Time and order estimation of paintings based on visual features and expert priors

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ABSTRACT

Time and order are considered crucial information in the art domain, and subject of many research efforts by historians. In this paper, we present a framework for estimating the ordering and date information of paintings and drawings. We formulate this problem as the embedding into a one dimension manifold, which aims to place paintings far or close to each other according to a measure of similarity. Our formulation can be seen as a manifold learning algorithm, albeit properly adapted to deal with existing questions in the art community. To solve this problem, we propose an approach based in Laplacian Eigenmaps and a convex optimization formulation. Both methods are able to incorporate art expertise as priors to the estimation, in the form of constraints. Types of information include exact or approximate dating and partial orderings. We explore the use of soft penalty terms to allow for constraint violation to account for the fact that prior knowledge may contain small errors. Our problem is tested within the scope of the PrintART project, which aims to assist art historians in tracing Portuguese Tile art "Azulejos" back to the engravings that inspired them. Furthermore, we describe other possible applications where time information (and hence, this method) could be of use in art history, fake detection or curatorial treatment.

Keywords: Time and order estimation ; partial order constraints; Manifold Learning; Computer Vision ; Metric Learning ; Spectral techniques

1. INTRODUCTION

A great deal of questions posed by art historians are intrinsically related to time estimation or chronologic ordering. Whether the subject is history, forensics, restoration or even inventory, the date on which an art piece was created is of utter importance. In spite of that, the general question of time estimation and ordering, aside from notable exceptions,[1–3] has lacked attention by either the image processing and computer vision community at large.

Tracing the roots of artistic work is one of the most central, and yet challenging, tasks in art history. In particular, this paper documents part of an effort to help art historians trace the artistic influences that shaped the Portuguese ceramic tile panels called "Azulejos". Though common in Europe and in the Mediterranean countries, Azulejos are quite expressive in the Iberian Peninsula and specially in Portugal[*]. Artists were commissioned to decorate bourgeois homes, public buildings, palaces and churches. In Portugal specifically, both by influence of the moors and the trends from central europe, these works culminated in very characteristic scenes with themes ranging from Religious to portraits of contemporary lifestyle or even pure geometric motifs. Fig. 1 shows one example, the church of Arraiolos, and a closeup view of one particular panel. The process of making these tiles typically starts with an engraving, imported from middle european famous artists of the time. These engravings were used as the main source of inspiration for the tile work, but artists seldom followed them precisely, often displacing characters, performing symmetry transformations or even merging more than one engraving (Fig. 2).

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[*]`http://mnazulejo.imc-ip.pt/`

The job of the art historian is to search for the prints and engravings that served as inspiration or which the panel painter copied from. As part of a multi-source automatic search system,[4,5] in this paper we present a framework to estimating the ordering and date information of images of paintings and drawings. We do this based on measures of similarity among them together with specific knowledge from experts. This time-ordering mechanism is one more component that helps pruning candidates among all possible prints that match a certain scene in the panel. For illustration purposes we use here one particular feature (color) and several types of engravings and tile panels, but the methodology is general for any type of image.

(a)

(b)

Figure 1. Examples of Portuguese Tile Art "Azulejos". (a) the Misericordia Church in Arraiolos, Portugal (b) a closer view of one particular panel. These panels were frequently copies of famous painting or printmaking.

We formulate the time-ordering problem as the embedding of the images into a one dimension manifold, which aims to place paintings far or close to each other according to a measure of similarity. Our formulation can be seen as a manifold learning algorithm,[6–8] albeit extended and properly adapted to deal with questions that exist in the art community.

To solve this problem, we propose two approaches that leverage the image similarity measures and expert priors to build a single timeline where the works are positioned. The first is based on Laplacian Eigenmaps, while the second is cast as a globally optimal convex optimization problem. Both methods are able to incorporate expertise as priors to the estimation, in the form of constraints. Types of information include approximate dating ("circa 1950" or "17th century"), partial orderings (painting A comes before B) or exact dating for different subsets of the paintings. We explore the use of soft penalty terms to allow for constraint violation to account for the fact that prior knowledge may contain small errors. While the first approach exhibits great computational efficiency its poor accuracy motivates the second approach, which demands much more computing power.

