Coordinated control of multiple vehicles with discrete-time periodic communications

João Almeida, Carlos Silvestre and António Pascoal

***Abstract*— This paper addresses the problem of coordinated path-following of networked autonomous vehicles with discrete-time periodic communications. The objective is to steer a group of autonomous vehicles along given spatial paths, while holding a desired inter-vehicle formation pattern. For a class of vehicles, we show how Lyapunov based techniques, graph theory, and results from networked systems can be brought together to yield a decentralized control structure where the dynamics of the cooperating vehicles and the constraints imposed by the topology of the inter-vehicle communications network are explicitly taken into account. Vehicle coordination is achieved by adjusting the speed of each vehicle along its path according to information exchanged periodically on the positions of a subset of the other vehicles, as determined by the communications topology adopted. Stability and convergence of the overall system are guaranteed and a criterion for selecting the coordination gains is presented.**

I. Introduction

Coordination of multi-agent systems is a topic of great interest in a wide range of scientific and technological areas that include biological and biologically inspired systems, artificial intelligence, and mobile robotics, to name but a few. Sharing information over a communication network is vital to achieving coordination, which brings about a number of challenging problems that arise from such practical constraints as limited bandwidths, time delays (latency), transmission noise, and intermittent failures. These problems are at the core of recent research efforts that aim to develop efficient multiple vehicle coordination systems in the presence of severe communication constraints.

In this paper, as a contribution to the study of general coordination systems, we address the problem of coordinated path following (CPF) where multiple vehicles are required to follow pre-defined spatial paths while keeping a desired inter-vehicle formation pattern in time. This problem arises, for example, in the operation of multiple autonomous underwater vehicles for fast acoustic coverage of the seabed. By imposing constraints on the inter-vehicle formation pattern, the efficacy of the task can be largely improved. In this case, the bandwidth available for communications is severely reduced. This, coupled with energy constraints and the need to avoid inter-vehicle message collisions, require that communications take place in the form of short and not too frequent burst sequences. A number of other realistic vehicle operation scenarios can also be envisioned that require coordinated motion control of marine or other types of vehicles, and where the inter-vehicle communications are naturally discrete in time [1].

We solve the CPF problem for a class of fully-actuated autonomous vehicles moving in two-dimensional space. Nevertheless, the results derived can be extended to the three-dimensional case. The solution adopted is based on Lyapunov stability theory and addresses explicitly the vehicle dynamics as well as the constraints imposed by the topology of the inter-vehicle communications network. The latter are tackled in the framework of graph theory, which allows for the consideration of communication topologies that contain unidirectional links. Each vehicle is equipped with a controller that makes the vehicle follow a predefined path. The speed of each vehicle is then adjusted so that the whole group of vehicles will keep a desired formation pattern. A supporting communications network affords the vehicles the medium over which information is exchanged to achieve synchronization. Because of bandwidth constraints, the information exchange between vehicles takes place at discrete time instants which, in this paper, we assume to occur at a fixed frequency. We further assume that the communication links established among vehicles are directed: a vehicle sends information to its neighbors but does not necessarily receive information back. We also assume that the transmission delay is negligible and that there are no packet collisions when the vehicles communicate simultaneously. Due to the absence of information in the intervals between transmission times, the control action of each vehicle runs in open loop, based on a simple model that predicts the evolution of the vehicle's neighbors. At transmission times, each vehicle sends information through the network that is used to achieve coordination and to update the models. This idea builds on previous work by Montestruque and Antsaklis on model-based networked systems [2]. With the control structure adopted, path-following (in space) and inter-vehicle coordination (in time) become essentially decoupled. The system that is obtained by putting together the path-following and vehicle coordination strategies takes a cascade form, where the output of the latter is an input to the former subsystem. The main result in this paper is that under mild assumptions on the connectivity of the graph induced by the communications network, and assuming periodic communications, stability and convergence of the proposed combined path-following/coordination system are guaranteed. We also suggest a design criterion to aid in the selection of the coordi-

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The authors are with the Institute for System and Robotics, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal.

E-mails: {jalmeida,cjs,antonio}@isr.ist.utl.pt

nation control gains. In the proposed technique, the gains of the distributed coordination controller are found by solving an optimization problem where the objective is to maximize the rate of convergence of the coordination subsystem. The paper builds upon and combines previous results on Path-Following control [3], Coordinated Path-Following [4], [5], [6], [7] and Networked Control Systems [2].

The paper is organized as follows. Section II describes the dynamic model of the autonomous vehicles considered. The coordinated path-following problem is formally stated in Section III. Section IV introduces the general structure of the proposed controller. Section V presents a solution to the path-following problem. Section VI offers a solution to the coordinated path-following problem with periodic communications. A gain selection criterion is also described for the coordination subsystem. Section VII gives an illustrative example where simulation results are presented. Finally, Section VIII contains the conclusions and directions for future work.

