EXPERIMENTAL EVALUATION OF SIMULTANEOUS 3D LOCALIZATION OF SENSOR NODES AND TRACKING MOVING TARGETS

Pinar Oguz Ekim, João Gomes, João Xavier, Paulo Oliveira

Institute for Systems and Robotics – Instituto Superior Técnico (ISR/IST), Technical University of Lisbon (UTL), Portugal
{*poguz, jpg, jxavier, pjcro*}*@isr.ist.utl.pt*

ABSTRACT

In this work we carry out an experimental performance characterization of a simultaneous localization and tracking (SLAT) algorithm for sensor networks, whose aim is to determine the positions of sensor nodes and a moving target in a network, given incomplete and inaccurate range measurements between the target and each of the sensors. To achieve this, we propose to iteratively maximize a likelihood function (ML) of positions given the observed ranges, which requires initialization with an approximate solution to avoid convergence towards local extrema. A modified Euclidean Distance Matrix (EDM) completion problem is solved for a block of target range measurements to approximately set up initial sensor/target positions, and the likelihood function is then iteratively refined through Majorization-Minimization (MM). To reduce the computational load, an incremental scheme is used whereby each new target or sensor position is estimated from range measurements, providing additional initialization for ML without the need for solving an expanded EDM completion problem. The proposed algorithms are experimentally evaluated with a series of 3D indoor tests for a range of operation of up to ten meters using a Crossbow Cricket location system and a robotic or human target. Centimetric accuracy is obtained under realistic conditions.

***Index Terms*—** Localization, tracking, sensor networks, Semidefinite Programming, real indoor experiments.

1. INTRODUCTION

This paper addresses the problem of tracking a moving device while localizing the sensor nodes of a Crossbow Cricket location system. Our motivating applications include human navigation and robotic navigation where the sensors do not have prior knowledge about their physical locations, particularly in an indoor environment where the Global Positioning System (GPS) does not work.

This research was partially supported by Fundação para a Ciência e a Tecnologia (FCT) through projects FCT [PEst-OE/EEI/LA0009/2011], PTDC/EEA-CRO/104243/2008, CMU-PT/SIA/0026/2009, and grant SFRH/BD/44771/2008.

There is a large and growing list of works related to location systems (mostly on target localization, but not on joint sensor/target localization) which rely on infrared (IR), radio frequency/received signal strength (RF/RSS), ultrawideband (UWB) and ultrasound (US) signals, etc. Many location systems rely on RSS, but these are impractical to use when indoor propagation conditions are complicated [1]. The overall performance of IR degrades under direct sunlight or high ambient temperature. On the other hand, these systems are appropriate for spaces in which other technologies do not perform properly [2]. The Active Bat [3] is based on ultrasonic pulses which rely on Time-of-flight measurements, requiring very accurate clock synchronization in the system. UWB based systems are an emerging indoor localization technology which provides an accuracy on the order of a few centimeters. However, they are still expensive [4]. In this paper, we employ a Cricket system, from Crossbow technologies, [5] which uses both ultrasound and RF signals to estimate the ranges between sensors and a target. These devices are inexpensive and easy to deploy; however, their operating range is limited, approximately ten meters.

This work experimentally evaluates the algorithms of [6, 7] for solving SLAT problems in real-world with several 3D indoor environments. The algorithms are based on plain ML estimation due to its asymptotically optimal performance. Since the ML estimator requires minimization of a nonconvex cost function, we propose a two-stage approach consisting of a *startup phase* whose main goal is to obtain an outline of the network configuration from a block of measurements (the term *batch* will be used for such a block from now on), followed by an *updating phase* where new target sightings are incrementally incorporated as the range measurements become available, while improving all previously determined locations. Each phase consists of an *initialization step* to calculate approximate locations, followed by an iterative *refinement step* of the likelihood function using MM. Local convergence to undesirable extrema in ML methods due to poor initialization is thus substantially alleviated.

The main contribution of this work are (i) the extension of the ML SLAT framework of [6] to 3D environments, while preserving its moderate complexity and modest prior knowledge, and (ii) experimental validation of the algorithms using a Cricket-based indoor positioning system.

In Section 2, the proposed method will be described. The practical indoor scenarios, their uncertainities and the accuracy of the algorithms are presented in Section 3. Finally, conclusions are rendered in Section 4.