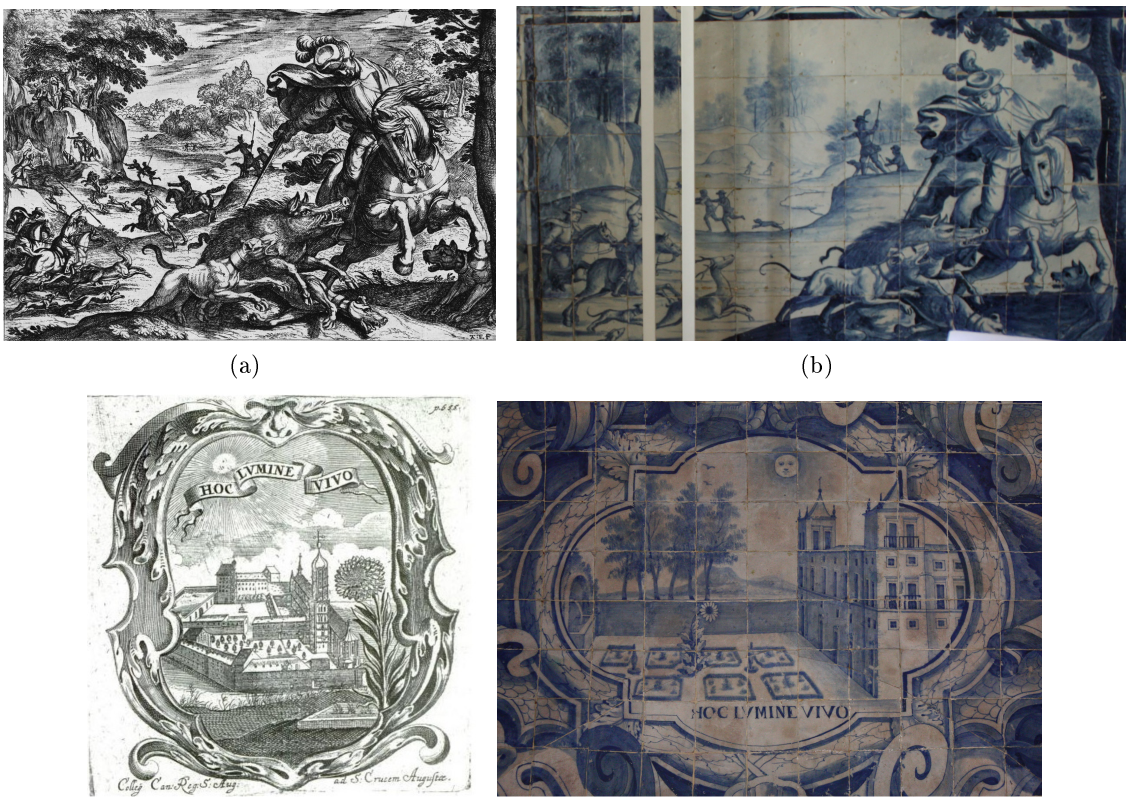
The construction of this timeline favors the positioning of paintings that are more similar (according to a given distance measure) closer to each other, while allowing dissimilar paintings to be positioned farther away, within the constraints imposed by prior expert knowledge. As such, we are able to relate the obtained positions to the single historical timeline in which the paintings were created.

In the case of Azulejos, the nature of the works makes it very hard to define a measure of similarity using traditional computer vision feature representations,[9,10] let alone estimate a date or possible ordering between them using only these measures. The lack of appropriate representations for art has already been identified in.[11–13] In this paper, we try to lessen the burden on the existence of appropriate feature representations and blend this information with the connoisseurship of art historians to provide an estimate for both time and order: while these are of significant importance to historians *per se*, we also argue they may aid existing techniques dealing with performing retrieval in the scenario where only "non-robust" feature descriptors are present, as is generally the case in Art and in Azulejos.[4]