II. Vehicle modeling

The kinematic and dynamic equations of a large class of vehicles are summarized next [8]. To this effect, consider an autonomous vehicle modeled as a rigid body subject to external forces and torques moving in a two dimensional space, as, for example, a surface vessel moving at sea. Let $\{\mathcal{I}\}$ be an inertial coordinate frame and $\{\mathcal{B}\}$ a body-fixed coordinate frame whose origin is located at the center of mass of the vehicle. The generalized position of the vehicle is $\boldsymbol{\eta} = (x, y, \psi)$, where (x, y) are the coordinates of the origin of $\{\mathcal{B}\}$ in $\{\mathcal{I}\}$ and ψ is the orientation of the vehicle (yaw angle) that parameterizes the body-to-inertial coordinate transformation matrix

$$J := J(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Denoting by $\boldsymbol{\nu} := (u, v, r)$ the generalized velocity of the vehicle relative to $\{\mathcal{I}\}$ expressed in $\{\mathcal{B}\}$, the following *kinematic* relations apply:

$$\dot{\boldsymbol{\eta}} = J\boldsymbol{\nu}\,, \qquad \dot{J} = rJS\,, \tag{1}$$

where S is a skew-symmetric matrix. We consider fully-actuated vehicles with *dynamic* equations of motion of the form [8]

$$M\dot{\boldsymbol{\nu}} = \boldsymbol{f}(\boldsymbol{\nu}, \boldsymbol{\eta}) + \boldsymbol{\tau} \tag{2}$$

where $M \in \mathbb{R}^{3\times 3}$ denotes a constant symmetric positive definite mass matrix, $\boldsymbol{\tau} = (\tau_u, \tau_v, \tau_r)$ is the generalized control input, and $\boldsymbol{f}(\boldsymbol{\nu}, \boldsymbol{\eta})$ represents all the remaining equivalent forces and torques acting on the body, related to Coriolis, centripetal and hidrodynamic damping effects. For marine vessels, M also includes the so-called hydrodynamic added-mass M_A, i.e., $M = M_{RB} + M_A$, where M_{RB} is the rigid-body mass matrix.

III. Problem statement

We now consider the problem of coordinated path-following (CPF). In the most general setup, we are given a set of $n \geq 2$ autonomous vehicles and a set of n spatial paths $\boldsymbol{\eta}_{di} = (x_{di}, y_{di}, \psi_{di})$ for $i = 1, 2, \ldots, n$. Each path $\boldsymbol{\eta}_{di}(\gamma_i)$ is parameterized by a continuous variable γ_i and it is required that vehicle i follow path $\boldsymbol{\eta}_{di}$. Following [5], the CPF problem is separated in two subproblems: a path-following problem for each vehicle, where we require the vehicle to follow a predefined desired spatial path, and a coordination problem for all vehicles addressing the constraints imposed by communication network topology.

The problem of path-following is formally stated next.

Path-following problem: Let $\boldsymbol{\eta}_{di}(\gamma_i) \in \mathbb{R}^3$ be a desired smooth path parameterized by a continuous variable $\gamma_i \in \mathbb{R}$ and $\upsilon_{di} \in \mathbb{R}$ a desired speed assignment for vehicle i. For each vehicle with equations of motion given by (1) and (2), design a feedback control law for $\boldsymbol{\tau}_i$ such that all closed-loop signals are bounded and as $t \to +\infty$ the position of the vehicle: i) converges to the desired path, i.e., $\|\boldsymbol{\eta}_i(t) - \boldsymbol{\eta}_{di}(\gamma_i(t))\| \to 0$; and, ii) satisfies a desired speed assignment along the path, i.e., $|\dot{\gamma}_i(t) - \upsilon_{di}(t)| \to 0$.

For the coordination problem, we start by introducing a measure of the degree of coordination of a fleet of vehicles. As in [6], this is done by reparameterizing each path $\boldsymbol{\eta}_{di}(\gamma_i)$ in terms of a conveniently defined variable ξ_i such that coordination is said to be achieved along the paths if $\xi_1 = \xi_2 = \ldots = \xi_n$. At this point, we formally define the “along-path” distances between vehicle i and j as $\xi_{i,j} = \xi_i - \xi_j$. Then, coordination is achieved if and only if $\xi_{i,j} = 0$ for all $i, j \in \{1, 2, \ldots, n\}$. Let the reparameterization of the path be represented by $\gamma_i = \gamma_i(\xi_i)$ and define $R_i(\xi_i) := \partial\gamma_i/\partial\xi_i$, which is assumed to be positive and bounded for all ξ_i. The dynamics of ξ_i and γ_i are related by

$$\dot{\gamma}_i = R_i(\xi_i)\dot{\xi}_i\,. \tag{3}$$

Suppose one vehicle, henceforth referred to as vehicle L, is elected as the “leader” and let the corresponding path η_{dL} be parameterized by $\gamma_L = \xi_L$. For this vehicle, $R_L(\xi_L) = 1$. Let υ_L be the desired constant speed assigned to the leader in advance, that is $\dot{\xi}_L = \upsilon_L$ in steady-state, known to all vehicles. From (3), it follows that the desired “along-path” speed of each vehicle equals $R_i(\xi_i)\upsilon_L$. It is important to point out that L can always be taken as a “virtual” vehicle that is added to the set of “real” vehicles as an expedient to simplify the coordination strategy.