2. PROBLEM FORMULATION

The network comprises sensors at unknown positions $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\} \in \mathbb{R}^3$, a set of reference sensors (anchors) at known positions $\{\mathbf{a}_1, \ldots, \mathbf{a}_l\} \in \mathbb{R}^3$, and unknown target positions $\{\mathbf{e}_1, \ldots, \mathbf{e}_m\} \in \mathbb{R}^3$. A central processing node has access to range measurements between target positions and all sensors/anchors, namely,

$$d_{ij} = \|\mathbf{x}_i - \mathbf{e}_j\| + w_{ij}, \qquad d_{kj} = \|\mathbf{a}_k - \mathbf{e}_j\| + w_{kj},$$

where w_{ij} and w_{kj} denote noise terms. If disturbances are Gaussian, independent and identically distributed, then maximizing the likelihood for the full batch of observations is equivalent to minimizing the cost function

$$\Omega(\mathbf{x}) = \sum_{(i,j)\in\mathcal{O}} (\|\mathbf{x}_i - \mathbf{e}_j\| - d_{ij})^2 + \sum_{(k,j)\in\mathcal{O}} (\|\mathbf{a}_k - \mathbf{e}_j\| - d_{kj})^2. \tag{1}$$

In (1), $\mathcal{O}$ is the index set for which pairwise range measurements are available. The set of unknown sensor and target positions is concatenated into column vector $\mathbf{x} \in \mathbb{R}^{3(n+m)}$, the argument of Ω. The goal of our SLAT approach is to find the set of coordinates in $\mathbf{x}$ which minimizes (1).

In the sequel, each section of the SLAT algorithm, i.e., startup initialization, MM refinement, and updating initialization will be discussed.

2.1. Startup Initialization: EDM-R

We assume to have a partial pre-distance matrix with zero diagonal entries and with certain nonnegative elements equal to the squares of range measurements between either sensor/target or anchor/target; the remaining elements are considered free. One of the methods to estimate the free elements is to resort to EDM completion, which is amenable to convex relaxation [8]. However, using the square of the measured distances destroys the gaussian noise assumption and increases the sensitivity to measurement errors. Thus, a modified EDM completion to plain distances [6] is applied as

$$\begin{array}{ll} \underset{\mathbf{E}}{\text{minimize}} & \sum_{(i,j)\in\mathcal{O}} (\sqrt{E_{ij}} - d_{ij})^2 \\ \text{subject to} & \mathbf{E} \in \mathcal{E},\ \mathbf{E}(\mathcal{A}) = \mathbf{A} \\ & \text{rank}(\mathbf{JEJ}) = 3 \end{array} \tag{2}$$

where $\mathcal{A}$ is the index set of anchor/anchor distances, $A_{ij} = \|\mathbf{a}_i - \mathbf{a}_j\|^2$, $\mathbf{J} = \mathbf{I}_{n+m+l} - \frac{1}{n+m+l}\mathbf{1}_{n+m+l}\mathbf{1}_{n+m+l}^T$ and element E_{ij} represents a squared pairwise distance ($\mathbf{1}$ is a vector of ones). $\mathcal{E}$ denotes the cone of EDM matrices, whose elements satisfy $E_{ii} = 0, E_{ij} = E_{ji} \geq 0$ and $-\mathbf{JEJ} \succeq 0$. Expanding the objective function in (2) results in

$$\begin{array}{ll} \underset{\mathbf{E}}{\text{minimize}} & \sum_{(i,j)\in\mathcal{O}} (E_{ij} - 2\sqrt{E_{ij}} d_{ij} + d_{ij}^2) \\ \text{subject to} & \mathbf{E} \in \mathcal{E},\ \mathbf{E}(\mathcal{A}) = \mathbf{A} \\ & \text{rank}(\mathbf{JEJ}) = 3. \end{array} \tag{3}$$

A SemiDefinite Relaxation (SDR) is obtained by introducing an epigraph-like variable $\mathbf{T}$ and dropping the rank constraint

$$\begin{array}{ll} \underset{\mathbf{E},\mathbf{T}}{\text{minimize}} & \sum_{(i,j)\in\mathcal{O}} (E_{ij} - 2T_{ij} d_{ij}) \\ \text{subject to} & T_{ij}^2 \leq E_{ij} \\ & \mathbf{E} \in \mathcal{E},\ \mathbf{E}(\mathcal{A}) = \mathbf{A}. \end{array} \tag{4}$$

