COOPERATIVE PATH-FOLLOWING OF MULTIPLE SURFACE VESSELS WITH PARAMETRIC MODEL UNCERTAINTY AND IN THE PRESENCE OF OCEAN CURRENTS

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Abstract: This paper addresses the problem of cooperative path-following of multiple autonomous vehicles. Stated briefly, the problem consists in steering a group of vehicles along specified paths, while keeping a desired spatial formation arrangement. For a given class of autonomous surface vessels, it is shown how Lyapunov based techniques and graph theory can be brought together to design a decentralised control structure where the vehicle dynamics and the constraints imposed by the topology of the inter-vehicle communication network are explicitly taken into account. Path-following for each vehicle amounts to reducing an appropriately defined geometric error to zero, in the presence of ocean currents and parametric model uncertainty. Vehicle cooperation is achieved by adjusting the speed of each vehicle along its path according to information exchanged on the positions of a subset of the other vehicles, as determined by the communication topology adopted. Global stability and convergence of the closed-loop system are guaranteed.

Keywords: Cooperative motion control, Path-following, Autonomous surface vehicles, Graph theory, Nonlinear control, Adaptive control.

1. INTRODUCTION

The topic of multi-agent system cooperation is of great interest in a wide range of scientific and technological areas such as mobile robotics, biology, and artificial intelligence, among others. Using a group of simple, and possibly inexpensive robots, not only allows for the execution of tasks too complex for a single robot, but also increases the efficiency and efficacy in the execution of those same tasks.

This paper addresses the problem of cooperative path-following (CPF) where multiple vehicles are required to follow pre-specified spatial paths while keeping a desired inter-vehicle formation pattern in time. This problem arises, for example, in the operation of multiple autonomous marine vehicles for fast acoustic coverage of large areas of the seabed. By imposing constraints on the inter-vehicle formation pattern, the efficacy of the task can be largely improved. A number of other realistic vehicle operation scenarios can also be envi-

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sioned that require cooperative motion control of marine or other types of vehicles.

The CPF problem is solved for a class of fully-actuated autonomous vehicles moving in two dimensional space. Nevertheless, the results presented here can be extended to the three dimensional case. The solution adopted is based on Lyapunov stability theory and addresses explicitly the vehicle dynamics as well as the constraints imposed by the topology of the inter-vehicle communication network, where the latter are tackled in the framework of graph theory. Each vehicle is equipped with a controller that makes the vehicle follow a predefined path. The speed of each vehicle is then adjusted so that the whole group of vehicles keeps a desired formation pattern. A supporting communication network provides the vehicles with the medium over which to exchange the information required for synchronization. The communication links established among vehicles are assumed to be bidirectional: one vehicle sends information to its neighbors and necessarily receives information back. We also assume that the transmission delay is negligible and that there are no collisions when vehicles communicate simultaneously. With the control structure adopted, path-following (in space) and inter-vehicle cooperation (in time) become essentially decoupled. The system that is obtained by putting together the path-following and vehicle cooperation strategies takes a interconnected form.

The paper builds upon and combines previous results on Path-Following control (Skjetne *et al.* 2005), and Cooperative Path-Following (Ghabcheloo *et al.* 2006, Aguiar *et al.* 2005). Its main contribution is the derivation of a cooperative path-following methodology for surface vessels with parametric model uncertainty and subject to unknown but constant ocean currents.

The paper is organised as follows. Section 2 describes the dynamic model of the autonomous vehicles considered. In Section 3, key concepts from graph theory are introduced. The problems of path-following and cooperation are formulated in Section 4. In Sections 5 and 6, a strategy for cooperative path-following is developed. Section 7 gives an illustrative example and contains simulation results. Finally, in Section 8 conclusions and directions for future work are presented.

2. VEHICLE MODELING

Consider an autonomous vehicle modelled as a rigid body subject to external forces and torques, as for example a surface vessel moving in the sea. Let $\{\mathcal{I}\}$ be an inertial coordinate frame and $\{\mathcal{B}\}$ a body-fixed coordinate frame with its origin at the centre of mass of the vehicle, as represented

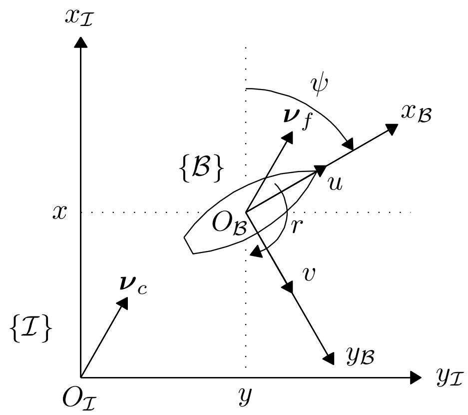

Fig. 1. Inertial and body-fixed coordinate frames.

