Coordinated Path-Following Control of Multiple Autonomous Underwater Vehicles

A. Pedro Aguiar, Reza Ghabcheloo, António M. Pascoal, and Carlos Silvestre
Institute for Systems and Robotics and Department of Electrical Engineering
Instituto Superior Tecnico
Lisbon, Portugal

ABSTRACT

The concept of multiple Autonomous Underwater Vehicles (AUVs) cooperatively performing a mission offers several advantages over single vehicles working in a non-cooperative manner such as increased efficiency, performance, reconfigurability, robustness and the emergence of new capabilities. This paper introduces the concept of coordinated path-following control of multiple AUVs. The vehicles are required to follow pre-specified spatial paths while keeping a desired inter-vehicle formation pattern in time. We show how Lyapunov-based techniques and graph theory can be brought together to yield a decentralized control structure where the dynamics of the cooperating vehicles and the constraints imposed by the topology of the inter-vehicle communications network are explicitly taken into account. *Path-following* for each vehicle amounts to reducing an appropriately defined geometric error to a small neighborhood of the origin. *Vehicle coordination* is achieved by adjusting the speed command of each vehicle along its path according to information on the positions of a subset of the other vehicles, as determined by the communications topology adopted. We illustrate our design procedure for underwater vehicles moving in three-dimensional space. Simulations results are presented and discussed.

KEY WORDS: Coordination of autonomous vehicles; Path-following; Autonomous Underwater Vehicle (AUV); Nonlinear control;

INTRODUCTION

The ever increasing sophistication of autonomous underwater vehicles (AUVs) is steadily paving the way for the execution of complex missions without direct supervision of human operator. A key enabling element for the execution of such missions is the availability of advance systems for motion control of AUVs. The past few decades have witnessed considerable interest in this area (Fossen, 1994; Leonard, 1995; Encarnação and Pascoal, 2000; Alonge *et al.*, 2001; Jiang, 2002; Pettersen and Nijmeijer, 2003; Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007; Aguiar and Pascoal, 2007*b*). The problems of motion control can be roughly classified into three groups: point stabilization, where the goal is to stabilize a vehicle at a given target point with a desired orientation; trajectory tracking, where the vehicle is required to track a time parameterized reference, and path-following, where the vehicle is required to converge to and follow a desired geometric path, without a timing law assigned to it. For underactuated AUVs, i.e., vehicles with a smaller number of control inputs than the number of independent generalized coordinates, motion control is still an active research topic.

Current research goes well beyond single vehicle control. In fact, recently there has been widespread interest in the problem of coordinated motion control of fleets of AUVs (Stilwell and Bishop, Dec. 2000; Encarnação and Pascoal, 2001; Lapierre *et al.*, 2003; Skjetne *et al.*, 2002; Ghabcheloo *et al.*, 2006*b*; Ihle *et al.*, Dec 2006). The concept of multiple AUVs cooperatively performing a mission offers several advantages (over single vehicles working in a non-cooperative manner) such as increased efficiency, performance, reconfigurability, robustness, and the emergence of new capabilities. Furthermore, each vehicle can in principle carry only a single dedicated sensor (per environmental variable of interest) making it less complex, and consequently increasing its reliability.

From a theoretical standpoint, the coordination of autonomous robotic vehicles involves the design of distributed control laws in the face of disrupted inter-vehicle communications, uncertainty, and imperfect or partial measurements. This is particularly significant in the case of underwater vehicles for two main reasons: *i)* the dynamics of marine vehicles are often complex and cannot be simply ignored or drastically simplified for control design purposes, and *ii)* underwater communications and positioning rely heavily on acoustic systems, which are plagued with intermittent failures, latency, and multi-path effects. It was only recently that these subjects have started to be formally tackled, and considerable research remains to be done to derive multiple vehicle control laws that can yield good performance in the presence of severe communication constraints.

Motivated by the above considerations, this paper introduces the concept of coordinated path-following (CPF) control of multiple AUVs. The vehicles are required to follow pre-specified spatial paths while keeping a desired inter-vehicle formation pattern in time. This problem arises, for example, in the operation of multiple AUVs for fast acoustic coverage of the seabed. In this application, two or more vehicles are required to fly above the seabed at the same or different depths, along geometrically similar spatial paths, and map the seabed using identical suites of acoustic sensors. Larger areas can be covered in a short period of time, by requiring that the vehicles traverse identical paths so that the projections of the acoustic beams on the seabed exhibit some overlapping. These objectives impose constraints on the inter-vehicle formation pattern. A number of other scenarios can also be envisioned that require coordinated motion control of marine vehicles (ASIMOV, 2000; Cardigos *et al.*, 2006).

In this paper, we solve the coordinated path-following problem for a class of underactuated underwater vehicles moving in either two or three-dimensional space. The solution adopted is rooted in Lyapunov-based theory and addresses explicitly the vehicle dynamics as well as the constraints imposed by the topology of the inter-vehicle communications network. The latter are tackled in the framework of graph theory (Godsil and Royle, 2001), which seems especially suitable to study the impact of communication topologies on the performance that can be achieved with coordination (Fax and Murray, 2002).