This method will be called EDM-R (EDM with ranges). How to estimate the spatial coordinates of the sensors and target positions from the solution EDM of (4) and the usage of anchors are discussed in [9]. Note that the Euclidean distance metric is invariant to global rotation, translation, and reflection, so is the function Ω in the absence of anchors. To remove most of those ambiguities in the solution, a minimum of $l = 4$ non collinear anchors must be considered. In our experiments EDM-R was reasonably fast (a few seconds) for scenarios with up to about 30 unknown positions. Next, a MM algorithm is proposed for iterative likelihood maximization.

2.2. Refinement Step: Majorization-Minimization

The key idea of MM is to find, at a certain point $\mathbf{x}^t$, a simpler function that has the same function value at $\mathbf{x}^t$ and anywhere else is larger than or equal to the objective function to be minimized. Such a function is called a majorization function. By minimizing the majorization function the next point of the algorithm is obtained, which also decreases the cost function.

Due to the space considerations the derivation of the MM iteration given in [9] is omitted. Briefly we define functions $f_{ij}(\mathbf{x}) = \|\mathbf{x}_i - \mathbf{e}_j\|$ and $g_{kj}(\mathbf{x}) = \|\mathbf{a}_k - \mathbf{e}_j\|$ in (1), expand the squares, and then use a linear lower bound for f_{ij} and g_{kj} to obtain a majorization function that is quadratic in $\mathbf{x}$ and easily minimized. Thus, each MM iteration boils down to solving a linear system of equations.

2.3. Updating Initialization: SLNN

Suppose that a batch of observations have been processed and a new target position is to be estimated. We could repeat MM refinement with EDM-R initialization acting on an expanded batch that concatenates all previous range measurements and those for the new target sighting. However, this would be computationally expensive due to the EDM completion step. Also, previously estimated positions would be ignored and could not contribute to computational complexity reduction. To alleviate the load, a simple methodology is proposed to obtain a good initial point for MM that avoids the EDM step. This consists of fixing the previous positions at their estimated values and only estimating the new target position. More precisely, define the cost function

$$\Psi(\mathbf{z}) = \sum_{i=1}^{n+l} (\|\mathbf{b}_i - \mathbf{z}\| - d_i)^2, \tag{5}$$

where $\mathbf{z}$ is the new target position, $\mathbf{b}_i$ denotes the previously estimated position of a sensor or anchor, and d_i is the corresponding range measurement. SLNN (Source localization with Nuclear Norm) is proposed to minimize (5), and the method is briefly summarized below [7]. As shown in [6], (5) is equivalent to

$$\begin{aligned} &\underset{\mathbf{z}, \mathbf{y}_i, \mathbf{u}_i}{\text{minimize}} && \textstyle\sum_{i=1}^{n+l} \|\mathbf{z} - \mathbf{y}_i\|^2 \\ &\text{subject to} && \mathbf{y}_i = \mathbf{b}_i + d_i \mathbf{u}_i, \quad \|\mathbf{u}_i\| = 1, \quad i = 1, \cdots, n+l \end{aligned} \tag{6}$$

where $\mathbf{z}, \mathbf{y}_i$ and $\mathbf{b}_i$ are vectors and $\mathbf{u}_i$ is a unit-norm vector, all in $\mathbb{R}^3$. Equivalently,

$$\begin{aligned} &\underset{\mathbf{z}, \mathbf{y}_i, \mathbf{u}_i}{\text{minimize}} && \|\mathbf{1}_{n+l}\mathbf{z}^T - \mathbf{Y}\|_F^2 \\ &\text{subject to} && \underbrace{\begin{bmatrix} \mathbf{y}_1^T \\ \vdots \\ \mathbf{y}_{n+l}^T \end{bmatrix}}_{\mathbf{Y}} = \underbrace{\begin{bmatrix} \mathbf{b}_1^T \\ \vdots \\ \mathbf{b}_{n+l}^T \end{bmatrix}}_{\mathbf{B}} + \mathbf{D} \underbrace{\begin{bmatrix} \mathbf{u}_1^T \\ \vdots \\ \mathbf{u}_{n+l}^T \end{bmatrix}}_{\mathbf{U}}, \quad \|\mathbf{u}_i\| = 1, \end{aligned} \tag{7}$$

