathcal{D}_{ij}^{c_e}) = 0 \times \mathcal{D}_j$. Then for every $X_j \in 0 \times \mathcal{D}_j$, there exists a $Y \in \mathcal{D}_{ij}^{c_e}$ such that $\pi_{0 \times TM_j}(Y) = X_j$. Let $\{X_j^1, X_j^2, \ldots, X_j^n\}$ be a basis for $0 \times \mathcal{D}_j$ and let $\{Y^1, Y^2, \ldots, Y^n\}$ be a set of linearly independent vector fields in $\mathcal{D}_{ij}^{c_e}$ such that $\pi_{0 \times TM_j}(Y^l) = X_j^l$ for $l = 1, 2, \ldots, n$. We then define f on the basis of $0 \times \mathcal{D}_j$ by $f(X_j^l) = Y^l$ which extends uniquely to a pointwise linear map from $0 \times TM_j$ to $TM_i \times TM_j$. We now show that f has the desired properties. Computing $\pi_{0 \times M_j} \circ f(X_j)$ we obtain $\pi_{0 \times TM_j}(Y) = X_j$, by definition of f, which shows $\pi_{0 \times TM_j} \circ f = id_{0 \times TM_j}$. Let now $\omega \in \Omega$. We want to show that $f^* \omega \in \mathcal{K}_j$,

[1] In case c_e is not real valued, $\mathrm{d}c_e$ is interpreted as a vector valued one-form.

that is $(f^*\omega)(X_j) = 0$ for every $X_j \in 0 \times \mathcal{D}_j$. Computing $(f^*\omega)(X_j)$ we get $\omega(f(X_j)) = \omega(Y)$ and since $Y \in \mathcal{D}_{ij}^{c_e}$ we conclude that $\omega(Y) = 0$, that is $f^*\omega \in \mathcal{K}_j$.

For the converse assume the existence of the map f. Since every vector Y in $\mathcal{D}_{ij}^{c_e}$ respects the kinematics of both agents, we necessarily have $\pi_{0\times TM_j}(Y) \in 0 \times \mathcal{D}_j$ which shows $\pi_{0\times TM_j}(\mathcal{D}_{ij}^{c_e}) \subseteq 0 \times \mathcal{D}_j$. We now show the reverse inclusion. Let $X_j \in 0 \times \mathcal{D}_j$, since $f(X_j) \in \mathcal{D}_{ij}^{c_e}$ we have:

$$\begin{aligned} \pi_{0\times TM_j} \circ f(X_j) &\in \pi_{0\times TM_j}(\mathcal{D}_{ij}^{c_e}) \\ \Leftrightarrow X_j &\in \pi_{0\times TM_j}(\mathcal{D}_{ij}^{c_e}) \end{aligned} \tag{3.1}$$

which shows $0 \times \mathcal{D}_j \subseteq \pi_{0\times TM_j}(\mathcal{D}_{ij}^{c_e})$ and allows to conclude $\pi_{0\times TM_j}(\mathcal{D}_{ij}^{c_e}) = 0 \times \mathcal{D}_j$ as desired. □

To state our main result in a concise way, we need some additional notation. Given a one-form $\omega \in \Omega$, which we may write as $\omega = a(x_i, x_j)\mathrm{d}x_i + b(x_i, x_j)\mathrm{d}x_j$ for smooth real valued maps a and b, we denote by $\omega|_i$ the form $a(x_i, x_j)\mathrm{d}x_i$ and similarly by $\omega|_j$ the form $b(x_i, x_j)\mathrm{d}x_j$. Decentralization can now be characterized by:

Theorem 3.2 *Let F be a formation graph modeling a formation with two agents (i and j) and a single constraint c_e associated with the undirected edge $e = (v_i, v_j)$. The constraint c_e can be realized as a directed edge from v_i to v_j (agent i has the responsibility of enforcing the constraint) iff the following condition holds for any $\omega \in \Omega$, smooth real valued maps r_l and one-forms $\alpha_l \in \Omega$, $\alpha_l \neq \omega$, $l = 1, 2, \ldots, k$:*

$$\text{if } \omega|_i = \sum_{l=1}^{k} r_l \alpha_l|_i \text{ then } \omega - \sum_{l=1}^{k} r_l \alpha_l \in \mathcal{K}_j \tag{3.2}$$