So far, the problem of coordination has been reduced to that of “aligning” the coordination states ξ_i. To go from this alignment to a more complex, possibly time-varying, spatial configuration, we introduce appropriate offsets in the desired positions of the vehicles relative to the mean point of the formation as defined with respect to the paths. To this effect, let $\boldsymbol{\xi} := [\xi_i]_{n\times 1}$ and define the formation mean point and offsets as $\bar{\xi} := \frac{1}{n}\mathbf{1}^\top\boldsymbol{\xi}$ and $\boldsymbol{\delta} := \boldsymbol{\xi} - \bar{\xi}\mathbf{1}$, respectively. Notice that $\mathbf{1}^\top\boldsymbol{\delta} = 0$. Let $\boldsymbol{\phi} = \boldsymbol{\phi}(t) \in \mathbb{R}^n$ represent a desired formation pattern, which might be time dependent

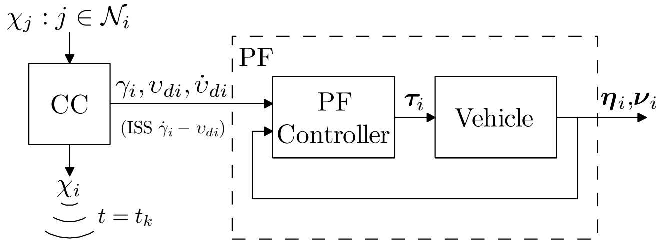

Fig. 1. Proposed control structure for coordinated path-following, represented for vehicle i.

and that verifies $\mathbf{1}^\top \boldsymbol{\phi} = 0$. The problem of coordination with time-varying pattern tracking is reduced to that of making $(\boldsymbol{\delta} - \boldsymbol{\phi}) \rightarrow \mathbf{0}$ as $t \rightarrow +\infty$. The speed assignment is now $\dot{\bar{\xi}} = \upsilon_L \Leftrightarrow \dot{\xi}_i - \dot{\phi}_i = \upsilon_L$. Again from (3), it follows that the desired "along-path" speeds for the vehicles are $\upsilon_{di} := R_i(\xi_i)(\upsilon_L + \dot{\phi}_i)$.

From a graph theoretical point of view, each vehicle is represented by a vertex and a communication link between two vehicles is represented by an arc (See [9] for an in-depth presentation of this subject). The communication links are assumed unidirectional, thereby inducing a directed graph. We consider time-invariant communication topologies and assume that the induced graph has at least one globally reachable vertex. The flow of information in an arc is directed from its head to its tail. The set of neighbors of vertex i is represent by $\mathcal{N}_i$ and it contains all vehicles from which vehicle i receives information.

For design purposes, we will take each $\dot{\xi}_i$ as a control input of the coordination dynamics (3). In order to satisfy the constraints imposed by the communication network, the control law for vehicle i must be decentralized, i.e, it may only depend on local states and/or on information exchange with its neighbors as specified by $\mathcal{N}_i$. The coordination problem is formulated as:

Coordinated problem: Design a decentralized feedback control law for $\dot{\boldsymbol{\xi}}$ such that $\boldsymbol{\delta} - \boldsymbol{\phi}$ tend asymptotically to zero and all vehicles travel at the desired "along-path" speed, that is, $\dot{\gamma}_i \rightarrow \upsilon_{di}$ for all i.

The coordination part of our CPF problem is closely related to agreement problems (see, e.g., [10]). In our case, all vehicles must agree on a common value, the mean point of the formation pattern, while they follow a desired speed profile under the constraint that the communications between vehicles be discrete-time periodic.

IV. Control structure (CPF)

In this section, we propose a control structure for the CPF problem. It is based on the work presented in [5] and is illustrated in Fig. 1 for the ith vehicle. The structure proposed shows a cascade of two subsystems: the path-following (PF) subsystem and the coordination control (CC) subsystem.

The PF subsystem is formed by the vehicle and a feedback controller designed to guide the vehicle to the desired path. The PF controller drives the vehicle through its command inputs $\boldsymbol{\tau}_i$ using a control law that depends on the vehicle's position $\boldsymbol{\eta}_i$ and velocity $\boldsymbol{\nu}_i$, and on signals provided by the CC subsystem.

The CC subsystem handles all the communications with neighboring vehicles and provides the PF subsystem with the inputs γ_i, υ_{di}, and $\dot{\upsilon}_{di}$. The exchange of information occurs at discrete-time instants $\{t_k\}$, that will henceforth be referred to as *update times*, of the form $\{t_k = hk + t_0 : k \in \mathbb{N}\}$ where h is the communication period in seconds. At these update times, a certain information variable χ_i, to be defined later, is sent by vehicle i to its adjacent vehicles and information variables $\chi_j : j \in \mathcal{N}_i$ from its neighbors are received. This information variables are the only data necessary to exchange among vehicles to achieve coordination.