in Fig. 1. The generalized position of the vehicle is $\boldsymbol{\eta} := (x, y, \psi)$, where (x, y) are the coordinates of the origin of $\{\mathcal{B}\}$ in $\{\mathcal{I}\}$, and ψ is the orientation of the vehicle (yaw angle) that parameterizes the matrix $J(\psi)$, transforming body coordinates into inertial coordinates, given by

$$J := J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Denote by $\boldsymbol{\nu} := (u, v, r)$ the generalized velocity of the vehicle relative to $\{\mathcal{I}\}$ expressed in $\{\mathcal{B}\}$. In general, the fluid is in motion. To take into account this motion, let $\boldsymbol{\nu}_f := (u_f, v_f, r_f)$ be the generalized velocity of the fluid relative to $\{\mathcal{I}\}$ expressed in $\{\mathcal{B}\}$. The fluid is assumed to be irrotational, that is, $r_f = 0$. Let $\boldsymbol{\nu}_c := (u_c, v_c, 0)$ represent the velocity of the ocean current, expressed in $\{\mathcal{I}\}$. The velocities $\boldsymbol{\nu}_f$ and $\boldsymbol{\nu}_c$ are related by $\boldsymbol{\nu}_f = J^\top \boldsymbol{\nu}_c$. The ocean current is assumed constant, that is, $\dot{\boldsymbol{\nu}}_c = 0$. Let $\boldsymbol{\nu}_r := \boldsymbol{\nu} - \boldsymbol{\nu}_f$ denote the relative velocity between the vessel and the fluid. Then, the following kinematic relations apply

$$\dot{\boldsymbol{\eta}} = J\boldsymbol{\nu}_r + \boldsymbol{\nu}_c \,, \tag{1a}$$

$$\dot{J} = rJS \,, \tag{1b}$$

where S is the skew-symmetric matrix

$$S = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (S^\top = -S).$$

We consider a fully-actuated vehicle with equations of motion of the form,

$$M\dot{\boldsymbol{\nu}}_r = \boldsymbol{\tau} - C(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{f}(\boldsymbol{\nu}_r) \tag{2}$$

where $M \in \mathbb{R}^{3\times 3}$, $M \succ 0$ denotes a constant symmetric positive definite mass matrix, $C(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r$ represent forces and torques that capture Coriolis and centripetal effects, $\boldsymbol{\tau} := (\tau_u, \tau_v, \tau_r)$ is the generalized control input consisting of forces τ_u, τ_v and torque τ_r, and $\boldsymbol{f}(\boldsymbol{\nu}_r)$ represents the hidrodynamic damping forces and torques acting on the body. For the special case of surface vessels, M also includes the so-called hydrodynamic added-mass M_A, i.e., $M = M_{RB} + M_A$, where M_{RB} is the rigid-body mass matrix. Equation (2) depends on a set of physical parameters, some of which are often only known with great uncertainty. In fact, while the coefficients of M and $C(\boldsymbol{\nu}_r)$ can be determined with good accuracy, some of the coefficients related to the hydrodynamic damping $\boldsymbol{f}(\boldsymbol{\nu}_r)$ are difficult to measure, so they should be considered unknown or uncertain and estimated by the adaptive controller to be designed. Let n_p be the total number of hydrodynamic coefficients or parameters, and let $\boldsymbol{\varphi} \in \mathbb{R}^{n_p}$ be the vector that represents them. It is assumed that these parameters are constant, that is $\dot{\boldsymbol{\varphi}} = \mathbf{0}$, and that $\boldsymbol{f}(\boldsymbol{\nu}_r)$ depends linearly on $\boldsymbol{\varphi}$. This implies that $\boldsymbol{f}(\boldsymbol{\nu}_r)$ can be expressed as $\Phi(\boldsymbol{\nu}_r)\boldsymbol{\varphi}$ with $\Phi(\boldsymbol{\nu}_r) \in \mathbb{R}^{3\times n_p}$. Therefore, (2) can be rewritten as

$$M\dot{\boldsymbol{\nu}}_r = \boldsymbol{\tau} - C(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \Phi(\boldsymbol{\nu}_r)\boldsymbol{\varphi}\,. \tag{3}$$

Define $\boldsymbol{\varphi}_k \in \mathbb{R}^{n_k}$ as the vector of known parameters and $\boldsymbol{\varphi}_u \in \mathbb{R}^{n_u}$ as the vector of uncertain parameters, such that $n_p = n_k + n_u$ and

$$\Phi(\boldsymbol{\nu}_r)\boldsymbol{\varphi} = \Phi_k(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_k + \Phi_u(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_u \tag{4}$$

where $\Phi_k(\boldsymbol{\nu}_r) \in \mathbb{R}^{3\times n_k}$ and $\Phi_u(\boldsymbol{\nu}_r) \in \mathbb{R}^{3\times n_u}$. Replacing (4) in (3) yields

$$M\dot{\boldsymbol{\nu}}_r = \boldsymbol{\tau} - C(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \Phi_k(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_k + \Phi_u(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_u\,.$$