where $\mathbf{D} = \text{diag}(d_1, \ldots, d_{n+l})$. For fixed $\mathbf{y}_i$, $\mathbf{u}_i$, the optimal $\mathbf{z}^T$ may be written compactly as

$$\mathbf{z}^T = (\mathbf{1}_{n+l}^T \mathbf{1}_{n+l})^{-1} \mathbf{1}_{n+l}^T \mathbf{Y} = \frac{1}{n+l} \mathbf{1}_{n+l}^T \mathbf{Y}. \tag{8}$$

Replacing this back in (7) to eliminate variable $\mathbf{z}$, the objective function becomes $\|\mathbf{\Pi Y}\|_F^2 = \text{tr}(\mathbf{Y}^T \mathbf{\Pi Y})$, where $\mathbf{\Pi}$ is a projection matrix. We can now eliminate variable $\mathbf{Y}$ and the first set of equality constraints, expand its definition in the objective function and ignore constant terms to obtain

$$\begin{aligned} &\underset{\mathbf{U}}{\text{minimize}} && 2\,\text{tr}(\mathbf{C}^T\mathbf{U}) - \tfrac{1}{n+l}\text{tr}(\mathbf{U}^T\mathbf{d}\mathbf{d}^T\mathbf{U}) \\ &\text{subject to} && \|\mathbf{u}_i\| = 1, \end{aligned} \tag{9}$$

where $\mathbf{C} = \mathbf{D\Pi B}$ and $\mathbf{d} = \mathbf{D1}_{n+l}$.

To rewrite the first term in the objective function of (9) in a form that is more amenable to SDR , $\mathbf{U}$ is replaced with the product $\mathbf{UV}$, where $\mathbf{V}$ is an 3×3 orthogonal matrix such that $\mathbf{V}^T\mathbf{V} = \mathbf{V}\mathbf{V}^T = \mathbf{I}_3$ and thus

$$\begin{aligned} &\underset{\mathbf{U}, \mathbf{V}}{\text{minimize}} && 2\,\text{tr}(\mathbf{C}^T\mathbf{UV}) - \tfrac{1}{n+l}\text{tr}(\mathbf{V}^T\mathbf{U}^T\mathbf{d}\mathbf{d}^T\mathbf{UV}) \\ &\text{subject to} && \|\mathbf{u}_i\| = 1, \quad \mathbf{V}^T\mathbf{V} = \mathbf{I}_3. \end{aligned} \tag{10}$$

Regarding (10), $\mathbf{V}$ may be interpreted as an inner optimization variable that, for each candidate $\mathbf{U}$, minimizes the value of the objective function. It is shown in [7] that the optimization problem (10) is equivalently rewritten as

$$\begin{aligned} &\underset{\mathbf{U}}{\text{maximize}} && 2\,\text{tr}\big((\mathbf{C}^T\mathbf{U}\mathbf{U}^T\mathbf{C})^{\frac{1}{2}}\big) + \tfrac{1}{n+l}\text{tr}(\mathbf{d}\mathbf{d}^T\mathbf{U}\mathbf{U}^T) \\ &\text{subject to} && \|\mathbf{u}_i\| = 1. \end{aligned} \tag{11}$$

The variable $\mathbf{W} = \mathbf{U}\mathbf{U}^T$ is introduced and the associated nonconvex constraint rank$(\mathbf{W}) = 3$ is ignored to obtain the SDR in standard form

$$\begin{aligned} &\underset{\mathbf{W}, \mathbf{Z}}{\text{maximize}} && 2\,\text{tr}(\mathbf{Z}) + \tfrac{1}{n+l}\text{tr}(\mathbf{d}\mathbf{d}^T\mathbf{W}) \\ &\text{subject to} && \mathbf{W} \succeq 0, \quad w_{ii} = 1 \\ & && \begin{bmatrix} \mathbf{C}^T\mathbf{WC} & \mathbf{Z} \\ \mathbf{Z} & \mathbf{I}_3 \end{bmatrix} \succeq 0, \quad \mathbf{Z} \succeq 0. \end{aligned} \tag{12}$$

The solution of our SDR is a $(n+l) \times (n+l)$ matrix $\mathbf{W}$ that should have approximately rank 3 when the relaxation is tight. How to build the $\mathbf{y}_i$ and, ultimately, the source position vector $\mathbf{z}$ from $\mathbf{W}$ is described in [7]. After an optimal target position is obtained, we return to the cost function (1) and apply MM to refine all the estimates. This incremental or time recursive procedure can be applied to either new targets or sensors.