The PF subsystem will be shown to be input-to-state stable (ISS) with respect to the input $\dot{\gamma}_i - \upsilon_{di}$ that, although not explicit in the structure, is present in the control laws used.

A sampled-data based approach to the problem of coordinated path-following was proposed in [11], where the variables parameterizing the paths (γ_i) evolve in a discrete fashion, and therefore the coordination control problem is posed in discrete-time. The authors also consider two different structures: a cascade connection similar to ours and a feedback interconnection where the CC subsystem receives a path error feedback from the PF subsystem. However, the types of communication links considered in [11] are only bidirectional.

V. Path-following

Central to the development of CPF strategies is the derivation of appropriate path-following control laws for each vehicle. In this section, we briefly describe a path-following controller for autonomous vehicles. See, for example, [12] and [3] for background material. The controller here presented is local to each vehicle so the index i will be omitted for the sake of simplicity.

Define the position error in the body-fixed frame as $\boldsymbol{z}_1 := J^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_d)$, and let $\zeta := \dot{\gamma} - \upsilon_d$ denote the "along-path" speed tracking error. Path-following is equivalent to driving $\boldsymbol{z}_1$ and ζ to zero. Applying backstepping design procedures, we obtain the feedback control law

$$\boldsymbol{\tau} = -\boldsymbol{z}_1 - K_2\boldsymbol{z}_2 - \boldsymbol{f} + M(\boldsymbol{\alpha}^t + \boldsymbol{\alpha}^\gamma \upsilon_d)\,,$$

where $\boldsymbol{z}_2 := \boldsymbol{\nu} - \boldsymbol{\alpha}$ is the velocity error of the vehicle and

$$\boldsymbol{\alpha} := J^\top \boldsymbol{\eta}_d^\gamma \upsilon_d - K_1\boldsymbol{z}_1\,,\ \boldsymbol{\alpha}^\gamma := K_1 J^\top \boldsymbol{\eta}_d^\gamma + J^\top \boldsymbol{\eta}_d^{\gamma^2} \upsilon_d\,,$$
$$\boldsymbol{\alpha}^t := -K_1(\boldsymbol{\nu} - rS\boldsymbol{z}_1) - rSJ^\top \boldsymbol{\eta}_d^\gamma \upsilon_d + J^\top \boldsymbol{\eta}_d^\gamma \dot{\upsilon}_d\,,$$

for which the time derivative of the Lyapunov function

$$V := \frac{1}{2}\boldsymbol{z}_1^\top \boldsymbol{z}_1 + \frac{1}{2}\boldsymbol{z}_2^\top M \boldsymbol{z}_2\,,$$

along the solutions of (1) and (2), takes the form

$$\dot{V} = -\boldsymbol{z}_1^\top K_1 \boldsymbol{z}_1 - \boldsymbol{z}_2^\top K_2 \boldsymbol{z}_2 + \mu\zeta\,,$$

where $\mu := -(\boldsymbol{\eta}_d^\gamma)^\top J \boldsymbol{z}_1 - (\boldsymbol{\alpha}^\gamma)^\top M \boldsymbol{z}_2$ admits the bound $|\mu| \leq \beta_1 \|\boldsymbol{z}_1\| + \beta_2 \|\boldsymbol{z}_2\|$ for some positive constants β_1 and β_2.

Lemma 1: The PF subsystem described is ISS with respect to input ζ.

VI. CPF WITH PERIODIC COMMUNICATION

We now offer a solution to the coordination problem. We start with a control law for $\dot{\xi}_i$ adapted from [7] and [6] that solves the coordination problem under continuous communication:

$$\dot{\xi}_i = \upsilon_L + \dot{\phi}_i - k_{ci} \sum_{j \in \mathcal{N}_i} (\xi_i - \phi_i - \xi_j + \phi_j) \,, \qquad (4)$$

where $k_{ci} > 0$ is an adjustable control gain and ϕ_i are the components of desired formation pattern represented by $\boldsymbol{\phi}$. Notice that the information required by vehicle i about its neighbors is $\chi_j := \xi_j - \phi_j$, that we refer to as *information state*, and not the coordination state ξ_j itself. The control law (4) can be rewritten as

$$\dot{\chi}_i = \upsilon_L - k_{ci} d_i \chi_i + k_{ci} \sum_{j \in \mathcal{N}_i} \chi_j \qquad (5)$$

where d_i is the number of neighbors of vehicle i (out-degree of vertex i). When using periodic communications, vehicle i does not receive $\chi_j : j \in \mathcal{N}_i$ between update times, so it needs to model their evolution in that interval. Let $\hat{\chi}^i_j$ represent local "replicas" of each χ_j as seen by vehicle i, that we refer to as *predictor states*. Analyzing (5), we see that if a steady-state condition is achieved, then $\dot{\chi}_i = \upsilon_L$ for all i. This suggests that the dynamics of χ_j can be predicted as $\dot{\hat{\chi}}^i_j = \upsilon_L$, thus yielding the controller

$$\dot{\chi}_i = \upsilon_L - k_{ci} d_i \chi_i + k_{ci} \sum_{j \in \mathcal{N}_i} \hat{\chi}^i_j \qquad \text{(control law)}\,, \qquad (6)$$

$$\dot{\hat{\chi}}^i_j = \upsilon_L \,, \ \text{for each } j \in \mathcal{N}_i \qquad \text{(model)}\,. \qquad (7)$$