To summarise, the equations of motion of the vehicle, with parametric model uncertainty and subjected to a constant ocean current, are

$$\dot{\boldsymbol{\eta}} = J\boldsymbol{\nu}_r + \boldsymbol{\nu}_c\,, \tag{5a}$$

$$M\dot{\boldsymbol{\nu}}_r = \boldsymbol{\tau} - C(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \Phi_k(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_k + \Phi_u(\boldsymbol{\nu}_r)\boldsymbol{\varphi}_u\,. \tag{5b}$$

3. GRAPH THEORY

Graph theory has become the tool *par excellence* to model constraints imposed by the nature of the inter-vehicle communication network in cooperating mission scenarios. In this section, some key concepts and properties of graph theory are recalled (see, e.g., (Godsil and Royle 2001)). Because in this paper all communication links are assumed to be bidirectional, the presentation is focused on undirected graphs. A graph $\mathcal{G} = \mathcal{G}(\mathcal{V}, \mathcal{E})$ consists of a finite set $\mathcal{V} = \{1, 2, \ldots, n\}$ of n vertices and a finite set $\mathcal{E}$ of m pairs of vertices $\{i, j\} \in \mathcal{E}$ named edges. If $\{i, j\}$ belongs to $\mathcal{E}$ then i and j are said to be adjacent. A path from i to j is a sequence of distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. If there is a path in $\mathcal{G}$ between any two vertices, then $\mathcal{G}$ is said to be connected. The adjacency matrix of a graph $\mathcal{G}$, denoted A, is a square matrix with rows and columns indexed by the vertices, such that the i, j-entry of A is one if $\{i, j\} \in \mathcal{E}$ and zero otherwise. The degree matrix Δ of a graph $\mathcal{G}$ is a diagonal matrix where the i, i-entry is equal to the number of adjacent vertices of vertex i. The Laplacian of a graph is defined as $L := \Delta - A$. By definition, the Laplacian is a symmetric matrix that satisfies $L\mathbf{1} = \mathbf{0}$ with $\mathbf{1} = [1]_{n\times 1}$, therefore 0 is eigenvalue of L with $\mathbf{1}$ being its associated eigenvector. If the graph is connected, then all other eigenvalues of the Laplacian are positive. This implies that for a connected graph, $\operatorname{rank} L = n - 1$ so there exists a matrix $G \in \mathbb{R}^{n\times(n-1)}$ such that $L = GG^\top$, where $\operatorname{rank} G = n - 1$ and $G^\top\mathbf{1} = \mathbf{0}$.

4. CONTROL PROBLEM FORMULATION

We now consider the problem of cooperative path-following (CPF). In the most general set-up, one is given a set of $n \geq 2$ autonomous vehicles and a set of n spatial paths $\boldsymbol{\eta}_{di}(\gamma_i)$, parameterized by some real variable γ_i, and require that vehicle i follow path $\boldsymbol{\eta}_{di}$. The CPF problem is formulated as two problems: a path-following problem for each vehicle, where we require each vehicle to follow a predefined desired path; and, a cooperation problem for all vehicles addressing the constraints imposed by the communication network topology.

In order to solve the path-following problem, and because there are uncertain parameters and the ocean current is assumed unknown, estimates of these quantities must be computed on-line by the controller to be implemented. For each vehicle, the estimate of the ocean current, expressed in $\{\mathcal{I}\}$, is denoted by $\hat{\boldsymbol{\nu}}_{ci} = (\hat{u}_{ci}, \hat{v}_{ci}, 0)$, where only the first two components need to be estimated since the fluid is assumed irrotacional. The estimates of the uncertain parameters are represented by $\hat{\boldsymbol{\varphi}}_i \in \mathbb{R}^{n_u}$. An adaptation law establishes how $\hat{\boldsymbol{\nu}}_{ci}$ and $\hat{\boldsymbol{\varphi}}_i$ should evolve in time in order to meet some appropriately defined goal. In our case, their evolution is determined by their time derivatives $\dot{\hat{\boldsymbol{\nu}}}_{ci}$ and $\dot{\hat{\boldsymbol{\varphi}}}_i$, respectively, which are chosen as functions of state variables, in order to guarantee that the vehicle follows the desired path. It is not required that the estimates converge to the actual values of the associated variables. For each vehicle, the problem of path-following is stated as follows:

Path-following problem: Let $\boldsymbol{\eta}_{di}(\gamma_i) \in \mathbb{R}^3$ be a desired path parameterized by a continuous variable $\gamma_i \in \mathbb{R}$, and $\upsilon_{di} \in \mathbb{R}$ a desired constant speed assignment for vehicle i. Suppose also that $\boldsymbol{\eta}_{di}(\gamma_i)$ is sufficiently smooth and its derivatives (with respect to γ_i) are bounded. Design a control law for $\boldsymbol{\tau}_i$ and adaptation laws for $\hat{\boldsymbol{\nu}}_{ci}$ and $\hat{\boldsymbol{\varphi}}_i$, such that: all closed-loop signals are bounded; the position of vehicle i converges to the desired path, i.e, $\|\boldsymbol{\eta}_i(t) - \boldsymbol{\eta}_{di}(\gamma_i(t))\| \to 0$ as $t \to +\infty$; and the desired speed assignment is satisfied along the path, i.e, $|\dot{\gamma}_i(t) - \upsilon_{di}| \to 0$ as $t \to +\infty$.