With the framework adopted, path-following (in space) and inter-vehicle coordination (in time) become essentially decoupled. Each vehicle is equipped with a controller that makes the vehicle follow a predefined path. The speed command for each vehicle is then adapted so that the whole group of vehicles keeps the desired formation pattern. A supporting communications network provides the fleet of vehicles with the medium over which to exchange the information that is required to synchronize the so-called coordination states.

The paper is organized as follows. Section II summarizes a model for a class of autonomous underwater vehicles and formulates the path-following and vehicle coordination problems. Section III presents solutions to the problems of single vehicle path-following as well as multiple vehicle coordinated path-following that takes into account the constraints imposed by the topology of the inter-vehicle communications network. Section IV illustrates the performance of the CPF control algorithm proposed through computer simulations. Finally, Section V contains the main conclusions and describes problems that warrant further research.

This paper builds upon and combine previous results obtained by the authors on path-following control (Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007) and coordination control (Ghabcheloo *et al.*, 2007; Ghabcheloo *et al.*, 2006*a*; Ghabcheloo *et al.*, 2006*c*; Aguiar and Pascoal, 2007*a*).

PROBLEM STATEMENT

Consider an underactuated autonomous underwater vehicle (AUV) not necessarily neutrally buoyant. Let $\{\mathcal{I}\}$ be an inertial coordinate frame and $\{\mathcal{B}\}$ a body-fixed coordinate frame whose origin is located at the center of mass of the vehicle. The configuration $(R, \mathbf{p})$ of the vehicle is an element of the Special Euclidean group $SE(3) := SO(3) \times \mathbb{R}^3$, where $R \in SO(3) := \{R \in \mathbb{R}^{3\times 3} : RR^T = I_3, \det(R) = +1\}$ is a rotation matrix that describes the orientation of the vehicle and maps body coordinates into inertial coordinates, and $\mathbf{p} \in \mathbb{R}^3$ is the position of the origin of $\{\mathcal{B}\}$ in $\{\mathcal{I}\}$. Denoting by $v \in \mathbb{R}^3$ and $\omega \in \mathbb{R}^3$ the linear and angular velocities of the vehicle relative to $\{\mathcal{I}\}$ expressed in $\{\mathcal{B}\}$, respectively, the following kinematic relations apply:

$$\dot{\mathbf{p}} = Rv, \tag{1a}$$
$$\dot{R} = RS(\omega), \tag{1b}$$

where

$$S(x) := \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}, \quad \forall x := (x_1, x_2, x_3)^T \in \mathbb{R}^3.$$

We consider AUVs with dynamic equations of motion of the following form:

$$\mathbf{M}\dot{v} = -S(\omega)\mathbf{M}v + f_v(v, R) + B_1 u_1, \tag{2a}$$
$$\mathbf{J}\dot{\omega} = -S(v)\mathbf{M}v - S(\omega)\mathbf{J}\omega + f_\omega(v, \omega, R) + B_2 u_2, \tag{2b}$$

where $\mathbf{M} \in \mathbb{R}^{3\times 3}$ and $\mathbf{J} \in \mathbb{R}^{3\times 3}$ include the so-called hydrodynamic added-mass M_A and added-inertia J_A matrices, that is, $\mathbf{M} = M_{RB} + M_A$, $\mathbf{J} = J_{RB} + J_A$. The symbols M_{RB} and J_{RB} denote the rigid-body mass and inertia matrices, respectively. The functions $f_v(\cdot)$ and $f_\omega(\cdot)$ capture hydrodynamic damping effects and restoring forces and moments, and are defined by

$$f_v = -D_v(v)v - \bar{g}_1(R), \quad f_\omega = -D_\omega(\omega)\omega - \bar{g}_2(R),$$

where

$$D_v(v) = \operatorname{diag}\{X_{v_1} + X_{|v_1|v_1}|v_1|, Y_{v_2} + Y_{|v_2|v_2}|v_2|, Z_{v_3} + Z_{|v_3|v_3}|v_3|\}$$
$$D_\omega(\omega) = \operatorname{diag}\{K_{\omega_1} + K_{|\omega_1|\omega_1}|\omega_1|, M_{\omega_2} + M_{|\omega_2|\omega_2}|\omega_2|, N_{\omega_3} + N_{|\omega_3|\omega_3}|\omega_3|\}$$

$$\bar{g}_1(R) = R^T \begin{pmatrix} 0 \\ 0 \\ W-B \end{pmatrix}, \qquad \bar{g}_2(R) = S(r_{\mathcal{B}})R^T \begin{pmatrix} 0 \\ 0 \\ B \end{pmatrix}$$

The gravitational and buoyant forces are given by $W = mg$ and $B = \rho g \nabla$, respectively, where m is the vehicle's mass, ρ is the mass density of the water and ∇ is the volume of displaced water. We assume that there are available a pure body-fixed control force $\tau_u := u_1$ in the $x_{\mathcal{B}}$ direction and two independent control torques $(\tau_q, \tau_r) := u_2$ about the $y_{\mathcal{B}}$ and $z_{\mathcal{B}}$ axes of the vehicle, that is,

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

We also assume that the metacentric height of the AUV is sufficiently large to provide adequate static stability in roll motion. The particulars of the AUV used in the simulations at the end of the paper are those of the *Sirene* underwater shuttle described in (Aguiar, 2002; Aguiar and Pascoal, 1997).