3. EXPERIMENTS

An efficient algorithmic framework was developed in our previous work [6]. However, this approach was restricted to 2D localization due to the use of a time recursive initialization algorithm that was specific to 2D (SLCP). The latter source localization algorithm is extended to 3D and higher dimensions in [7] and it is summarized in Section 2.3. In both papers the algorithms were only tested in simulation in MATLAB, under fully controlled conditions. Additionally, the combination of those algorithms were not tested in 3D neither in real nor simulated environments. Therefore, the present paper illustrates the performance of these algorithms in a real world experimental setup, where the statistics of perturbations are unknown and the true locations of sensors are only approximately known (to an accuracy on the order of 2 cm).

To estimate ranges, we used a Cricket system in which a mobile Cricket Node (beacon/target) simultaneously emits a radio and an ultrasound pulse every second. Since the difference in arrival time of these two pulses to a sensor node (listener) is proportional to the range between the sensor node and the target, Crickets compute ranges from these arrival times [5]. No range measurements between listener nodes were collected. The measurements were transmitted to a desktop machine, which processed them using batch or time-recursive algorithms.

Since the algorithm scheme entirely relies on distance estimates, any inaccuracies or spurious estimates will result in erroneous positions. Therefore, in the sequel the uncertainties in the setup are described. The line-of-sight operating range of ultrasonic listener-beacon pairs is around 10 meters, when both the listener and the beacon are facing each other. It is observed in [10] that approximately within 5 m range when a listener and beacon face each other at $0-40^\circ$ angles, the error in range estimation remains quite stable within 2 cm boundaries. From 40° on up to 75°, the error rises to 9 cm. From 75° onwards, the listener is no longer able to detect the ultrasonic signal. Similar behavior was experienced in our setup , for up to 4 m range measurements it was observed that the uncertainities in measurements can go up to 6 cm due to the variable facing angle between sensors and the beacon along the trajectory. Secondly, the ultrasound sensor on a Cricket occupies an area of 1 cm x 2 cm on the circuit board, so it is difficult to estimate the ground truth for its location below those dimensions. Additionally, the anchor nodes are normally assumed to be fixed at known positions. However, in practice, there are uncertainties in anchor node positions due to imperfect deployment etc.

In the following, results will be presented relative to tracking a Pioneer P3-DX robot and a human in a test environment shown in Figure 1. The metric for accuracy is root mean square error (RMSE), defined as $\sqrt{\frac{1}{n+m}\sum_{i=1}^{n+m}\|\mathbf{x}_i-\hat{\mathbf{x}}_i\|^2}$, where $\mathbf{x}_i$ and $\hat{\mathbf{x}}_i$ denote the i-th true and estimated sensor or target positions, respectively.

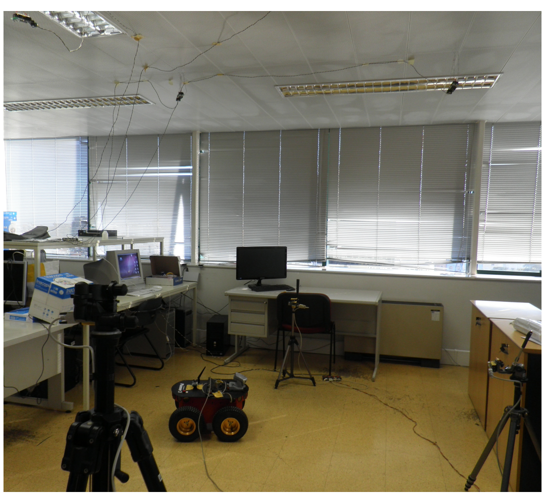

Fig. 1: The test enviroment and Pioneer P3-DX: 4 Cricket sensors on the ceiling and 4 on top of tripods distributed over the lab in a 4m x 4m area. The robot or a human carries the beacon.