As will become clear, the cooperation problem will be solved by adjusting the speeds of the vehicles as functions of the "along-path" distances between them. We start by introducing a measure of the degree of cooperation of a fleet of vehicles. Following (Ghabcheloo *et al.* 2006), this is done by reparameterizing each path $\boldsymbol{\eta}_{di}(\gamma_i)$ in terms of a conveniently defined variable ξ_i such that cooperation is achieved along the paths if $\xi_1 = \xi_2 = \ldots = \xi_n$. At this point, the "along-path" distances between vehicle i and j are formally defined as $\xi_{i,j} := \xi_i - \xi_j$. Then, cooperation is achieved if and only if $\xi_{i,j} = 0$ for all $i, j \in \{1, 2, \ldots, n\}$. Let the reparameterization of the path be represented by $\gamma_i = \gamma_i(\xi_i)$ and define $R_i(\xi_i) := \partial\gamma_i/\partial\xi_i$, which is assumed positive and bounded for all ξ_i. The dynamics of ξ_i are given by

$$\dot{\xi}_i = \frac{\dot{\gamma}_i}{R_i(\xi_i)} \,. \tag{6}$$

Suppose one vehicle, henceforth referred to as vehicle $\mathcal{L}$, is elected as the "leader" and let the corresponding path $\eta_{d\mathcal{L}}$ be parameterized by $\gamma_{\mathcal{L}} = \xi_{\mathcal{L}}$. For this vehicle, $R_{\mathcal{L}}(\xi_{\mathcal{L}}) = 1$. Let $v_{\mathcal{L}}$ be the desired constant speed assigned to the leader in advance, that is $\dot{\xi}_{\mathcal{L}} = v_{\mathcal{L}}$ in steady-state, known to all vehicles. From (6), it follows that the desired speed of each vehicle is $v_{di} = R_i(\xi_i)v_{\mathcal{L}}$. Define the "along-path" speed tracking error of vehicle i as

$$\zeta_i := \dot{\gamma}_i - R_i(\xi_i)v_{\mathcal{L}} \,, \tag{7}$$

whose dynamic equation is

$$\dot{\zeta}_i = u_i = \ddot{\gamma}_i - \frac{\mathrm{d}}{\mathrm{d}t}\left(R_i(\xi_i)v_{\mathcal{L}}\right) . \tag{8}$$

Using (7), the dynamics of $\dot{\xi}_i$ can be rewritten as

$$\dot{\xi}_i = \frac{\zeta_i}{R_i(\xi_i)} + v_{\mathcal{L}} \,. \tag{9}$$

To write the above dynamic equations in vector form, define $\boldsymbol{\xi} := [\xi_i]_{n\times 1}$, $\boldsymbol{\zeta} := [\zeta_i]_{n\times 1}$, $\boldsymbol{u} = [u_i]_{n\times 1}$ and $R(\boldsymbol{\xi}) := \mathrm{diag}[R_i(\xi_i)]_{n\times n}$, to obtain

$$\dot{\boldsymbol{\zeta}} = \boldsymbol{u} \,, \tag{10a}$$

$$\dot{\boldsymbol{\xi}} = R(\boldsymbol{\xi})^{-1}\boldsymbol{\zeta} + v_{\mathcal{L}}\mathbf{1} \,. \tag{10b}$$

The objective is to derive a control strategy for $\boldsymbol{u}$ to make $\xi_1 = \ldots = \xi_n$. From a graph theoretical point of view, each vehicle is represented by a vertex and a communication bidirectional link between two vehicles is represented by an edge between the corresponding vertices. We consider time-invariant communication topologies and assume that the induced graph is connected. The set of neighbors of vertex i is represent by $\mathcal{N}_i$ and contains all vertices adjacent to i. In other words, it represents the set of vehicles that vehicle i communicates with. In order to satisfy the constraints imposed by the communication network, the control law for vehicle i can only depend on local states or on information exchange with its neighbors. The cooperation error is defined as $\boldsymbol{\theta} := G^\top\boldsymbol{\xi} \in \mathbb{R}^{n-1}$, where G is obtained from the decomposition of the Laplacian of the induced graph, as indicated in Section 3. Given the properties of G, $\boldsymbol{\theta} = \mathbf{0}$ is equivalent to $\xi_{i,j} = 0$ for all i, j. The dynamics (10) rewritten in the error space are

$$\dot{\boldsymbol{\zeta}} = \boldsymbol{u} \,, \tag{11a}$$

$$\dot{\boldsymbol{\theta}} = G^\top R(\boldsymbol{\xi})^{-1}\boldsymbol{\zeta} \,, \tag{11b}$$

where the fact that $G^\top\mathbf{1} = \mathbf{0}$ was used. The cooperation problem is formulated as:

Cooperation problem: Consider the cooperation system with dynamics (11). Design a feedback control law for $\boldsymbol{u}$, where each u_i is a function of local states and the variables $\xi_j : j \in \mathcal{N}_i$, so that $\boldsymbol{\theta}$ and $\boldsymbol{\zeta}$ are driven asymptotically to zero.