For an underactuated vehicle restricted to move on a planar surface, the same equations of motion (1)–(2) apply without the first two right-hand-side terms in (2b). Also, in this case, $(R, p) \in SE(2)$, $v \in \mathbb{R}^2$, $\omega \in \mathbb{R}$, $g_v \in \mathbb{R}^2$, $G_\omega \in \mathbb{R}$, $u_\omega \in \mathbb{R}$, with all the other terms in (2) having appropriate dimensions, and the skew-symmetric matrix $S(\omega)$ is given by $S(\omega) = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix}$. For simplicity, in what follows, we restrict our attention to the three-dimensional case. However, all results are directly applicable to the two-dimensional case.

For our purposes, we consider a fleet of $n \geq 2$ vehicles. For $i \in \mathcal{I} := \{1, .., n\}$ we let $\mathbf{p}_i(t) \in \mathbb{R}^3$ and $\mathbf{p}_{d_i}(\gamma_i) \in \mathbb{R}^3$ denote the position of vehicle i and its assigned (desired) path, where the latter is parameterized by $\gamma_i \in \mathbb{R}$. We further let $v_{r_i}(\gamma_i) \in \mathbb{R}$ denote the desired speed assignment for vehicle i defined in terms of parameters γ_i. Finally, $u_i = [u_{1,i}, u_{2,i}]$ and $e_{p_i} := \mathbf{p}_i - \mathbf{p}_{d_i}$ denote the control vector and the path-following position error, respectively, of vehicle i.

Equipped with above notation, the problems of *path-following* (PF) and *coordinated path-following* (CPF) are defined next.

Path-following problem. *Let $p_{d_i}(\gamma_i) \in \mathbb{R}^3$ be a desired path parameterized by a continuous variable $\gamma_i \in \mathbb{R}$, and $v_r(\gamma_i) \in \mathbb{R}$ a desired reference speed for the vehicle i. Design feedback control laws for u_i such that all the closed-loop signals are bounded, the position of the vehicle converges to and remains inside a tube centered around the desired path, and the vehicle travels at a desired speed assignment v_r, that is, $\dot{\gamma} - v_r \to 0$ as $t \to \infty$.*

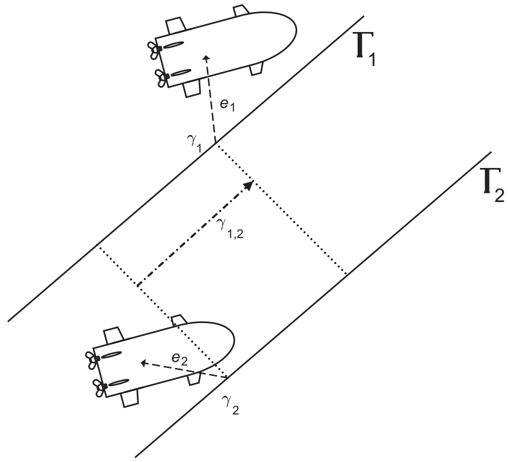

Fig. 1. Along-path distances: straight lines

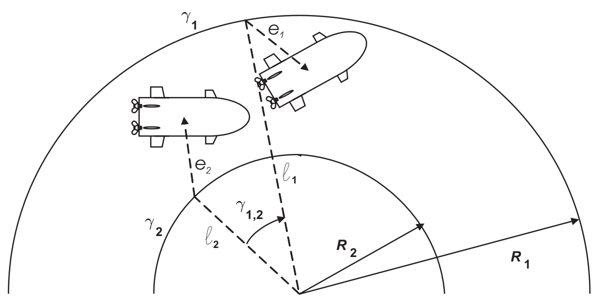

Fig. 2. Along-path distances: circumferences

Assuming a path-following controller has been implemented for each vehicle, it now remains to coordinate (that is, synchronize) the entire group of vehicles so as to achieve a desired formation pattern compatible with the paths adopted. As will become clear, this will be achieved by adjusting the desired speeds of the vehicles as functions of the "along-path" distances among them. To better grasp the key ideas involved in the computation of these distances, consider for example the case of vehicles maneuvering along parallel translations of the same straight line. See Figure 1 for the case of two vehicles. Suppose the objective is to align the vehicles along a perpendicular to both paths (so-called "in-line" formation pattern).