Proof: Assume that decentralization is possible. Then by Lemma 3.1 we know that there exists a map $f : 0 \times TM_j \to TM_i \times TM_j$ such that $\pi_{0\times TM_j} \circ f = id_{0\times TM_j}$ and $f^*\Omega \subseteq \mathcal{K}_j$. We start by showing that $f^*\beta$, $\beta \in \Omega$ admits a simple form.

$$\begin{aligned} f^*(\beta) &= f^*(\beta|_i + \beta|_j) \\ &= f^*(\beta|_i) + f^*(\beta|_j) \\ &= f^*(\beta|_i) + f^*\pi^*_{0\times TM_j}(\beta|_j) \\ &= f^*(\beta|_i) + (\pi_{0\times TM_j} \circ f)^*(\beta|_j) \\ &= f^*(\beta|_i) + (id_{0\times TM_j})^*(\beta|_j) \\ &= f^*(\beta|_i) + \beta|_j \end{aligned} \tag{3.3}$$

Let now $\omega \in \Omega$ and $\alpha_l \in \Omega$ for $l = 1, 2, \ldots, k$. Since $f^*\omega \in \mathcal{K}_j$ and $f^*\alpha_l \in \mathcal{K}_j$ it follows by linearity that $f^*(\omega - \sum_{l=1}^k r_l\alpha_l) \in \mathcal{K}_j$. However by (3.3) $f^*(\omega - \sum_{l=1}^k r_l\alpha_l) = f^*(\omega|_i - \sum_{l=1}^k r_l\alpha_l|_i) + \omega|_j - \sum_{l=1}^k r_l\alpha_l|_j$ and since $\omega - \sum_{l=1}^k r_l\alpha_l = 0$, by assumption, we have $f^*(\omega - \sum_{l=1}^k r_l\alpha_l) = \omega|_j - \sum_{l=1}^k r_l\alpha_l|_j \in \mathcal{K}_j$ as desired.

For the converse, assume that the condition in the Theorem holds. We will construct a map f with the conditions specified by Lemma 3.1. Instead of defining f we will equivalently define f^* and in virtue of (3.3) it suffices to define f^* on $\beta|_i$ to define it for $\beta \in \Omega$. Let $\{\omega_1, \omega_2, \ldots, \omega_p\}$ be a basis for Ω. Then we define $f^*\omega_1|_i$ to be $-\omega_1|_j$ and it follows by (3.3) that $f^*\omega_1 = -\omega_1|_j + \omega_1|_j = 0 \in \mathcal{K}_j$. For ω_n, with $n = 2, 3, \ldots, p$ we define $f^*\omega_n|_i$ to be $-\omega_n|_j$ if $\omega_n|_i$ is linearly independent of the one-forms $\omega_1|_i, \omega_2|_i, \ldots, \omega_{n-1}|_i$, otherwise $f^*\omega_n|_i$ is already defined to be $f^*(\sum_l r_l\omega_l|_i)$. As f^* was defined on a basis of Ω it extends uniquely by linearity to all of Ω. Let us now show that f has the desired properties. The equality $\pi_{0\times TM_j} \circ f = id_{0\times TM_j}$ holds by construction so that we only have to show that $f^*\Omega \subseteq \mathcal{K}_j$. For ω_1 it holds by definition of f^* and for ω_n we have $f^*\omega_n \in \mathcal{K}_j$ if $\omega_n|_i$ is linearly independent of the forms $\omega_1|_i, \omega_2|_i, \ldots, \omega_{n-1}|_i$. Otherwise we have:

$$\begin{aligned} f^*\omega_n &= f^*\omega_n|_i + \omega_n|_j \\ &= f^*(\sum_l r_l\omega_l|_i) + \omega_n|_j \\ &= -\sum_l r_l\omega_l|_j + \omega_n|_j \\ &= -\sum_l r_l\omega_l + \omega_n \in \mathcal{K}_j \end{aligned} \tag{3.4}$$

where the last equality holds in virtue of assumption (3.2). □

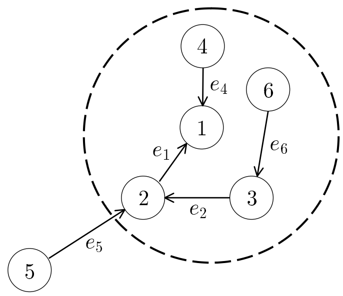

Figure 2: Formation where agents within larger circle are considered as a macro-agent.

We have thus solved Problem 2.2. For more details on how to use the previous result in real applications we urge the reader to consult Section 5 where we present several examples. We also note that condition 3.2 is simple to check in applications since it amounts to determine linear independence of one-forms and this can be efficiently done in a symbolic algebra computer package.

3.1 From two agents to acyclic formations

We now briefly describe how Theorem 3.2 can be applied to acyclic formations containing more then two agents. Given one such formation, to analyze a specific undirected edge $e = (v_a, v_b)$, one regards all the agents to which there exists a path in the formation graph from v_a passing trough edge e, as leaders of agent a. Furthermore, the leaders and the formation constraints between then, can be regarded as a macro-agent, see for example Figure 2. This means that in this context, the codistribution:

$$\sum_{i \in I} \pi_i^* \mathcal{K}_i + \sum_{e' \in E'} \mathrm{d}c_{e'}$$

represents the kinematics of such a macro-agent on the state space:

$$M = \prod_{i \in I} M_i$$

where I is the set of leaders and E' the set of edges between them. This reformulation allows to cast any acyclic formation in the setup required to apply Theorem 3.2 by considering the formation with the macro agent and agent a and the single edge e between then. Note that if one wants to determine if *several* undirected edges linking agent a to other agents can be decentralized, one has to consider as leaders, the union of the leader obtained by each edge as a macro-agent, as well as to consider the several edges as a single constraint between the two agents.

4 Decentralization with partial observations

When the formation agents only have restricted access to state information, additional constraints must be taken into account to determine how the edges can be implemented at the level of controllers. In this section we extend the results of the previous section to explicitly accommodate partial observations. Towards this objective, we equip each agent i with an observation function $h_i : \prod_{i \in I} M_i \longrightarrow O_i$. Here, $\prod_{i \in I} M_i$ represents the state space of all agents in the formation, and O_i the observation space. Given a constraint c_e, with $e = (v_i, v_j)$, agent i must be able to observe the constraint, in order to enforce it. By observing the constraint we mean:

Definition 4.1 *Let $e = (v_i, v_j)$ be an edge between agent i and j, and c_e the associated constraint. We say that constraint c_e is observable by agent i iff there exist a map $\overline{c_e} : O_i \longrightarrow \mathbb{R}^n$ making the following diagram commutative:*

$$\begin{array}{ccc} M_i \times M_j & \xrightarrow{\ c_e\ } & \mathbb{R}^n \\ {\scriptstyle h_i|_{M_i \times M_j}} \downarrow & \nearrow_{\overline{c_e}} & \\ O_i & & \end{array} \qquad (4.1)$$

or equivalently $c_e = \overline{c_e} \circ h_i|_{M_i \times M_j}$.