This paper considers the case where the matrix $R(\boldsymbol{\xi})$ is constant. While restrictive, it still allows to consider paths where there is no need for reparameterization ($\xi_i = \gamma_i$) or it amounts to a constant scaling. In the next sections, a controller is designed that solves both problems presented, thereby solving the CPF problem.

5. PATH-FOLLOWING

The path-following controller here developed follows the lines of those presented in (Encarnação and Pascoal 2001) and (Skjetne *et al.* 2005). Standard backstepping techniques are used to derive the controller, by iteratively introducing control-Lyapunov functions. The controller is local to each vehicle, so the index i will be omitted for the sake of simplicity.

Step 1. Coordinate transformation: Define the position error in the body-fixed frame as $\boldsymbol{z}_1 := J^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_d)$ and the current estimation error as $\tilde{\boldsymbol{\nu}}_c := \boldsymbol{\nu}_c - \hat{\boldsymbol{\nu}}_c$. The dynamic equation of $\boldsymbol{z}_1$ is

$$\begin{aligned}
\dot{\boldsymbol{z}}_1 &= \dot{J}^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_d) + J^\top(\dot{\boldsymbol{\eta}} - \boldsymbol{\eta}_d^\gamma\dot{\gamma}) \\
&= -rSJ^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_d) + J^\top(J\boldsymbol{\nu}_r + \boldsymbol{\nu}_c - \boldsymbol{\eta}_d^\gamma\dot{\gamma}) \\
&= -rSz_1 + \boldsymbol{\nu}_r + J^\top(\hat{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\dot{\gamma}) + J^\top\tilde{\boldsymbol{\nu}}_c
\end{aligned}$$

where $\boldsymbol{\eta}_d^\gamma = \partial\boldsymbol{\eta}_d/\partial\gamma$ and ζ is the speed tracking error defined in (7).

Step 2. Convergence of $\boldsymbol{z}_1$: Define a first control-Lyapunov function as

$$V_1 := \frac{1}{2}\boldsymbol{z}_1^\top\boldsymbol{z}_1 \,,$$

whose time derivative is

$$\dot{V}_1 = \boldsymbol{z}_1^\top\left(\boldsymbol{\nu}_r + J^\top(\hat{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma v_d)\right) + \boldsymbol{z}_1^\top J^\top\left(\tilde{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\zeta\right) \tag{12}$$

where the fact that $\boldsymbol{z}_1^\top S\boldsymbol{z}_1 = 0$ for all $\boldsymbol{z}_1$ is used. Define the velocity error $\boldsymbol{z}_2 := \boldsymbol{\nu}_r - \boldsymbol{\alpha}$ where $\boldsymbol{\alpha} := -J^\top(\hat{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma v_d) - K_1\boldsymbol{z}_1$, and rewrite (12) as

$$\dot{V}_1 = -\boldsymbol{z}_1^\top K_1\boldsymbol{z}_1 + \boldsymbol{z}_1^\top\boldsymbol{z}_2 + \boldsymbol{z}_1^\top J^\top(\tilde{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^\gamma\zeta) \,.$$

Step 3. Backstepping for $\boldsymbol{z}_2$: First, the time derivative of $\boldsymbol{\alpha}$ is decomposed in three terms,

$$\dot{\boldsymbol{\alpha}} = \boldsymbol{\sigma} + \boldsymbol{\alpha}^{\gamma}\dot{\gamma} - K_1 J^\top \tilde{\boldsymbol{\nu}}_c$$

where the functions $\boldsymbol{\sigma}$ and $\boldsymbol{\alpha}^{\gamma}$ are defined as

$$\begin{aligned}\boldsymbol{\sigma} &:= -K_1(\boldsymbol{\nu}_r - rS\boldsymbol{z}_1 + J^\top \hat{\boldsymbol{\nu}}_c) \\ &\quad + rSJ^\top(\hat{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^{\gamma} \upsilon_d) - J^\top \dot{\hat{\boldsymbol{\nu}}}_c\,, \\ \boldsymbol{\alpha}^{\gamma} &:= K_1 J^\top \boldsymbol{\eta}_d^{\gamma} + J^\top \boldsymbol{\eta}_d^{\gamma^2} \upsilon_d\,,\end{aligned}$$

with $\boldsymbol{\eta}_d^{\gamma^2} = \partial^2 \boldsymbol{\eta}_d / \partial \gamma^2$. Notice that $\boldsymbol{\sigma}$ depends on $\dot{\hat{\boldsymbol{\nu}}}_c$ that is still undefined. The dynamic equation for $\boldsymbol{z}_2$ is then

$$\begin{aligned}M\dot{\boldsymbol{z}}_2 &= \boldsymbol{\tau} - C\boldsymbol{\nu}_r + \Phi_k \boldsymbol{\varphi}_k + \Phi_u \boldsymbol{\varphi}_u - M\left(\boldsymbol{\sigma} + \boldsymbol{\alpha}^{\gamma}\dot{\gamma}\right) \\ &\quad + MK_1 J^\top \tilde{\boldsymbol{\nu}}_c\,.\end{aligned}$$