Let $\Gamma_i; i = 1, 2$ denote the path to be followed by vehicle i and s_i denote the abscissa of the associated "virtual target" $p_{d_i}(\gamma_i)$ being tracked along Γ_i. Since the position $p_i(t)$ of vehicle i approaches $p_{d_i}(\gamma_i)$, the vehicles become or asymptotically synchronize if $\gamma_{ij}(t) := \gamma_i(t) - \gamma_j(t)\ \forall i, j \in \mathcal{I}$. This shows that in the case of translated straight lines $\gamma_{i,j} = s_i - s_j$ is a good measure of the along-path distances among the vehicles. Similarly, in the case of scaled circumferences, Figure 2, an appropriate measure of the distances among the vehicles is angle $\gamma_{12} = s_1/R_1 - s_2/R_2$. In both cases, we say that the vehicles are coordinated if the corresponding along-path distance is zero, that is, $\gamma_i - \gamma_j = 0$. Coordination is achieved by adjusting the "desired speed" of each vehicle i as a function the along-path distances $\gamma_{ij}; j \in \mathcal{N}_i$ where $\mathcal{N}_i$ denotes the set of vehicles that vehicle i communicates with. For more general types of paths and coordination patterns, an adequate choice of the path parametrizations will allow for the conclusion that the vehicles are *coordinated*, or, in equivalent terms are *synchronized* or have reached *agreement*, if and only if

$$\gamma_{i,j} = 0, \quad \forall i, j \in \mathcal{I}$$

see (Ghabcheloo *et al.*, 2007; Egerstedt and Hu, Dec. 2001). Since the objective of the coordination is to coordinate variables γ_i, we will refer to them as *coordination states*. We will further require that the formation as a whole (group of multiple vehicles) travel at an assigned speed profile v_r while coordinated, that is, asymptotically $\dot{\gamma}_i = v_r;\ \forall i \in \mathcal{I}$. This issue requires clarification. Note that the desired speed assignment is given in terms of the time derivatives of the coordination states, γ_i, not the actual time derivative of the positions (speeds) of the vehicles undergoing the synchronization. In the limit, assuming the vehicles have reached their paths, their speeds degenerate into $\frac{ds_i}{dt}; i \in \mathcal{I}$ in such a way as to $\frac{d\gamma_i}{dt} = \frac{d\gamma_i}{ds_i}\frac{ds_i}{dt} = v_r$, that is, $\frac{ds_i}{dt} = v_r/\frac{d\gamma_i}{ds_i}$ which yields $\frac{ds_i}{dt} = R_i v_r$ for the example of the circumferences given above. With this speed assignment one does not have to specify the actual speed of the vehicles, but rather those of their coordination states which are equal and degenerate simply into v_r no matter what type of coordination is under study.

Coordination problem. *For each vehicle $i \in \mathcal{I} := \{1, ..., n\}$ derive a control law for the speed command as a function of γ_i and γ_j, $j \in \mathcal{N}_i$ such that $\gamma_i - \gamma_j, \forall i, j \in \mathcal{I}$ approach zero as $t \to \infty$ and the formation travels at an assign speed v_r, that is, $|\dot{\gamma}_i - v_r|$ tends to zero.*

The coordinated path-following (CPF) problem is the combination of the two previous stated problems.

MAIN RESULTS

This section offers a solution to the coordinated path-following problem for a group of n AUVs modeled by (1)–(2).

Path-following controller

In (Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007) we proposed a solution to the path-following problem for underactuated autonomous vehicles described by (1)–(2) in the presence of possibly large modeling parametric uncertainty. The path-following controller was designed combining Lyapunov-based techniques with adaptive switching supervisory control, and yields global boundedness and convergence of the position error to a small neighborhood, and robustness to parametric modeling uncertainty.

In this section, we briefly discuss the results presented in (Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007) to solve the path-following problem. Let

$$e_i := R_i'\big[p_i(t) - p_{d_i}(\gamma_i(t))\big]$$

be the path-following error of the vehicle i expressed in its body-fixed frame. Borrowing from the techniques of backstepping, in (Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007) a feedback law for u_{1_i}, u_{2_i} was derived that makes the time-derivative of the Lyapunov function

$$V_i := \frac{1}{2}e_i'e_i + \frac{1}{2}\varphi_i'\mathbf{M}_i^2\varphi_i + \frac{1}{2}z_{2_i}'\mathbf{J}_i z_{2_i} + \frac{1}{2}\eta_i^2$$

take the form

$$\begin{aligned}\dot{V}_i = &-k_{e_i}\, e_i'\mathbf{M}_i^{-1}e_i + e_i'\delta_i - \varphi_i'K_{\varphi_i}\varphi_i - z_{2_i}'K_{z_2 i}z_{2_i}\\ &+ \eta_i(\mu_i(\cdot) + \ddot{\gamma}_i - v_r^{\gamma}\dot{\gamma}_i)\end{aligned}$$

where φ_i and z_{2_i} are linear and angular velocity errors (see (Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007) for details), k_{e_i}, K_{φ_i}, $K_{z_2 i}$ are positive definite matrices, δ_i is a small constant vector,

$v_r^\gamma := \frac{\partial v_r}{\partial \gamma}$, and μ_i captures the terms associated to the speed error $\eta_i := \dot{\gamma}_i - v_r$.