2. PREVIOUS LITERATURE

Literature on order estimation is very scarce in the fields of computer vision and image analysis. This is due to the ill-posedness of the problem, explained by the fact that images can be considered samplings of the high-dimensional plenoptic manifold, meaning they vary significantly due to pose, illumination and location. This fact makes it cumbersome to design representations that measure similarity between images taken in general scenarios, so the majority of existing applications focus on more constrained settings (usually, fixing location and slightly varying the pose[10]). Very recently, the vision community has started to address the link between pictures and their sequentiality. Schindler *et al.*[1] proposed a generative model for sequence estimation. Their method was able to establish an ordering of photographs of the Manhattan bay over the 20th century. This ordering allowed them to model a 3D reconstruction of the evolution of the bay along time. More recently, Kim *et al.*[2] proposed a method that models the temporal evolution of appearances attached to given words along time in the world wide web. This, for example, allows one to follow trends on how people graphically represent a given concept or company. As we have mentioned before, these methods rely heavily on the assumption that there is a strong measure of similarity between images. While this may be true in the setting of photography, paintings usually exhibit a high degree of abstraction and significant variance in the colors and techniques used to represent the same motifs. Therefore, the use of these descriptors has to be tailored to the (usually small) collection of paintings in question. With the assumption that a robust similarity metric exists not holding in general, these approaches become thus infeasible in the context of art. Worthy of note, however, is recent work in Fractal Analysis by Taylor *et al.*,[3] which briefly mentions using their fractal dimension description to measure the evolution of Pollock's paintings along the years of his career.

On the other hand, literature on dimensionality reduction applied to images is abundant in the last years.



(a) (b) (c) (d)

Figure 2. Comparison between an Azulejo and the engraving that served as an inspiration. Note that despite the similarity, there are leaps of interpretation between the two representations, such as change of pose or relative position of elements. This increases the difficulty of designing similarity measures a computer can understand.

People have used Principal Component Analysis and nonlinear manifold techniques such as ISOMAP, Laplacian Eigenmaps or Maximum Variance Unfolding to make sense of data that are embedded in very high dimensions,[14] as these techniques bring it back to its intrinsic dimension. Using the framework that is a common stepping stone to many of these methods (Multidimensional Scaling), Kruskal has looked into the ordering problem in the sixties, in the form of an isotonic regression.[15] Here, we wish to find a least squares fit to data according to constraints imposing partial ordering over the variables. This problem was originally used to find embeddings whose magnitude order of distances between points is proportional to the magnitude order of dissimilarity between data points in the original high-dimensional space. On the next section, we show how it is possible to bridge the gap between these two bodies of knowledge and machine learning techniques to perform the estimation of date and chronological order.

3. PROBLEM FORMULATION

The problem of finding an ordering using image similarity and expert priors can be cast as creating a mapping of images to a single temporal line. We would like to use similarity between images of paintings as seen by a computer to connect the dots between pieces of information art historians currently have on a subset of those paintings. To tackle this problem, we formulate it as finding a single dimension embedding of the data, as

$$\begin{array}{ll} \text{minimize} & \sum_{i,j} W_{ij}(t_i - t_j)^2 \\ s.t. & t_i > t_{i-1}, \forall i \in \mathcal{I} \\ & t_k = C_k, \forall k \in \mathcal{K}. \end{array} \tag{1}$$

Here, we wish to position images i and j in the timeline as t_j and t_i, according to a measure of similarity W_{ij} between them, partial ordering constraints in set $\mathcal{I}$ and known dates in set $\mathcal{K}$ (C_k are known constants). This formulation penalizes positioning i and j very far away when their similarity is high, while allowing a more distant placement of dissimilar images. As this process tries to capture the variation of the images as a whole onto a single dimension space, we assume that the metric chosen in $\mathbf{W} = [W_{ij}]$ yields a strong correlation to the time evolution of the works whose date and order we wish to estimate.