The uncertain parameters estimation error is defined as $\tilde{\boldsymbol{\varphi}} := \boldsymbol{\varphi}_u - \hat{\boldsymbol{\varphi}}$. Define a second control-Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}\boldsymbol{z}_2^\top M \boldsymbol{z}_2 = \frac{1}{2}\boldsymbol{z}_1^\top \boldsymbol{z}_1 + \frac{1}{2}\boldsymbol{z}_2^\top M \boldsymbol{z}_2\,,$$

whose time derivative is

$$\begin{aligned}\dot{V}_2 &= -\boldsymbol{z}_1^\top K_1 \boldsymbol{z}_1 + \boldsymbol{z}_1^\top J^\top(\tilde{\boldsymbol{\nu}}_c - \boldsymbol{\eta}_d^{\gamma}\zeta) + \tilde{\boldsymbol{\varphi}}^\top \Phi_u^\top \boldsymbol{z}_2 + \\ &\quad + \boldsymbol{z}_2^\top \big(\boldsymbol{z}_1 + \boldsymbol{\tau} - C\boldsymbol{\nu}_r + \Phi_k \boldsymbol{\varphi}_k + \Phi_u \hat{\boldsymbol{\varphi}} + \\ &\quad - M(\boldsymbol{\sigma} + \boldsymbol{\alpha}^{\gamma}\upsilon_d)\big) - \boldsymbol{z}_2^\top M \boldsymbol{\alpha}^{\gamma}\zeta\,. \qquad (13)\end{aligned}$$

Using the feedback law

$$\boldsymbol{\tau} = -\boldsymbol{z}_1 - K_2\boldsymbol{z}_2 + C\boldsymbol{\nu}_r - \Phi_k\boldsymbol{\varphi}_k - \Phi_u\hat{\boldsymbol{\varphi}} + M(\boldsymbol{\sigma} + \boldsymbol{\alpha}^{\gamma}\upsilon_d) \qquad (14)$$

with $K_2 \succ 0$, and replacing in (13), yields

$$\dot{V}_2 = -\boldsymbol{z}_1^\top K_1 \boldsymbol{z}_1 - \boldsymbol{z}_2^\top K_2 \boldsymbol{z}_2 + \boldsymbol{\rho}^\top \tilde{\boldsymbol{\nu}}_c + \boldsymbol{\pi}^\top \tilde{\boldsymbol{\varphi}} + \mu\zeta \qquad (15)$$

where the following auxiliary functions are used: $\mu = -(\boldsymbol{\eta}_d^{\gamma})^\top J \boldsymbol{z}_1 - (\boldsymbol{\alpha}^{\gamma})^\top M \boldsymbol{z}_2$, $\boldsymbol{\pi} = \Phi_u^\top \boldsymbol{z}_2$, and $\boldsymbol{\rho} = J(\boldsymbol{z}_1 + K_1 M \boldsymbol{z}_2)$.

Step 4. Adaptation laws for $\hat{\boldsymbol{\nu}}_c$ and $\hat{\boldsymbol{\varphi}}$: Define a third control-Lyapunov function as

$$\begin{aligned}V_3 &= V_2 + \frac{1}{2}\tilde{\boldsymbol{\nu}}_c^\top \Sigma^+ \tilde{\boldsymbol{\nu}}_c + \frac{1}{2}\tilde{\boldsymbol{\varphi}}^\top \Gamma^{-1}\tilde{\boldsymbol{\varphi}} \qquad (16) \\ &= \frac{1}{2}\boldsymbol{z}_1^\top \boldsymbol{z}_1 + \frac{1}{2}\boldsymbol{z}_2^\top M \boldsymbol{z}_2 + \frac{1}{2}\tilde{\boldsymbol{\nu}}_c^\top \Sigma^+ \tilde{\boldsymbol{\nu}}_c + \frac{1}{2}\tilde{\boldsymbol{\varphi}}^\top \Gamma^{-1}\tilde{\boldsymbol{\varphi}}\end{aligned}$$

where Σ^+ is the pseudoinverse of Σ, with

$$\Sigma = \begin{bmatrix}\Sigma_1 & 0 \\ 0 & 0\end{bmatrix}$$

and $\Sigma_1 \succ 0$. The time derivative of (16) is

$$\begin{aligned}\dot{V}_3 &= -\boldsymbol{z}_1^\top K_1 \boldsymbol{z}_1 - \boldsymbol{z}_2^\top K_2 \boldsymbol{z}_2 + \mu\zeta + \tilde{\boldsymbol{\nu}}_c^\top(\boldsymbol{\rho} - \Sigma^+ \dot{\hat{\boldsymbol{\nu}}}_c) \\ &\quad + \tilde{\boldsymbol{\varphi}}^\top(\boldsymbol{\pi} - \Gamma^{-1}\dot{\hat{\boldsymbol{\varphi}}})\,. \qquad (17)\end{aligned}$$