At this point we remark that if all that is required is to solve a pure path-following problem then one can utilize the freedom of assigning a feedback law to $\ddot{\gamma}_i$ in order to make $\dot{V}_i$ negative definite (see details in (Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007)). This strategy must be modified to address coordination as follows: select the update law for $\ddot{\gamma}_i$ as in (Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007) but adding the additional term $\tilde{v}_{r_i}$, that is,

$$\ddot{\gamma}_i = -\mu_i + v_r^\gamma - k_{\gamma_i}\eta_i + \tilde{v}_{r_i}, \quad k_{\gamma_i} > 0 \tag{3}$$

The signal $\tilde{v}_{r_i}$ is a correction term for $\dot{\gamma}_i$ that will be exploited to achieve synchronization of the AUVs (coupling with the coordination controller to be designed below). At the path-following control level, this signal is viewed as an external disturbance as clearly shown in the following lemma.

The following result holds.

Lemma 1: The feedback laws for u_{1_i}, u_{2_i} for each vehicle i obtained in (Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007) together with (3) solve robustly the path-following problem, that is, all the states of the closed-loop are bounded and the path-following error $e_{p_i} := \mathbf{p}_i - \mathbf{p}_{d_i}$ and speed error $\eta_i := \dot{\gamma}_i - v_r(\gamma_i)$ are input-to-output practically stable (IOpS) (Sontag and Wang, 1996). In particular, the Lyapunov function $V_{PF} := \sum_{i=1}^n V_i$ satisfies

$$\dot{V}_{PF} \le -\lambda_{PF} V_{PF} + \rho_{PF}|\tilde{v}_r|^2 + \epsilon,$$

for some $\epsilon, \lambda_{PF}, \rho_{PF} > 0$, where $\tilde{v}_r := [\tilde{v}_{r_i}]_{i\in\mathcal{I}} := (\tilde{v}_{r_1}, \ldots, \tilde{v}_{r_n})'$. Moreover, one can have the gain $\frac{\rho_{PF}}{\lambda_{PF}}$ and ϵ small at will by making the path-following control gains sufficiently large. □

Coordinated controller

In this section we develop the coordinated controller. To this effect, we first recall some key concepts from algebraic graph theory.

Let $\mathcal{N}_i$ be the index set of the vehicles that vehicle i communicates with (the so called neighboring set of vehicle i). We assume that the communication links are bidirectional, that is, $i \in \mathcal{N}_j \Leftrightarrow j \in \mathcal{N}_i$. Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be the undirected graph induced by the inter-vehicle communication network, with $\mathcal{V}$ denoting the set of n nodes (each corresponding to a vehicle) and $\mathcal{E}$ the set of edges (each standing for a data link). Nodes i and j are said to be adjacent if there is an edge between them. A path of length r between node i and node j consist of $r+1$ consecutive adjacent nodes. We say that $\mathcal{G}$ is connected when there exists a path connecting every two nodes in the graph. The adjacency matrix of a graph, denoted A, is a square matrix with rows and columns indexed by the nodes such that the i,j-entry of A is 1 if $j \in \mathcal{N}_i$ and zero otherwise. The degree matrix D of a graph $\mathcal{G}$ is a diagonal matrix where the i,i-entry equals $|\mathcal{N}_i|$, the cardinality of $\mathcal{N}_i$. The Laplacian of a graph is defined as $L := D - A$. Thus, L is symmetric and its every row sum equals zero, that is, $L\mathbf{1} = \mathbf{0}$, where $\mathbf{1} := [1]_{n\times 1}$ and $\mathbf{0} := [0]_{n\times 1}$. If $\mathcal{G}$ is connected, L has a simple eigenvalue at zero with an associated eigenvector $\mathbf{1}$ and the remaining eigenvalues are all positive.

Consider now the coordination control problem with a communication topology defined by a graph $\mathcal{G}$. Using a Lyapunov-based design, we propose a decentralized feedback law for $\tilde{v}_{r_i}$ as a function of the information obtained from the neighboring vehicles. Following (Ghabcheloo *et al.*, 2006*c*), we introduce the error vector

$$\xi := L_K\gamma, \quad L_K := I - \frac{1}{\mathbf{1}'K^{-1}\mathbf{1}}\mathbf{1}\mathbf{1}'K^{-1},$$

where $\gamma := [\gamma_i]_{i\in\mathcal{I}}$, $\mathbf{1} := [1]_{i\in\mathcal{I}}$, and $K > 0$ is a diagonal matrix. This coordination error vector satisfies the following key properties (Ghabcheloo *et al.*, 2006*c*):

1) L_K has $n-1$ eigenvalues at 1 and a single eigenvalue at zero with right and left eigenvectors $\mathbf{1}$ and $K^{-1}\mathbf{1}$, respectively such that $L_K\mathbf{1} = \mathbf{0}$ and $\mathbf{1}'K^{-1}L_K = \mathbf{0}'$.
2) $\xi = \mathbf{0} \Leftrightarrow \gamma \in \operatorname{span}\{\mathbf{1}\}$ and $|\gamma_i - \gamma_j|^2 \le 2|\xi|^2$.
3) $L_K K L = KL$.
4) $L L_K = L$.
5) $\nu' L_K K^{-1} L_K \nu \le \nu' K^{-1}\nu$; $\forall \nu \in \mathbb{R}^n$.
6) Let

$$\lambda_{2,m} := \min_{\substack{\mathbf{1}'\nu=0\\ \nu'\nu\neq 0}} \frac{\nu' L \nu}{\nu'\nu}, \qquad \lambda_m := \min_{\substack{\mathbf{1}'K^{-1}\nu=0\\ \nu'\nu\neq 0}} \frac{\nu' L \nu}{\nu'\nu}$$

If $\mathcal{G}$ is connected, then $\lambda_m = \frac{(\mathbf{1}'K^{-1}\mathbf{1})^2}{n\mathbf{1}'K^{-2}\mathbf{1}}\lambda_{2,m} > 0$.