However, this is not sufficient to achieve coordination due to initial conditions that do not match the desired formation pattern. To overcome this problem, a reset is made to the predictor states when information is exchanged. We therefore add the following condition to the controller:

$$\hat{\chi}^i_j(t_k) = \chi^i_j(t_k^-), \ \text{for each } j \in \mathcal{N}_i \qquad \text{(update)}\,, \qquad (8)$$

where the notation $x(t^-)$ stands for the left limit or limit from below, i.e., $x(t^-) = \lim_{s \nearrow t} x(s)$. Because all $\hat{\chi}^i_j$ are initialized with the same value $\chi_j(t_0)$, and because we assume there is absolute synchronization with respect to update times, vehicles that model the same predictor state have equal values, i.e, $\hat{\chi}^{i_1}_j = \hat{\chi}^{i_2}_j$ for all i_1, i_2 and j. Therefore, we do not need to refer to $\hat{\chi}^{i_1}_j$ and $\hat{\chi}^{i_2}_j$ as different states, we simply refer to them as $\hat{\chi}_j$.

Defining $\hat{\boldsymbol{\chi}} := \boldsymbol{\xi} - \boldsymbol{\phi} = [\hat{\chi}_i]_{n\times 1}$, and the diagonal matrix $K_c := \mathrm{diag}[k_{ci}]_{n\times n}$ (where $k_{cr} = 0$ if $d_r = 0$ for some $1 \leq r \leq n$), (6)-(8) can be written in vector form as

$$\dot{\boldsymbol{\chi}} = \upsilon_L \mathbf{1} - K_c D \boldsymbol{\chi} + K_c A \hat{\boldsymbol{\chi}} \qquad \text{(control law)}\,, \qquad (9a)$$

$$\dot{\hat{\boldsymbol{\chi}}} = \upsilon_L \mathbf{1} \qquad \text{(model)}\,, \qquad (9b)$$

$$\hat{\boldsymbol{\chi}}(t_k) = \boldsymbol{\chi}(t_k^-) \qquad \text{(update)}\,, \qquad (9c)$$

where $D = \mathrm{diag}[d_i]_{n\times n}$ and A are the out-degree and adjacency matrices associated to the communications graph, respectively.

A. Error space and dynamics

The Laplacian of a graph is defined as $\mathcal{L} := D - A$. If a graph contains a globally reachable vertex, then L can be decomposed as $\mathcal{L} = FG$ where $F \in \mathbb{R}^{n\times(n-1)}$, $G \in \mathbb{R}^{(n-1)\times n}$, $\mathrm{rank}\, F = \mathrm{rank}\, G = n-1$ and $G\mathbf{1} = \mathbf{0}$ (See [13] for algebraic properties of the Laplacian). We define the coordination error as

$$\boldsymbol{\theta} := G(\boldsymbol{\xi} - \boldsymbol{\phi}) = G\boldsymbol{\chi} \in \mathbb{R}^{n-1}\,.$$

Since $G\mathbf{1} = \mathbf{0}$ and using the definitions of formation mean point and offsets of Section III, we have

$$G(\boldsymbol{\xi} - \boldsymbol{\phi}) = G(\boldsymbol{\delta} + \bar{\xi}\mathbf{1} - \boldsymbol{\phi}) = G(\boldsymbol{\delta} - \boldsymbol{\phi})\,.$$

Because $\mathbf{1}^\top \boldsymbol{\delta} = \mathbf{1}^\top \boldsymbol{\phi} = 0$, $\boldsymbol{\delta} - \boldsymbol{\phi}$ is normal to the null space of G. We conclude that $\boldsymbol{\theta} = \mathbf{0}$ if and only if $(\boldsymbol{\delta} - \boldsymbol{\phi}) = \mathbf{0}$. Let

$$\tilde{\boldsymbol{\chi}} := \boldsymbol{\chi} - \hat{\boldsymbol{\chi}} \in \mathbb{R}^n$$

represent the predictor state error. If $\tilde{\boldsymbol{\chi}} = \mathbf{0}$, then the information states are coherent, i.e, the predictor states equal the actual states. Considering (9), the error dynamics for $\boldsymbol{\theta}$ and $\tilde{\boldsymbol{\chi}}$ are given by