3.1 Laplacian Eigenmaps

We recall that if we remove the partial orderings and equality constraints in (1), this formulation is the already known Laplacian Eigenmaps for manifold learning[6]

$$\begin{array}{ll} \text{minimize} & \sum_{i,j} W_{ij}(t_i - t_j)^2 \\ s.t. & \mathbf{t}^\top \mathbf{D} \mathbf{t} = 1, \end{array} \tag{2}$$

where $\mathbf{D}$ is a diagonal matrix comprising the row sums of $\mathbf{W}$. This problem is not convex, being solved by reducing it to a Generalized eigenvalue problem of the form

$$(\mathbf{D} - \mathbf{W})\mathbf{t} = \lambda \mathbf{D}\mathbf{t} \tag{3}$$

and taking the minimum eigenvector. Due to the structure of the graph laplacian $\mathbf{D} - \mathbf{W}$, the last eigenvalue is always zero, leading to a trivial solution where all elements of $\mathbf{t}$ are zero. The best cost solution here is, as opposed to typical optimization settings, undesirable. Therefore, we avoid it by using the solution given by the smallest non-zero eigenvalue. Having said this, we note that the problem (2), when added the constrains in (1), turns into a cumbersome one. Due to the fact mentioned above, if we solve (1) using convex optimization techniques, the solution obtained will be the minimum deviation from the trivial solution that is feasible with the imposed constraints, thus with no practical usability. Based on the intractability of the problem as is, we propose two formulations: On the one hand, we propose a greedy approach, where we use the solution obtained from Laplacian Eigenmaps and then enforce the order constraints using isotonic regression. On the other hand, we propose an approach based on metric learning and Maximum Variance Unfolding. While the first approach exhibits great computational efficiency, its poor accuracy motivates the second approach, which is the solution of a convex problem but demands much more computing power. We now focus the remainder of this section in reviewing Maximum Variance Unfolding and Metric Learning and describing how to concatenate them into a method that finds the time and order manifold.

3.2 Convex Formulation

Weinberger and Saul[8] proposed a manifold learning technique alternative to Laplacian Eigenmaps (2). Maximum Variance Unfolding (also known as Semidefinite Embedding) is formulated as

$$\begin{array}{ll} \text{maximize} & \text{trace}(\mathbf{T}^\top \mathbf{T}) \\ s.t. & \frac{1}{N}\sum_{i=1}^{N} t_i = 0 \\ & ||\mathbf{t}_i - \mathbf{t}_j|| = ||\mathbf{x}_i - \mathbf{x}_j||. \end{array} \tag{4}$$

Here, we intend to learn a Gramian matrix $\mathbf{G} = \mathbf{T}^\top \mathbf{T}$ that maximizes the pairwise distances between two images that have little similarity, as expressed by the norm of their differences $||\mathbf{x}_i - \mathbf{x}_j||$. By applying Multidimensional Scaling on the obtained Gramian $\mathbf{G}$, we are able to recover a one dimensional embedding, similar to (2), that preserves distances found on the data manifold. However, as we stated before, the feature domain $\mathbf{x}_i$ will not be discriminative enough to create an embedding representing time by itself, due to the high variability present in art images. To lessen the burden on the representation, we need to introduce, as in (1), constraints limiting the number of possible solutions of $\mathbf{G}$ to matrices respecting the knowledge imposed by art historians. As such, we use the ideas presented on metric learning to transform the metric defining the distance between $\mathbf{x}_i$ and $\mathbf{x}_j$, giving more weight to some dimensions than others when measuring the distance between them. This is appropriate since we initially assumed features being used have some correlation with time, but live in the midst of severe noise. In the context of ranking queries in information retrieval systems, Schultz and Joachims proposed a framework to learn a modified Mahalanobis distance metric based on pairwise distance constraints, as

$$\begin{array}{ll} \text{minimize} & \left(||\mathbf{A}||_F + \gamma \sum_{ijk} \xi_{ijk}\right), \\ s.t. & \mathbf{A} \succeq 0 \\ & d_{\mathbf{A}}(\mathbf{x}_i, \mathbf{x}_j) < d_{\mathbf{A}}(\mathbf{x}_i, \mathbf{x}_k) - 1 + \xi_{ijk}, \forall i, j, k \\ & \xi_{ijk} \geq 0, \forall ijk \end{array} \tag{5}$$

where $d_{\mathbf{A}}(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i - \mathbf{x}_j)^\top \mathbf{A}(\mathbf{x}_i - \mathbf{x}_j)$. This formulation is finding the lowest rank matrix $\mathbf{A}$ being able to model all the constraints adequately, with violations allowed by slack variables ξ.