Exploiting the results above we now propose the coordination feedback law using a Lyapunov-based strategy.

With the path-following control law proposed in the previous section, the dynamics of the coordination subsystem can be written in vector form as

$$\dot{\gamma} = \omega, \tag{4a}$$
$$\dot{\omega} = f_\gamma(\cdot) + \tilde{v}_r, \tag{4b}$$

where $\gamma := [\gamma_i]_{i\in\mathcal{I}}$, $\omega := [\dot{\gamma}_i]_{i\in\mathcal{I}}$, $f_\gamma(\cdot) := [-\mu_i + v_r^\gamma - k_{\gamma_i}\eta_i]_{i\in\mathcal{I}}$, and $\tilde{v}_r := [\tilde{v}_{r_i}]_{i\in\mathcal{I}}$. Let $\xi_1 := L_K\gamma$ and consider the control Lyapunov function

$$V_1 := \frac{1}{2}\xi_1' K^{-1}\xi_1,$$

whose time-derivative along the solution of (4) is given by

$$\dot{V}_1 = \xi_1' K^{-1} L_K \omega.$$

We can regard ω as a virtual control input that one would use to make $\dot{V}_1$ negative. To accomplish this, and taking into account that the final feedback law must be decentralized, we introduce a second error vector $\xi_2 := \omega + KL\xi_1$ and re-write

$$\dot{V}_1 = -\xi_1' L \xi_1 + \xi_1' K^{-1} L_K \xi_2.$$

Consider the augmented Lyapunov function

$$V_2 := V_1 + \frac{1}{2}\xi_2' K_2^{-1}\xi_2 = \frac{1}{2}\xi_1' K^{-1}\xi_1 + \frac{1}{2}\xi_2' K_2^{-1}\xi_2,$$

where $K_2 > 0$ is a diagonal matrix. The time-derivative of V_2 along the solution of (4) can be written as

$$\dot{V}_2 = -\xi_1' L\xi_1 + \xi_2'\big(-L_K K^{-1}\xi_1 + K_2^{-1} f_\gamma(\cdot) + K_2^{-1}\tilde{v}_r + K_2^{-1}KL\dot{\gamma}\big).$$

Selecting the decentralized feedback law

$$\tilde{v}_r = -KL\dot{\gamma} - K_2(\omega + KL\gamma), \tag{5}$$

where $K_2 > 0$ is a diagonal matrix, we obtain

$$\dot{V}_2 = -\xi_1' L \xi_1 - \xi_2\xi_2 - \xi_2' L_K K^{-1}\xi_1 + \xi_2' K_2^{-1} f_\gamma(\cdot).$$

Applying Young's inequality, it is straightforward to conclude that for sufficiently large gain K_2, $\xi := \operatorname{col}(\xi_1, \xi_2)$ is input-to-state stable (ISS) (Sontag and Wang, 1996) with state ξ and input $f_\gamma(\cdot)$. The following result applies.

Lemma 2: The coordination feedback law (5) solves robustly the coordination problem that is, the coordination subsystem (4) in the

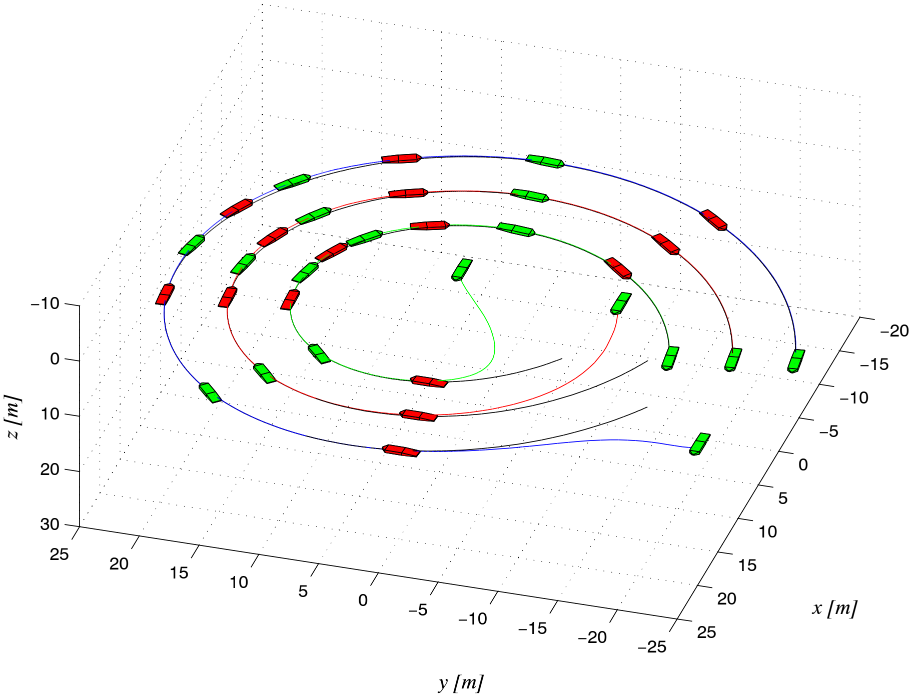

Fig. 3. Coordinated path-following of 3 AUVs.