$$\begin{aligned} \dot{\tilde{\boldsymbol{\chi}}} &= -K_c D\boldsymbol{\chi} + K_c A \hat{\boldsymbol{\chi}} = -K_c D\boldsymbol{\chi} + K_c A(\boldsymbol{\chi} - \tilde{\boldsymbol{\chi}}) \\ &= -K_c \mathcal{L}\boldsymbol{\chi} - K_c A\tilde{\boldsymbol{\chi}} = -K_c F\boldsymbol{\theta} - K_c A \tilde{\boldsymbol{\chi}}\,, \\ \dot{\boldsymbol{\theta}} &= G(-K_c\mathcal{L}\boldsymbol{\chi} - K_c A\tilde{\boldsymbol{\chi}}) = -GK_cF\boldsymbol{\theta} - GK_cA\tilde{\boldsymbol{\chi}}\,, \end{aligned}$$

where we used the fact that $\mathcal{L}\boldsymbol{\chi} = FG\boldsymbol{\chi} = F\boldsymbol{\theta}$. Defining the aggregated state variable $\boldsymbol{z} := (\boldsymbol{\theta}, \tilde{\boldsymbol{\chi}})$, the above error dynamics can be written as

$$\begin{cases} \dot{\boldsymbol{z}} = \Lambda \boldsymbol{z}, & t \in [t_k, t_{k+1}) \\ \boldsymbol{z}(t) = (\boldsymbol{\theta}(t^-), \mathbf{0}), & t = t_k \end{cases} \qquad (10)$$

where

$$\Lambda := \begin{bmatrix} -GK_cF & -GK_cA \\ -K_cF & -K_cA \end{bmatrix}. \qquad (11)$$

The dynamic system (10) is a linear impulsive system, whose time response is given in the next proposition.

Proposition 1 ([2]): The system described by (10) with initial conditions $\boldsymbol{z}(t_0) = (\boldsymbol{\theta}(t_0), \mathbf{0})$ has the time response

$$\boldsymbol{z}(t) = e^{\Lambda(t-t_k)} \Phi^k \boldsymbol{z}(t_0)$$

for $t \in [t_k, t_{k+1})$, where

$$\Phi := \begin{bmatrix} I_{n-1} & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h} \begin{bmatrix} I_{n-1} & 0 \\ 0 & 0 \end{bmatrix}. \qquad (12)$$

Note that if

$$e^{\Lambda h} = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}, \ \text{then}\ \Phi = \begin{bmatrix} E_{11} & 0 \\ 0 & 0 \end{bmatrix},$$

with $E_{11} \in \mathbb{R}^{(n-1)\times(n-1)}$. Therefore n of the eigenvalues of Φ are 0, while the remainder correspond to the eigenvalues of E_{11}. The next theorem establishes a sufficient and necessary condition for asymptotic stability of (10).

Theorem 1 ([2]): For the system described by (10), the origin is a globally exponentially stable equilibrium point if and only if Φ is a *convergent matrix* (all its eigenvalues are strictly inside the unit circle).

B. Main result

We start by showing that the interconnection of the n CC subsystems is globally asymptotically stable (GAS). This proof involves checking that the condition of Theorem 1 is always satisfied.

First, a closed-form expression for E_{11} is derived by showing that Λ is similar to a diagonal matrix, which allows us to easily compute the matrix exponential $e^{\Lambda h}$.

Lemma 2: Consider the matrix Λ defined in (11). Let $P, \Delta \in \mathbb{R}^{n\times n}$ be defined as

$$\Delta := \begin{bmatrix} -K_c D & 0_{n\times(n-1)} \\ 0_{(n-1)\times n} & 0_{n-1} \end{bmatrix}, P := \begin{bmatrix} G & I_{n-1} - GD^+F \\ I_n & -D^+F \end{bmatrix}.$$

where $D^+ = \operatorname{diag}(d_1^+, \dots, d_n^+)$ stands for the pseudoinverse of D and

$$d_i^+ := \begin{cases} d_i^{-1}, & \text{if } d_i > 0 \\ 0, & \text{if } d_i = 0 \end{cases} \quad \text{for } i = 1, 2, \dots, n.$$

Then P is nonsingular with inverse given by

$$P^{-1} = \begin{bmatrix} D^+F & I_{n-1} - D^+(D-A) \\ I_{n-1} & -G \end{bmatrix}$$

and $\Lambda = P\Delta P^{-1}$.

Using Lemma 2, the matrix exponential results in

$$e^{\Lambda h} = Pe^{\Delta h}P^{-1} = P \begin{bmatrix} e^{-K_c D h} & 0_{n\times(n-1)} \\ 0_{(n-1)\times n} & I_{n-1} \end{bmatrix} P^{-1}$$

which gives the following closed-form expression for E_{11}:

$$E_{11} = I_{n-1} - G\left(I_n - e^{-K_c D h}\right) D^+ F\,.$$

Theorem 2: For any communication graph with at least one globally reachable vertex, fixed period $h > 0$, and controller gains $k_{ci} > 0$ for all i, the matrix Φ defined in (12) is a convergent matrix.

The next theorem states that the control structure proposed in Section IV, together with the control laws developed in Sections V and VI, solve the CPF problem presented in Section III.

Theorem 3: The overall system formed by the interconnection of the n combined PF/CC systems is GAS.