We note that both (5) and a relaxed version of (4) are convex optimization problems, so we formulate an analogous version of (1) by concatenating both techniques as

$$\begin{array}{ll} \text{maximize} & \text{trace}(\mathbf{G}) - \lambda \left(||\mathbf{A} - \mathbf{I}||_F + \gamma \sum_{ijk} \xi_{ijk}\right) \\ s.t. & \mathbf{1}^\top \mathbf{G} \mathbf{1} = 0 \\ & \mathbf{e}_i^\top \mathbf{G} \mathbf{e}_i + \mathbf{e}_j^\top \mathbf{G} \mathbf{e}_j - 2\mathbf{e}_i^\top \mathbf{G} \mathbf{e}_j = \mathbf{x}_i \mathbf{A} \mathbf{x}_j, \\ & \mathbf{G} \succeq 0 \\ & \mathbf{A} \succeq 0 \\ & d_{\mathbf{A}}(\mathbf{x}_i, \mathbf{x}_j) < d_{\mathbf{A}}(\mathbf{x}_i, \mathbf{x}_k) - 1 + \xi_{ijk}, \forall i, j, k \\ & \xi_{ijk} \geq 0, \forall ijk. \end{array} \tag{6}$$

Note that this formulation differs from the objective in (5). Here, we are learning a metric that is as close as possible to the original Euclidean metric but obeys the imposed constraints known *a priori*. Hence, we minimize the norm of the difference to the identity matrix instead of the Frobenius norm of $\mathbf{A}$. The resulting formulation is a Semi-definite Program, therefore being solvable by convex optimization techniques.

4. EXPERIMENTS

The experiments performed in this section are done using CVX[16,17] convex optimization package. For baseline comparison, we also provide results given by the original formulation of Laplacian Eigenmaps.

4.1 PrintART dataset

We test our concept on the PrintART project dataset.[4,18] This dataset consists of 307 images of engravings annotated by an art historian (annotated data includes pose of main characters in the scene, theme, year, author) portuguese tile mosaics. We try to estimate the order between a set of 16 engravings from the PrintART dataset, 33 Azulejos and 33 paintings from Pablo Picasso collected from the Internet (for examples of the elements in each group, see Fig. 3).

(a) (b) (c)

Figure 3. Examples for the classes used: (a) Engravings , typically B&W; (b) Azulejos (blue) and (c) Picasso paintings.

We construct a similarity matrix between all the images using as features histograms of color in the HSV space, with 10 equally spaced bins on each channel. Since the dataset is comprised of images with different resolutions, we perform an L1 normalization in each histogram. The final descriptor is a 30 dimension vector, with which we construct a similarity matrix using the relation $W_{ij} = \exp(-||\mathbf{x}_i - \mathbf{x}_j||^2/\sigma)$, where σ is set to $\sigma = 1$. Although these sets differ significantly in their creation date, the obtained similarity matrix (Fig. 4) shows only a partial clustering of these sets, with several inter cluster similarity terms having the same order of magnitude as their inter-cluster counterparts.

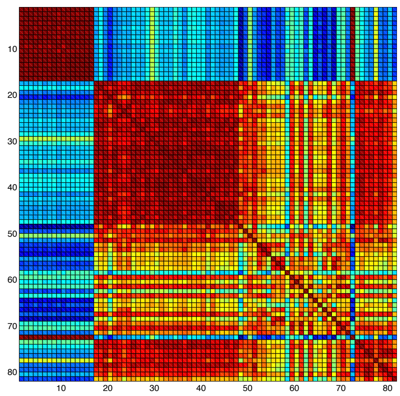


Figure 4. Similarity matrix used in Laplacian Eigenmaps (Red means higher similarity). The engravings cluster (indices 1—16 is the only one distinct from the others (Azulejos, from 17 to 50, Non Blue Period Picassos from 51 to 72 and Blue Period Picassos from 73 to 82).