closed-loop with (5) is input-to-state stable (ISS). In particular, the Lyapunov function $V_{CC} := V_2$ satisfies

$$\dot{V}_{CC} \leq -\lambda_{CC} V_{CC} + \rho_{CC} V_{PF},$$

for some $\lambda_{CC}, \rho_{CC} > 0$. Moreover, one can have the gain $\frac{\rho_{CC}}{\lambda_{CC}}$ small at will by making the coordinated control gains sufficiently large. □

Coordinated path-following controller

Using the results in Lemmas 1–2, the properties of the communication system described above, and applying the small-gain theorem (Jiang *et al.*, 1994) we conclude the following result.

Theorem 1: Consider the overall closed-loop system Σ_{CL} composed by n AUVs of the form (1)–(2) and the CPF controller given by the feedback laws for u_{1_i}, u_{2_i} for each vehicle i obtained in (Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007) together with (3) and the coordinated controller (5). For sufficiently large path-following control gains or coordination control gains, the overall closed-loop system solves robustly the CPF problem, that is, the path-following errors, speed errors, and coordination errors are IOpS. In particular, the composite Lyapunov function $V_{CPC} := V_{PF} + V_{CC}$ satisfies

$$V_{CPC}(t) \leq c V_{CPC}(t_0) e^{-\lambda_{CPC}(t-t_0)} + \epsilon$$

for some $c, \lambda_{CPC}, \epsilon > 0$. Furthermore, by appropriate choice of the controller parameters, any desired values for λ_{CPC} and ϵ are possible. □

AN ILLUSTRATIVE EXAMPLE

This section illustrates the application of the previous results to coordinate three AUVs moving in three-dimensional space. The AUVs are required to follow paths of the form

$$\mathbf{p}_{d_i}(\gamma_i) = \left[c_i \cos(\tfrac{2\pi}{T}\gamma_i + \phi_d), c_i \sin(\tfrac{2\pi}{T}\gamma_i + \phi_d), d\gamma_i\right],$$

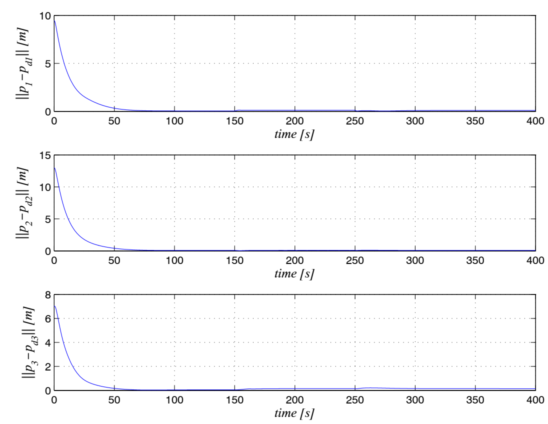

Fig. 4. Path-following errors.

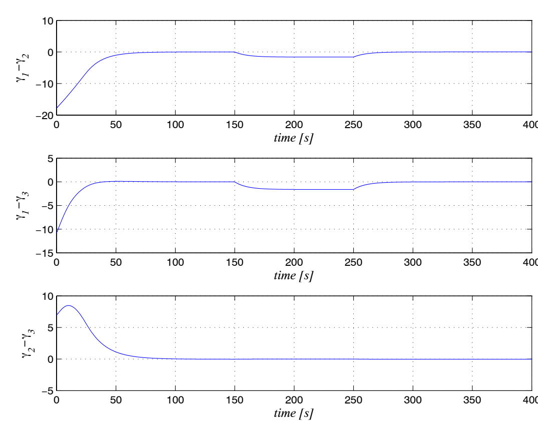

Fig. 5. Vehicle coordination errors.

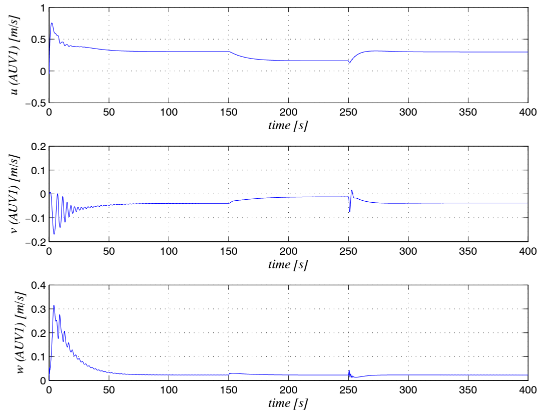

Fig. 6. Time evolution of linear velocities in $x-$direction (surge) u, $y-$direction (sway) v, and $z-$direction (heave) w of AUV1.