The previous definition formalizes the intuitive idea that the constraint is observable if agent i by making use of its observation space O_i can determine if the constraint is satisfied or not. This is the case when the constraint c_e can be defined on the observation space as $\overline{c_e}$, agreeing with the original constraint in the sense that $c_e = \overline{c_e} \circ h_i|_{M_i \times M_j}$. Observational constraints can be characterized as follows[2]:

Proposition 4.2 *Let $e = (v_i, v_j)$ be an edge between agent i and j, and c_e the associated constraint. Assume that c_e is a constant rank map and $h_i|_{M_i \times M_j}$ is a surjective, constant rank map, then constraint c_e is observable by agent i iff:*

$$Ker(Th_i|_{M_i \times M_j}) \subseteq Ker(Tc_e) \tag{4.2}$$

Proof: Assume that the constraint is observable, then there exists a map $\overline{c_e}$ such that $c_e = \overline{c_e} \circ h_i|_{M_i \times M_j}$. It follows by differentiation that $Tc_e = T\overline{c_e} \circ Th_i|_{M_i \times M_j}$ and if $X \in Ker(Th_i|_{M_i \times M_j})$, then:

$$\begin{aligned} Th_i|_{M_i \times M_j} \cdot X &= 0 \\ \Leftrightarrow T\overline{c_e} \circ Th_i|_{M_i \times M_j} \cdot X &= 0 \\ \Leftrightarrow Tc_e \cdot X &= 0 \end{aligned}$$

which shows that $X \in Ker(Tc_e)$, that is $Ker(Th_i|_{M_i \times M_j}) \subseteq Ker(Tc_e)$. To show the converse we note that since c_e and $h_i|_{M_i \times M_j}$ are constant rank maps their preimages $c_e^{-1}(a)$ and $h_i|_{M_i \times M_j}^{-1}(a)$ define submanifolds of $M_i \times M_j$ for each regular value $a \in \mathbb{R}^n$. Since $Ker(Tc_e)$ and $Ker(T_i|_{M_i \times M_j})$ are the tangent spaces to those manifolds, inclusion (4.2) implies that:

$$h_i|_{M_i \times M_j}^{-1}(a) \subseteq c_e^{-1}(a) \tag{4.3}$$

and this allows to define $\overline{c_e}$ by $\overline{c_e}(o) = c_e(o')$ for any $o' \in h_i|_{M_i \times M_j}^{-1}(o)$ which satisfies $c_e = \overline{c_e} \circ h_i|_{M_i \times M_j}$ in virtue of (4.3). □

The previous Proposition combined with Theorem 3.2 allows to give the following solution to Problem 2.3:

[2] We can regard O_i as the quotient manifold induced by the surjection $h_i|_{M_i \times M_j}$. In this case it suffices to ensure that $h_i|_{M_i \times M_j}(x_i, x_j) = h_i|_{M_i \times M_j}(x_i', x_j')$ implies $c_e(x_i, x_j) = c_e(x_i', c_j')$ to ensure the existence of the map $\overline{c_e}$. However this condition may be more difficult to check in concrete applications.

Theorem 4.3 *Let F be a formation graph modeling a formation with two agents (i and j) and a single constraint c_e associated with the single edge $e = (v_i, v_j)$. If agent i has partial observations defined by h_i the constraint c_e can be realized as a directed edge from v_i to v_j (agent i has the responsibility of enforcing the constraint) iff the following conditions hold for any $\omega \in \Omega$, any smooth real valued maps r_l, and any one-forms $\alpha_l \in \Omega$, $\alpha_l \neq \omega$ and $l = 1, 2, \ldots, k$:*

if $\omega|_i = \sum_{l=1}^{k} r_l \alpha_l|_i$ then $\omega - \sum_{l=1}^{k} r_l \alpha_l \in \mathcal{K}_j$

and $Ker(Th_i|_{M_i \times M_j}) \subseteq Ker(Tc_e)$.

The previous result can also be applied to formation with more then two agents, by considering macro-agents as described in the previous section.