With this similarity matrix, we run Laplacian Eigenmaps. Without any ordering constraints, the resulting embedding (Fig. 5), due to the ambiguity present in the color representation used, severely violates the known ordering that Picassos are later than Azulejos and these are later than Engravings.

On the other hand, we provide to our method some of the prior knowledge that engravings should be prior to azulejos and the latter prior to Picasso's paintings. Note that we do not provide this to all paintings, but only for a small subset of the pieces, resulting in 11 constraints of the form $d_{\mathbf{A}}(\mathbf{x}_i, \mathbf{x}_j) < d_{\mathbf{A}}(\mathbf{x}_i, \mathbf{x}_k) - 1 + \xi_{ijk}$. The obtained similarity matrix is shown in Fig. 6. In this case, the similarity matrix exhibits a clearer separation between the four clusters (Engravings, Azulejos, Non Blue Period Picassos and Blue Period Picassos). Using this similarity matrix, we see that ordering that results thereof (Fig. 7) gets corrected in its majority to the expected one.

It should be noted here that albeit our method achieves a significant improvement over Laplacian Eigenmaps alone, it still fails to provide a perfect ordering. This is due to the fact that the used image description is not informative enough to allow a separation of pathological cases (Fig. 8) such as the Guernica, which is black and white and will therefore be always positioned always closer to the engravings. Nevertheless, the results are improvement over the baseline with only a small number of constraints added, a result which restates the informativeness of expert priors.

5. OTHER POSSIBLE APPLICATIONS FOR TIME AND ORDER ESTIMATION

The example shown before is only a small selection of many problems where time and order estimation is important. In this section, we briefly describe other possible applications where time and order may be helpful.

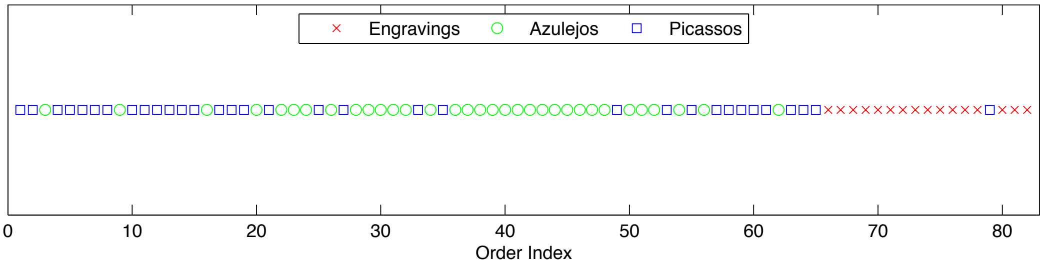


Figure 5. Result of embedding using Laplacian Eigenmaps with features as color histograms. The ordering Engravings → Azulejos → Picassos is severely violated, due to the bad separation of the clusters in Fig. 4.

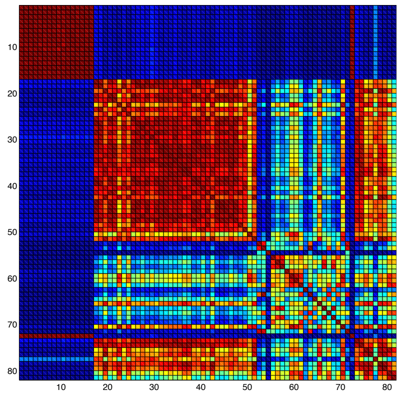


Figure 6. Similarity matrix obtained by our method (Red means higher similarity). When compared, Fig. 6, our method achieves a more evident separation between Azulejos and Picasso works not from the Blue period.