Using the adaptation laws

$$\dot{\hat{\boldsymbol{\nu}}}_c = \Sigma\boldsymbol{\rho}\,, \qquad (18a)$$

$$\dot{\hat{\boldsymbol{\varphi}}} = \Gamma\boldsymbol{\pi}\,, \qquad (18b)$$

and replacing in (17), yields

$$\dot{V}_3 = -\boldsymbol{z}_1^\top K_1 \boldsymbol{z}_1 - \boldsymbol{z}_2^\top K_2 \boldsymbol{z}_2 + \mu\zeta\,. \qquad (19)$$

6. COOPERATIVE PATH-FOLLOWING

So far, the path-following design established the following control-Lyapunov function for the ith vehicle:

$$V_i := \frac{1}{2}\boldsymbol{z}_{1i}^\top \boldsymbol{z}_{1i} + \frac{1}{2}\boldsymbol{z}_{2i}^\top M_i \boldsymbol{z}_{2i}\,,$$

whose time derivative is

$$\dot{V}_i = -\boldsymbol{z}_{1i}^\top K_{1i}\boldsymbol{z}_{1i} - \boldsymbol{z}_{2i}^\top K_{2i}\boldsymbol{z}_{2i} + \mu_i\zeta_i\,.$$

The design is now continued taking into account the cooperation dynamics (11).

Step 5. Control law for $\boldsymbol{u}$: The solution developed here is based on the work in (Ghabcheloo *et al.* 2006, Aguiar *et al.* 2005). Define a composite control-Lyapunov function, combining terms from path-following and cooperation, as

$$V_c := \underbrace{\sum_{i=1}^{n}\frac{d_i}{e_i}V_i}_{\text{path-following}} + \underbrace{\frac{1}{2}\boldsymbol{\theta}^\top\boldsymbol{\theta} + \frac{1}{2}\boldsymbol{\zeta}^\top E^{-1}\boldsymbol{\zeta}}_{\text{cooperation}} \qquad (20)$$

where $E = \operatorname{diag}[e_i]_{n\times 1}$ with $e_i > 0$ and $d_i > 0$ for all i. Let $D = \operatorname{diag}[d_i]_{n\times 1}$ and $\boldsymbol{\mu} = [\mu_i]_{n\times 1}$. The time derivative of V_c is

$$\begin{aligned}\dot{V}_c &= \boldsymbol{\zeta}^\top\left(E^{-1}\boldsymbol{u} + R^{-1}G\boldsymbol{\theta} + E^{-1}D\boldsymbol{\mu}\right) \\ &\quad - \sum_{i=1}^{n}\frac{d_i}{e_i}\left(\boldsymbol{z}_{1i}^\top K_{1i}\boldsymbol{z}_{1i} + \boldsymbol{z}_{2i}^\top K_{2i}\boldsymbol{z}_{2i}\right)\,. \qquad (21)\end{aligned}$$

Defining the following feedback law for $\boldsymbol{u}$,

$$\boldsymbol{u} = -ER^{-1}G\boldsymbol{\theta} - D\boldsymbol{\mu} - B\boldsymbol{\zeta} \qquad (22)$$

where $B = \operatorname{diag}[b_i]_{n\times 1}$ with $b_i > 0$ for all i, and replacing in (21), yields

$$\dot{V}_c = -\boldsymbol{\zeta}^\top E^{-1}B\boldsymbol{\zeta} - \sum_{i=1}^{n}\frac{d_i}{e_i}\left(\boldsymbol{z}_{1i}^\top K_{1i}\boldsymbol{z}_{1i} + \boldsymbol{z}_{2i}^\top K_{2i}\boldsymbol{z}_{2i}\right). \qquad (23)$$

Rewriting the control law (22) from the point of view of vehicle i results in

$$u_i = -d_i\mu_i - b_i\zeta_i - \frac{e_i}{R_i}\sum_{j\in\mathcal{N}_i}(\xi_i - \xi_j)\,. \qquad (24)$$

Clearly, it is a decentralised control law that satisfies the constraints imposed by the communication network. Using (20) as candidate Lyapunov function, which has a negative semidefinite time derivative (23), and resorting to LaSalle's invariance principle (see, e.g., (Khalil 2002)), it is possible to prove global asymptotic stability of the closed-loop system.

Theorem 1. The feedback law for $\boldsymbol{\tau}_i$ given by (14) and the adaptation laws for $\hat{\boldsymbol{\nu}}_{ci}$ and $\hat{\boldsymbol{\varphi}}_i$ given by (18a) and (18b), respectively, together with (22), solve the path-following and cooperation problems. Moreover, $\hat{\boldsymbol{\nu}}_{ci} \to \boldsymbol{\nu}_{ci}$ as $t \to +\infty$.