C. Gain Selection

The proof of Theorem 2 shows that as $k_{ci} \to +\infty$, the eigenvalues of E_{11} tend to the eigenvalues of $I_n - D^+\mathcal{L}$ apart from the zero eigenvalue. When $k_{ci} = 0$ for all i, then $E_{11} = I_{n-1}$ and all eigenvalues are one. Due to the continuity of the eigenvalues of a matrix with respect to its entries, we conclude that the eigenvalues of E_{11} start off from one and tend to "limit" values. Let $\rho(E_{11})$ represent the spectral radius of E_{11}. The spectral radius of a non-symmetric matrix is a continuous function of its entries but is neither convex nor locally Lipschitz. Minimizing $\rho(E_{11})$ is equivalent to maximizing the rate of convergence of the coordination system at update times. To this effect, define $\boldsymbol{k}_c := \operatorname{diag}(K_c) \in \mathbb{R}^n$. We then formulate the optimization problem

$$\boldsymbol{k}_c^* = \begin{array}{c} \arg\min \\ \boldsymbol{k}_c > 0 \end{array} \rho\big(E_{11}(\boldsymbol{k}_c)\big)\,. \tag{13}$$

Because of the properties of $\rho(E_{11})$, there is no efficient way of solving this problem. However, we present a possible method for handling this difficulty by considering a simplification of the optimization variable $\boldsymbol{k}_c$. Given the special structure of E_{11}, a simplification can be made that reduces the complexity of (13). The gain matrix K_c is restricted to the be of the form $K_c = k_c h^{-1} D^+$, where $k_c > 0$. For this type of gain matrix, the eigenvalues of E_{11} move along straight lines from their initial to their final positions. Using the change of variables $y = 1 - e^{-k_c}$, the optimal gain k_c is obtained by solving

$$y^* = \begin{array}{c} \arg\min \\ E_{11} = I_{n-1} - yGD^+F \\ 0 \le y \le 1 \end{array} \rho(E_{11}(y))\,. \tag{14}$$

Due to the special structure of E_{11}, Problem (14) is equivalent to

$$y^* = \begin{array}{c} \arg\min \\ |1 - y\lambda_i| \le t, i = 1, 2, \dots, n-1 \\ 0 \le y \le 1 \end{array} t \tag{15}$$

where $\lambda_i \in 1 - \sigma(D^+\mathcal{L})\backslash\{0\}$. The optimal gain is given by $k_c^* = \min\{-\ln(1-y^*), k_{max}\}$ where k_{max} is a positive constant used to limit the value of k_c^* when y^* is close to one. Problem (15) belongs to a class of problems known as second-order cone programming (SOCP, see, e.g, [14]) for which a global minimum can easily be computed.

VII. Illustrative Example

We consider a group of 3 identical vehicles whose kinematics and dynamics equations of motion can be written as in (1)-(2), with

$$M = \operatorname{diag}(m_u, m_v, I_r),\ \boldsymbol{f}(\boldsymbol{\nu}, \boldsymbol{\eta}) = D(\boldsymbol{\nu})\boldsymbol{\nu} + C(\boldsymbol{\nu})\boldsymbol{\nu}$$
$$D(\boldsymbol{\nu}) = \operatorname{diag}(X_u + X_{|u|u}|u|, Y_v + Y_{|v|v}|v|, N_r + N_{|r|r}|r|),$$
$$C(\boldsymbol{\nu}) = \begin{bmatrix} 0 & -m_v r & 0 \\ m_u r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In the simulations presented, the physical parameters are

$X_u = -1\,\mathrm{kg/s}$	$Y_{\|v\|v} = -200\,\mathrm{kg}$	$m_u = 500\,\mathrm{kg}$
$X_{\|u\|u} = -25\,\mathrm{kg/m}$	$N_r = -0.5\,\mathrm{kgm^2/s}$	$m_v = 1000\,\mathrm{kg}$
$Y_v = -10\,\mathrm{kg/s}$	$N_{\|r\|r} = -1500\,\mathrm{kgm^2}$	$I_r = 700\,\mathrm{kgm^2}$

The communications graph is a directed circular graph and $h = 10\,\mathrm{s}$. Each vehicle is required to follow a "U" like path formed by three different sections: i) a straight line with an orientation of $0°$; ii) a half circumference arc traversed clockwise; iii) and, a straight line with an orientation of $-180°$. The vehicles are required to keep an in-line formation pattern, $\boldsymbol{\phi}_0 = (0, 0, 0)$ until $t_1 = 150\,\mathrm{s}$ when the formation starts to change into a triangle like shape $\boldsymbol{\phi}_1 = \frac{1}{3}(5, -10, 5)$.The initial conditions of each vehicle are $\boldsymbol{\eta}_1(t_0) = (-5, -5, \pi/3)$, $\boldsymbol{\eta}_2(t_0) = (-5, 15, -\pi/4)$, $\boldsymbol{\eta}_3(t_0) = (5, 17, 2\pi/3)$, $u_i(t_0) = v_i(t_0) = 0\,\mathrm{m/s}$ and $r_i(t_0) = 0\,\mathrm{rad/s}$ for $i = 1, 2, 3$. The initial condition for $\boldsymbol{\gamma}$ is chosen so that for each vehicle i, γ_i yields the closest point on the respective path. This gives $\boldsymbol{\gamma}(t_0) = (-5, -5, 5)\,[\mathrm{m}]$.