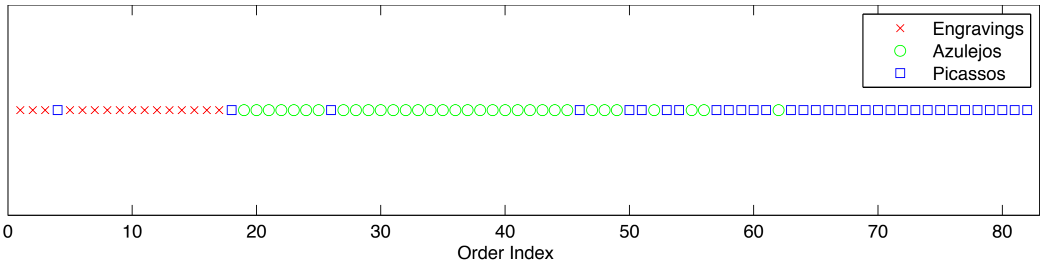


Figure 7. Result of embedding using our method, using HSV color histograms as features. When compared to Fig. 5, the ordering has been practically restored to Engravings → Azulejos → Picassos using a small number of constraints.

Art history: Influence trees Estimating order may enlighten difficult questions in the design of Master-Pupil trees, providing some lead as to which artist influenced whom. Analogous to Kim *et al.*,[2] we could also follow characteristics of important trends (or styles) and their respective evolution on given Art History periods.

Falsification detection Time is yet another prior on which to determine whether a given painting is a falsification or not. For example, a painting that has been determined to be outside the known scope of an artist's career is less likely to be attributed to him. Since this problem usually involves considerable risk and reward factors,[12] several sources of information should be collected. Time and date estimation pose as a non-destructive method of obtaining factors that might shed light on a decision.

Application to Curatorial Treatment Burns *et al.*[19] proposed a method to backtrack a painting's original color by looking at the profile of degradation ("aging spectrums") of the color pigments. To do this, however, one needs to have an estimate of when the artist painted the masterpiece, something that is not available in several cases. This could pose as an alternative to yield this information, so color rejuvenation or even virtual rendering of the painting with the original colors can be made available to smaller museums, where the option of recurring to expensive methods such as multispectra, fluoroesence and ultra-violet imaging is limited by their budget.

6. CONCLUSIONS

We presented two methods that recur to image measurements and prior expert knowledge to aid art historians in answering the dating and ordering question for a set of paintings where some have an unknown date. Besides the importance of solving the ordering problem by itself, we have presented several application domains where the solution for this problem is either needed of would significantly benefit their outcome. In particular, we motivated this problem as an extra layer of information in the scope of doing semantic and textual retrieval of engravings that inspired a given tile artpiece, in the context of the PrintART project.

The methods presented constitute a trade-off between accuracy and computational complexity. While the generalized eigenvalue solution provided by laplacian eigenmaps has low performance, the solution provided by the Semi-definite program in (6) has a significant setback in regards to computational complexity, given currently solvers for this class of optimization problems. We should also note that although these methods relieve the burden on appropriate image representations, this is still a striking issue when the images are art pieces. However, we have shown that in this domain, we usually possess considerable prior knowledge given by the art historians that might help disambiguate the majority of cases where the former are not discriminative enough.

Besides proper image descriptors, further work should focus on solving the partial ordering constraints as a generalized eigenvalue problem, to achieve both performance and computational complexity simultaneously.

(a) (b)

Figure 8. Picasso's pathological cases using color information alone: (a) La Vie (1903), a work from the blue period is bound to get mixed in with Azulejos ; (b) Guernica (1937), being black and white, is closer to engravings.

Additionally, the question of whether ordering constraints can be extended to higher-dimensional embeddings and whether they represent useful information in the context of paintings (and images) should be investigated.

REFERENCES

[1] Schindler, G. and Dellaert, F., "Probabilistic temporal inference on reconstructed 3d scenes," in [*CVPR*], 1410–1417, IEEE (2010).

[2] Kim, G., Xing, E. P., and Torralba, A., "Modeling and analysis of dynamic behaviors of web image collections," in [*ECCV (5)*], Daniilidis, K., Maragos, P., and Paragios, N., eds., *Lecture Notes in Computer Science* **6315**, 85–98, Springer (2010).