5 Examples

In this section we present several examples to illustrate the results of the previous sections. Consider a unicycle type robot with kinematics described by:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ u \end{bmatrix}$$

where (x, y) represents the coordinates of the robot center of mass in same inertial frame, and θ represents the robot heading angle. As inputs, (u, v) represent linear and angular velocity, respectively. This kinematics model can equivalently be described as the kernel of the following one-form:

$$\sin\theta\,\mathrm{d}x - \cos\theta\,\mathrm{d}y \tag{5.1}$$

which, thus, defines the codistribution $\mathcal{K}_i$ of any agent i of unicycle type. Consider now the formation displayed on the left of Figure 3. We assume that all the agents have kinematics defined by codistributions of the form (5.1). The constraint associated to edge e_1 is given by $c_{e_1} = x_1 - x_3 - K$, where K is a positive scalar while the constraint associated to edge e_2 is given by $c_{e_2} = y_2 - y_3 - K$. To determine if edge e_1 can be decentralized one

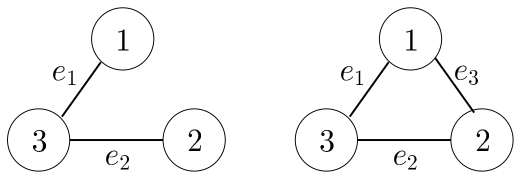

Figure 3: Tree agents in different formations.

computes the codistribution Ω which is given by:

$$\begin{aligned}\Omega &= \pi_1^* \mathcal{K}_1 + \pi_3^* \mathcal{K}_3 + \mathrm{d}c_{e_1} \\ \Omega &= span\{\omega_1, \omega_2, \omega_3\} \\ \omega_1 &= \sin\theta_1 \,\mathrm{d}x_1 - \cos\theta_1 \,\mathrm{d}y_1 \\ \omega_2 &= \sin\theta_3 \,\mathrm{d}x_3 - \cos\theta_3 \,\mathrm{d}y_3 \\ \omega_3 &= \mathrm{d}x_1 - \mathrm{d}x_3\end{aligned}$$

We can see by inspection, that $\omega_1|_3$, $\omega_2|_3$ and $\omega_3|_3$ are linearly independent. Theorem 3.2 now ensures that the edge is decentralizable. Similarly one can show that the edge e_2 is decentralizable. To determine if *both* edges are decentralizable, that is, if agent 3 has the capability of fullfiling the constraints associated with edges e_1 and e_2, one considers agent 1 and 2 as a macro-agent. In this case Ω is given by $\Omega = span\{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ where $\omega_1 = \sin\theta_1 \,\mathrm{d}x_1 - \cos\theta_1 \,\mathrm{d}y_1$, $\omega_2 = \sin\theta_2 \,\mathrm{d}x_2 - \cos\theta_2 \,\mathrm{d}y_2$, $\omega_3 = \sin\theta_3 \,\mathrm{d}x_3 - \cos\theta_3 \,\mathrm{d}y_3$, $\omega_4 = \mathrm{d}x_1 - \mathrm{d}x_3$ and $\omega_5 = \mathrm{d}y_2 - \mathrm{d}y_3$. Applying Theorem 3.2 one determines that $\omega_3|_3 = \cos\theta_3\omega_5|_3 - \sin\theta_3\omega_4|_3$ and $\omega_3 - \cos\theta_3\omega_5 + \sin\theta_3\omega_4 = -\cos\theta_3 \mathrm{d}y_2 + \sin\theta_3 \mathrm{d}x_1$ which cannot be written as a linear combination of ω_1 and ω_3. This implies, by Theorem 3.2, that edges e_1 and e_2 cannot be decentralized simultaneously. This agrees with our intuition, since if agents 1 and 2 move arbitrarily, then agent 3 cannot enforce both constraints.