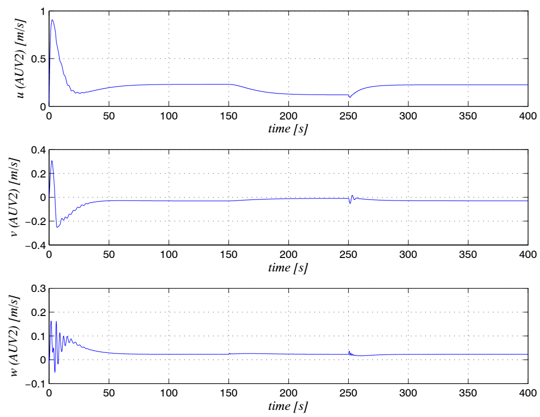

Fig. 7. Time evolution of linear velocities in $x-$direction (surge) u, $y-$direction (sway) v, and $z-$direction (heave) w of AUV2.

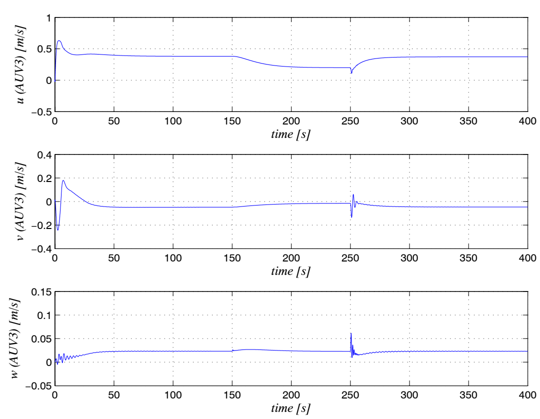

Fig. 8. Time evolution of linear velocities in $x-$direction (surge) u, $y-$direction (sway) v, and $z-$direction (heave) w of AUV3.

with $c_1 = 20\,\mathrm{m}$, $c_2 = 15\,\mathrm{m}$, $c_3 = 25\,\mathrm{m}$, $d = 0.05\,\mathrm{m}$, $T = 400$, and $\phi_d = -\frac{3\pi}{4}$. The initial conditions are $\mathbf{p}_1 = (10\,\mathrm{m}, -15\,\mathrm{m}, -5\,\mathrm{m})$, $\mathbf{p}_2 = (5\,\mathrm{m}, 0\,\mathrm{m}, 0\,\mathrm{m})$, $\mathbf{p}_3 = (20\,\mathrm{m}, -25\,\mathrm{m}, 5\,\mathrm{m})$, $R_1 = R_2 = R_3 = I$, and $v_1 = v_2 = v_3 = \omega_1 = \omega_2 = \omega_3 = \mathbf{0}$. The reference speed was set to $v_r = 1[\mathrm{sec}^{-1}]$. The vehicles are also required to keep a formation pattern that consists of having them aligned along a straight line in the plane. Furthermore, AUV 1 is allowed to communicate with AUVs 2 and 3, but the latter two do not communicate between themselves directly. To further illustrate the behavior of the proposed CPF control architecture, we also force the following scenario: from $t = 150\,s$ to $t = 250\,s$, AUV 1 is only able to follow the path with $\dot{\gamma}_1 = 0.5$. Figure 3 shows the trajectories of the AUVs. Figures 4–5 illustrate the evolution of the path-following errors $e_{p_i} := \mathbf{p}_i - \mathbf{p}_{d_i}$ and coordination errors $\gamma_i - \gamma_j$. Before $t = 150\,s$, clearly, the vehicles adjust their speeds to meet the formation requirements and the coordination errors converge to zero. When AUV 1 is forced to slow down during $t \in [150, 250]$ (without transmitting explicitly to its neighborhoods its new reference velocity), the coordination errors increase but remain bounded, which means that the AUVs 2 and 3 were capable of adjusting their velocities. This behavior can also be seen in Figures 6–8 that display the time evolution of the linear velocities of the AUVs.

CONCLUSION

The paper addressed the problem of steering a group of autonomous underwater vehicles along given spatial paths, while holding a desired inter-vehicle formation pattern (coordinated path-following). A solution was derived that builds on recent results on path-following control (Aguiar and Hespanha, 2004; Aguiar and Hespanha, 2007) and state-agreement (coordination) control (Ghabcheloo *et al.*, 2006*c*; Aguiar and Pascoal, 2007*a*) obtained by the authors. The solution proposed builds on Lyapunov based techniques and addresses explicitly the constraints imposed by the topology of the inter-vehicle communications network. Furthermore, it leads to a decentralized control law whereby the exchange of data among the vehicles is kept at a minimum. Simulations illustrated the efficacy of the solution proposed. Further work is required to extend the methodology proposed to address the problems of robustness against temporary communication failures, the fact that communications do not occur in a continuous manner, and the cost of exchanging information. Preliminary results regarding these issues can be found in (Aguiar and Pascoal, 2007*a*), where we incorporated in each vehicle a logic-based communication system that decides when to transmit information to the neighbors by comparing its actual state with its estimate "as perceived" by the neighboring system, and transmitting data when the "difference" between the two exceeds a certain level. Thus, the systems communicate at discrete instants of time, asynchronously.

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