[3] Taylor, R. P., Guzman, R., Martin, T. P., Hall, G. D. R., Micolich, A. P., Jonas, D., Scannell, B. C., Fairbanks, M. S., and Marlow, C. A., "Authenticating pollock paintings using fractal geometry," *Pattern Recognition Letters* **28**(6), 695–702 (2007).

[4] Carneiro, G. and Costeira, J. P., "The automatic annotation and retrieval of digital images of prints and tile panels using network link analysis algorithms," *Computer Vision and Image Analysis of Art II* (2011).

[5] Nuno Pinho da Silva, Manuel Marques, G. C. J. a. P. C., "Explaining scene composition using kinematic chains of humans: application to portuguese tiles history," *IS&T SPIE Computer Vision and Image Analysis of Art II* (2011).

[6] Belkin, M. and Niyogi, P., "Laplacian Eigenmaps for Dimensionality Reduction and Data Representation," *Neural Computation* **15**, 1373–1396 (June 2003).

[7] Cox, T. and Cox, M., [*Multidimensional Scaling*], Chapman & Hall, London (2001).

[8] Weinberger, K. Q. and Saul, L. K., "Unsupervised learning of image manifolds by semidefinite programming," in [*Computer Vision and Pattern Recognition, 2004. CVPR 2004. Proceedings of the 2004 IEEE Computer Society Conference on*], *Computer Vision and Pattern Recognition, 2004. CVPR 2004. Proceedings of the 2004 IEEE Computer Society Conference on* **2**, II–988–II–995 Vol.2 (2004).

[9] Wang, J. Z., Wiederhold, G., Firschein, O., and Wei, S. X., "Content-Based Image Indexing and Searching Using Daubechies' Wavelets," *Int. J. on Digital Libraries* **1**(4), 311–328 (1997).

[10] Lowe, D. G., "Object recognition from local scale-invariant features," in [*Computer Vision, 1999. The Proceedings of the Seventh IEEE International Conference on*], *Computer Vision, IEEE International Conference on* **2**, 1150–1157 vol.2, IEEE Computer Society, Los Alamitos, CA, USA (August 1999).

[11] Hughes, J. M., Graham, D. J., and Rockmore, D. N., "Stylometrics of artwork: uses and limitations," *IS&T SPIE Computer Vision and Image Analysis of Art* (2010).

[12] C. Richard Johnson, J., Hendriks, E., Berezhnoy, I. J., Brevdo, E., Hughes, S. M., Daubechies, I., Li, J., Postma, E., and Wang, J. Z., "Image processing for artist identification: Computarized analysis of vicent van gogh's painting brushstrokes," *IEEE Signal Processing Magazine* **37** (2008).

[13] Stork, D. G., "Computer vision and computer graphics analysis of paintings and drawings: An introduction to the literature," *CAIP* (2009).

[14] Tenenbaum, J. B., Silva, V., and Langford, J. C., "A Global Geometric Framework for Nonlinear Dimensionality Reduction," *Science* **290**, 2319–2323 (December 2000).

[15] Kruskal, J., "Nonmetric multidimensional scaling: A numerical method," *Psychometrika* **29**, 115–129 (1964). 10.1007/BF02289694.

[16] Grant, M. and Boyd, S., "CVX: Matlab software for disciplined convex programming, version 1.21." `http://cvxr.com/cvx` (Oct. 2010).

[17] Grant, M. and Boyd, S., "Graph implementations for nonsmooth convex programs," in [*Recent Advances in Learning and Control*], Blondel, V., Boyd, S., and Kimura, H., eds., *Lecture Notes in Control and Information Sciences*, 95–110, Springer-Verlag Limited (2008). `http://stanford.edu/~boyd/graph_dcp.html`.

[18] PrintART project, "printart: where computer vision meets art." `http://printart.isr.ist.utl.pt/` (Oct. 2010).

[19] Berns, R. S., Byrns, S., Casadio, F., Fiedler, I., Gallagher, C., Imai, F. H., Newman, A., Rosen, M., and Taplin, L. A., "Rejuvenating the appearance of seurat's a sunday on la grande jatte – 1884 using color and imaging science techniques – a simulation," (2005).