Consider now the formation displayed in the right of Figure 3 that represents the previous situation with the added edge e_3 between agents 1 and 2 and the associated constraint:

$$c_{e_3} = \begin{bmatrix} x_1 - x_2 - K \\ y_1 - y_2 - K \end{bmatrix}$$

In this case we consider that agent 3 has only access to partial information modeled by the observation map $h_3(x_1, y_1, \theta_1, x_2, y_2, \theta_2, x_3, y_3, \theta_3) = (x_1, y_1, x_2, \theta_2, x_3, y_3)$. Although edges e_1 and e_2 can be simultaneously decentralized if one consider full information, the same is no longer true in this case since $Ker(Th_3|_{M\times M_3}) = Ker(Th_3)$ is not contained in $Ker(Tc_{e_1e_2})$, where we have denote by $c_{e_1e_2}$ the constraint induced by c_{e_1} and c_{e_2} between agent 3 and the macro-agent constituted by agents 1 and 2 on state space $M = M_1 \times M_2$. However, replacing constraint c_{e_2} by the constraint:

$$\begin{bmatrix} x_1 - x_3 - K \\ y_1 - y_3 - K \end{bmatrix}$$

renders the edges decentralizable and does not change the trajectories of the individual agents. This observation motivates another interesting problem: given a formation, determine new inter-agent constraints, that maximize decentralization and do not change the agents individual trajectories.

Consider now a formation between a mobile robot of unicycle type and an aircraft, as displayed in Figure 4. We consider that the aircraft is working in a degraded mode of operation due to a failure in the rudder. As such, its motion is restricted to a vertical plane where a unicycle type model is valid. These considerations lead to the codistribution $\mathcal{K}_1 = span\{\sin\theta_1 \mathrm{d}y_1 - \cos\theta_1 \mathrm{d}z_1, \mathrm{d}x_1\}$. We will consider that agent 2, the mobile robot, is of unicycle type an moves on a horizontal surface. This leads to the codistribution $\mathcal{K}_2 = span\{\sin\theta_2 \mathrm{d}y_2 - \cos\theta_2 \mathrm{d}x_2, \mathrm{d}z_2\}$. The two agents have to satisfy the formation constraint given by $c_e = (x_1 - x_2)^2 + (z_1 - z_2)^2 - K^2$ which forces then keep a fixed distance of K on the vertical plane defined by $y_2 = c$, $c \in \mathbb{R}$. We will first try to determine if the constraint can be implemented by a directed edge from agent 2 (the mobile robot) to agent 1 (the aircraft). Computing Ω one determines that if $\theta_2 = 0$ we have $(\mathrm{d}c_2)|_2 = -2(z_1 - z_2)(\mathrm{d}z_2)|_2 + 2(x_1 - x_2)(\sin\theta_2 \mathrm{d}y_2 - \cos\theta_2 \mathrm{d}x_2)|_2$. Theorem 3.2 now implies that we must have $\mathrm{d}c_2 + 2(z_1 - z_2)(\mathrm{d}z_2) - 2(x_1 - x_2)(\sin\theta_2 \mathrm{d}y_2 - \cos\theta_2 \mathrm{d}x_2) \in \mathcal{K}_1$ which holds

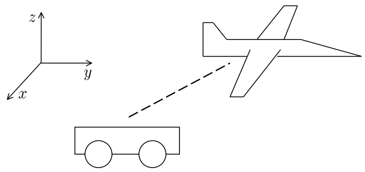

Figure 4: A mobile robot and an aircraft in formation.

only for $\theta_1 = \pi/2 + \upsilon\pi$, $\upsilon \in \mathbb{Z}$. We then see that the constraint is not decentralizable for all state space configurations of the agents. Trying to decentralize the constraint as an arrow from agent 1 to agent 2 produces similar results, which shows that this constraint requires coordination among the agents to be implemented. Since no decentralized solution is possible, complex coordination mechanisms are necessary as opposed to simple feedback control laws.

6 Conclusions

In this paper we began to explore the connections between formation topology, low level implementation and sensing requirements. We studied the problem of determining in which conditions it is possible to implement a given constraint between agents as a directed edge, thereby assigning the responsibilities of the constraint enforcement to a single agent. Simple sufficient and necessary conditions were given for edge decentralization. Furthermore we also solve this problem in the presence of partial observations. Many other interesting problems remain to be addressed, in particular the problem of determining new constraints allowing for a more decentralized implementation and yet providing similar trajectories for the individual agents.

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