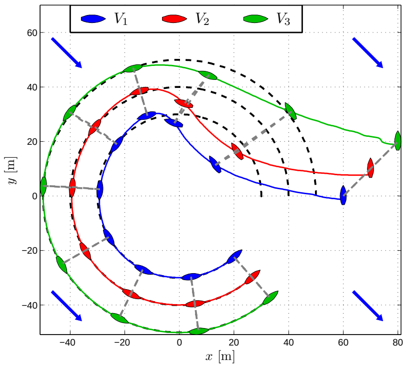

Fig. 2. Desired and actual vehicles' trajectories.

7. AN ILLUSTRATIVE EXAMPLE

Consider a group of $n = 3$ identical vehicles where the communication network induces a graph with the adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

The vehicles' equations of motion can be written as in (5), where

$$M = \mathrm{diag}(m_u, m_v, I_r),\ C(\boldsymbol{\nu}_r) = \begin{bmatrix} 0 & -m_v r & 0 \\ m_u r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{\varphi} = \left(X_u, X_{|u|u}, Y_{uv}, Y_{ur}, N_r, N_{|r|r}\right),$$
$$\Phi(\boldsymbol{\nu}_r) = \begin{bmatrix} u_r & |u_r|u_r & 0 & 0 & 0 & 0 \\ 0 & 0 & u_r v_r & u_r r & 0 & 0 \\ 0 & 0 & 0 & 0 & r & |r|r \end{bmatrix}^\top.$$

The matrices Φ_k and Φ_u are partitions of Φ compatible with the following choice of uncertain parameters: $\boldsymbol{\varphi}_u = (X_{|u|u}, Y_{uv}, Y_{ur}, N_{|r|r})$. In the simulation presented here, the physical parameters are

$$\begin{aligned} &X_u = -0.5\,\mathrm{Kg/s},\ X_{|u|u} = -23.1\,\mathrm{Kg/m}, \\ &Y_{uv} = -926.0\,\mathrm{Kg/m},\ Y_{ur} = 227.6\,\mathrm{Kg}, \\ &N_r = -0.26\,\mathrm{Kgm^2/s},\ N_{|r|r} = -1764.2\,\mathrm{Kgm^2}, \\ &m_u = 421.8\,\mathrm{Kg},\ m_v = 1008.1\,\mathrm{Kg},\ I_r = 690.5\,\mathrm{Kgm^2} \end{aligned}$$

The desired paths are concentric circumferences centred at the origin and with radii 30 m, 40 m and 50 m. Matrix C equals $\frac{1}{3}\mathrm{diag}(3,4,5)$. The path-following controller gains are $K_{1i} = 0.0375 I_3$ and $K_{2i} = 200 I_3$. The initial conditions for each vehicle are $(x_i, y_i, \psi_i) = (50 + 10i\,\mathrm{m}, 10(i-1)\,\mathrm{m}, \frac{\pi}{2}\,\mathrm{rad})$ and $u_i = v_i = r_i = 0$. The reference velocity is $v_{\mathcal{L}} = 0.4\,\mathrm{m/s}$. The ocean current is $\boldsymbol{\nu}_{ci} = (1\,\mathrm{m/s}, -1\,\mathrm{m/s}, 0)$, and the adaptation gains are $\Sigma_i = \mathrm{diag}(10^{-3}, 10^{-3}, 0)$ and $\Gamma_i = \mathrm{diag}(50, 10^2, 10^2, 10^2)$. The estimates for both the current and the parameters are initialised with a 30% error (each vehicle has an independent initialization). The cooperation gains are $E = 0.1 I_3$, $B = I_3$ and $D = 0.01 I_3$. Fig. 2 illustrates the trajectory of each vehicle. Fig. 3 shows the temporal evolution of the errors $\boldsymbol{\theta}(t)$, $\boldsymbol{\zeta}(t)$ and $\|\boldsymbol{z}_{1i}(t)\|$.

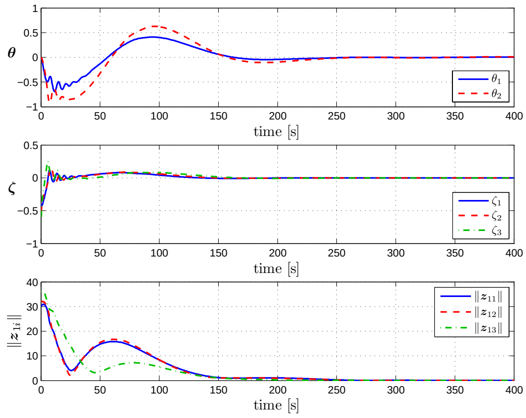

Fig. 3. Evolution of cooperation, "along-path" speed, and path-following errors.

8. CONCLUSIONS AND FUTURE WORK

This paper addressed the problem of cooperative path-following (CPF). Nonlinear control techniques were used to solve the CPF problem for surface vessels with parametric model uncertainty and subjected to an unknown but constant ocean current. Future work involves extending the results to a broader class of vehicles, studying cooperation strategies under more challenging mission scenarios involving communication failures and time delays, and analysing the performance of the resulting cooperation control laws.

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