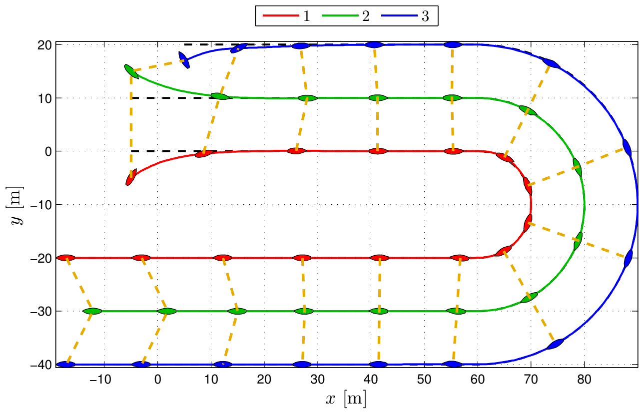

Fig. 2. Trajectory of each vehicle in the 2D-plane.

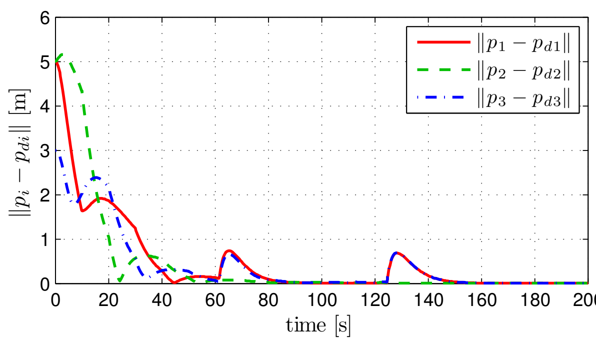

Fig. 3. Time evolution of the path-following position errors.

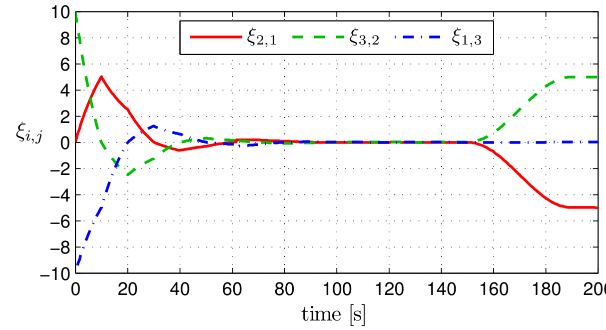

Fig. 4. Time evolution of the "along-path" distances.

The reference speed is set to $v_L = 1\,\mathrm{s}^{-1}$. The scale radii are $R_i = 1\,\mathrm{m}$ for the straight lines and $R_i = i\,\mathrm{m}$ for the half circumferences. The PF gains are $K_1 = 0.15 I_3$ and $K_2 = 200 I_3$. The optimum gain is $k_c^* = 0.6931$, yielding the eigenvalues $\sigma(E_{11}^*) = \{0.25 \pm 0.433i\}$, with spectral radius $\rho(E_{11}^*) = 0.5$, and $K_c^* = 0.0693 I_3$. Fig. 2 illustrates the trajectories of the vehicles in the two-dimensional space. Fig. 3 shows the time evolution of the position errors of each vehicle, given by $\|p_i(t) - p_{di}(\gamma_i(t))\|$, where $p_i(t) = (x_i(t), y_i(t))$ and $p_{di}(t) = (x_{di}(\gamma_i(t)), y_{di}(\gamma_i(t)))$. The small peaks at $t = 60\,\mathrm{s}$ and $t = 125\,\mathrm{s}$ steam from the fact that the path is not differentiable when changing from straight line to circumference and vice-versa. Fig. 4 presents the time evolution of the "along-path" distances. As can be seen, the evolution of $\xi_{i,j}$ agrees with the initial in-line formation and after $t = t_1$ tends to the triangle formation defined by ϕ_1.

VIII. Conclusions and Future Work

We addressed the problem of coordinated path-following (CPF) for a group of autonomous vehicles using periodic communications. We proposed and analyzed a decentralized control structure formed by a cascade of two subsystems named path-following (PF) and coordination control (CC). Stability and convergence of the overall system are guaranteed for any fixed update period, as long as the graph induced by the communication network has at least one globally reachable vertex. We also suggested a criterion for selecting the coordination control gains.

Future work includes the study of the applicability of the proposed control structure to address coordination problems involving time delays, communication failures, and asynchronous updates. The study of an interconnected control structure with a feedback term from the PF subsystem to the CC subsystem will also be studied, as it makes the overall system more robust to isolated vehicle failures and yields smoother convergence of the vehicles to the paths.

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