Experiment 1: In this experiment, 8 listeners were deployed around the lab and on the ceiling in a 4 m x 4 m area and a beacon was attached to a Pioneer P3-DX mobile robot programmed to follow a circular trajectory. As the robot moves, the beacon periodically emits signals which allow some of the sensors to measure their distances to the robot. Through the trajectory of 1.5 minutes, 18 positions of the robot are observed. For the ground truth, positions of sensors and beacon/robot are measured manually. On average 7 sensors heard beacon signals from each target position. The algorithm is able to accurately localize a network of 4 anchors, 4 sensors and 18 target positions with the RMSE of 4.1 cm. The geometrical configuration of these sensor and target positions has intrinsic reflection ambiguity that are not resolved with anchors because the robot moves in the same z-coordinate through the circular trajectory. Therefore, some of the sensor positions are estimated at the mirror of their real positions with respect to the robot z-coordinate. For this particular setup, the intrinsic reflection ambiguity is readily solved by projecting them to the positive z-axis with respect to the estimated robot z-coordinate.

Figure 2 shows sensor/anchor positions and the 3D nominal and estimated target trajectories. Dark symbols represent estimated positions from the full batch (18 target positions), while red symbols display the estimated positions from an initial batch (first 15 target positions), followed by three time recursive updates for the three last target positions. At each time recursive update, it is assumed that a new target range measurement is obtained by the sensors and the new position is estimated by fixing the previously estimated positions while minimizing (12). The newly estimated target position and all positions estimated previously are given as an initial point to start MM. Therefore, Figure 2 presents not only the last three target positions estimated in updating initializations, but also all positions refined by MM step.

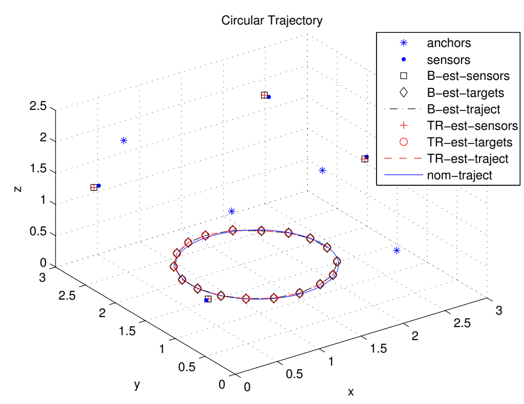

Fig. 2: Circular trajectory of Pioneer P3-DX. B-est and TR-est refer to estimation from full batch and the time recursive procedure, respectively.

Experiment 2: Within this experiment, the robot moves along a straight line, followed by a half circle, a straight line, and a full circle for approximately 2 minutes, generating 24 target positions. The RMSE of this setup (4 anchors, 4 sensors and 24 target positions) is 3.95 cm which is slightly better than the previous experiment due to the larger batch dimension (24 target positions). Figure 3 shows the sensor/anchor node positions as well as, the nominal and estimated target trajectories for both the batch and time recursive approaches. The latter pertains to the last three target positions, as described in experiment 1. The time recursive procedure attains the same accuracy as the batch algorithm, with the advantage of lower computational complexity.

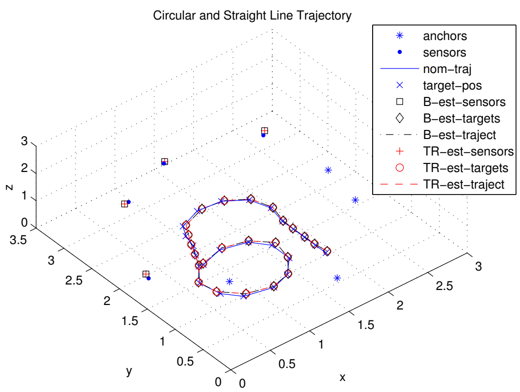

Fig. 3: Piecewise linear/circular trajectory of Pioneer P3-DX. B-est and TR-est refer to estimation from full batch and the time recursive procedure, respectively.

Experiment 3: In this experiment, a human carries the beacon, generating 18 target sightings along a somewhat erratic trajectory (the human moves to the positions that have ground truths). Figure 4 depicts the sensor/anchor positions, the nominal and the estimated target trajectories. Since the human carries the beacon in different z-coordinates, there is no symmetry, thus no intrinsic ambiguity in this case. The RMSE of this network of 4 anchors, 4 sensors and 18 target positions is 3.88 cm.

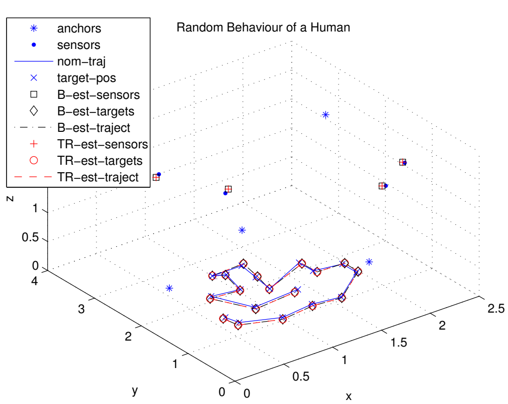

Fig. 4: Trajectory of a human target.

4. CONCLUSION

A ML-based technique is presented to solve the SLAT problem using a two-phase approach. A MM method is proposed to iteratively maximize the non-convex likelihood function, for which a good initial point is obtained from two initialization schemes based on EDM completion and source localization that bypass the need for strong priors on sensor/target positions. With this methodology, a good initialization and a scalable solution are guaranteed for SLAT problems. In our experiments, it is observed that the range estimation accuracy depends on the distance and the facing angle of sensor-target pair, which affects the accuracy of the algorithms. Additionally, for particular setups, intrinsic ambiguities can not be resolved with anchors. It is demonstrated experimentally that the proposed scheme can track a target and localize sensors to within about 4 cm accuracy in a 3D indoor environment using mixed ultrasound-RF ranging.

5. REFERENCES

[1] L. M. Ni, Y. Liu, Y. C. Lau, and A. P. Patil, “Landmarc: Indoor location sensing using active RFID,” *Wireless Networks*, vol. 10, pp. 701–710, 2004.

[2] R. Want, A. Hopper, V. Falcao, and J. Gibbons, “The active badge location system,” *ACM Transactions on Information Systems (TOIS)*, vol. 10, pp. 91–102, 1992.

[3] A. Ward, A. Jones, and A.Hopper, “A new location technique for the active office,” *IEEE Personal Communications*, vol. 4, pp. 42–47, October 1997.

[4] A. Conti, D. Dardari, and M. Win, “Experimental results on cooperative UWB based positioning systems,” in *Proceedings of the IEEE International Conference on Ultra-Wideband (ICUWB’08)*, 2008, vol. I, pp. 191–195.

[5] N. Priyantha, A. Chakraborty, and H. Balakrishnan, “The Cricket location support system,” in *Proceedings of the ACM International Conference on Mobile Computing and Networking (MobiCom’00)*, 2000.

[6] P. Oğuz-Ekim, J. Gomes, J. Xavier, and P. Oliveira, “Robust localization of nodes and time-recursive tracking in sensor networks using noisy range measurements,” *IEEE Transactions on Signal Processing*, vol. 59, pp. 3930–3942, August 2011.

[7] P. Oğuz-Ekim, J. Gomes, J. Xavier, and P. Oliveira, “Approximate maximum likelihood source localization from range measurements through convex relaxation,” *submitted to IEEE Transactions on Signal Processing*, arXiv:1111.6755v1, 2011.

[8] J. Dattorro, *Convex Optimization and Euclidean Distance Geometry*, Meboo publishers, 2005.

[9] P. Oğuz-Ekim, J. Gomes, J. Xavier, and Oliveira P., “ML-based sensor network localization and tracking: Batch and time-recursive approaches,” in *Proceedings of the 17th European Signal Processing Conference (EUSIPCO’09)*, 2009.

[10] M. Popa, J. Ansari, J. Riihijarvi, and P. Mahonen, “Combining Cricket system and inertial navigation for indoor human tracking,” in *Proceedings of the IEEE Wireless Communication and Networking Conference (WCNC’08)*, 2008, pp. 3063–3068.