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Dynamic Positioning and Way-Point Tracking of Underactuated AUVs in the Presence of Ocean Currents

A. PEDRO AGUIAR†* and ANTÓNIO M. PASCOAL†

† ISR/IST Institute for Systems and Robotics, Instituto Superior Técnico,
Torre Norte 8, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

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This paper addresses the problem of dynamic positioning and way-point tracking of underactuated autonomous underwater vehicles (AUVs) in the presence of constant unknown ocean currents and parametric modeling uncertainty. A nonlinear adaptive controller is proposed that steers an AUV along a sequence of way-points consisting of desired positions (x, y) in a inertial reference frame, followed by vehicle positioning at the final target point. The controller is first derived at the kinematic level assuming that the ocean current disturbance is known. An exponential observer for the current is then designed and convergence of the resulting closed-loop system trajectories is analyzed. Finally, integrator backstepping and Lyapunov based techniques are used to extend the kinematic controller to the dynamic case and to deal with model parameter uncertainty. Simulation results with a dynamic model of an underactuated autonomous underwater shuttle for the transport of benthic[1] labs are presented and discussed.

1 Introduction

The problem of autonomous underwater vehicle (AUV) control continues to pose considerable challenges to system designers, especially when the vehicles are underactuated and exhibit large parameter uncertainty. From a conceptual standpoint, the problem is quite rich and the tools used to solve it must necessarily borrow from solid nonlinear control theory. However, the interest in this type of problem goes well beyond the theoretical aspects because it is well rooted in practical applications that constitute the core of new and exciting underwater mission scenarios, as the following example shows.

Over the past few years, there has been renewed interest in the development of stationary benthic stations to carry out experiments on the biology, geochemistry, and physics of deep sea sediments and hydrothermal vents *in situ*, over long periods of time. However, classical methods of deploying and servicing benthic laboratories are costly and require permanent support from specialized crews resident on board manned submersibles or surface ships [9]. To overcome some of the above mentioned problems, a European team consisting of IFREMER (FR), IST (PT), THETIS (GER), and VWS (GER), developed a prototype autonomous underwater shuttle vehicle named *Sirene* to automatically transport and position a large range of stationary benthic laboratories on the seabed, at a desired target point, down to depths of 4000 meters [5]. In a typical mission scenario (see Figure 1), the *Sirene* vehicle and the laboratory are first coupled together and launched from a support ship. Then, the ensemble descends in a free-falling trajectory (under the action of a ballast weight) at a speed in the range from 0.5 to 1 m/s. At approximately 100 m above the seabed, the *Sirene* releases its ballast and the weight of the all ensemble becomes neutral. At this point, the operator onboard the support ship instructs the vehicle to progress at a fixed speed, along a path defined by a number of selected way-points, until it reaches a vicinity of the desired target point. Afterwards, *Sirene* maneuvers to acquire the final desired heading and lands smoothly on target, after which it uncouples itself from the benthic laboratory and returns to the surface. The benthic laboratory can be easily recovered at a later time by sending an acoustic signal to the vehicle that triggers the release of a weight and forces the laboratory to re-surface slowly.

*Corresponding author. Email: pedro@isr.ist.utl.pt
[1]benthos - bottom of the ocean

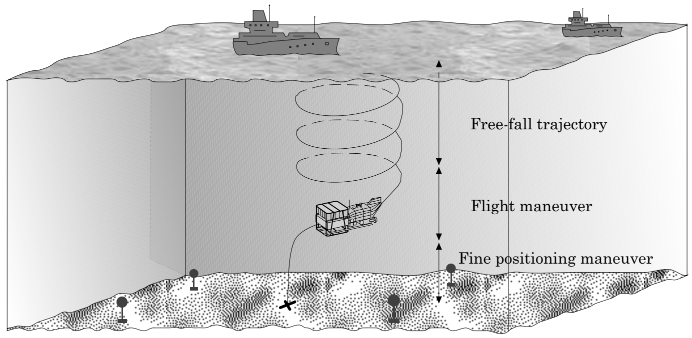

Figure 1. Mission scenario.

The *Sirene* Autonomous Underwater Vehicle (AUV) – depicted in Figure 2 – has an open-frame structure and is 4.0 m long, 1.6 m wide, and 1.96 m high. Its dry weight is 4000 Kg and its maximum operating depth is 4000 m. The vehicle is equipped with two back thrusters for surge and yaw motion control in the horizontal plane and one vertical thruster for depth control. Roll and pitch motion are left uncontrolled, since the metacentric height[2] is sufficiently large (36 cm) to provide adequate static stability. The AUV has no side thruster, thus making it underactuated. In the figure, the vehicle carries a representative benthic lab which is cubic-shaped and has a volume of approximately $2.3m^3$.

The problem of steering an underactuated AUV like *Sirene* to a point with a desired orientation (i.e., pose control) has only recently received special attention in the literature (cf., e.g., [14; 16; 17; 7; 8; 2] and references therein). This task raises some challenging questions in control system theory because, in addition to being underactuated, the vehicle exhibits complex hydrodynamic effects that must necessarily be taken into account during the controller design phase. Namely, the vehicle exhibits sway and heave velocities that generate non-zero angles of sideslip and attack, respectively. This rules out any attempt to design a steering system for the AUV that would rely on its kinematic equations only.

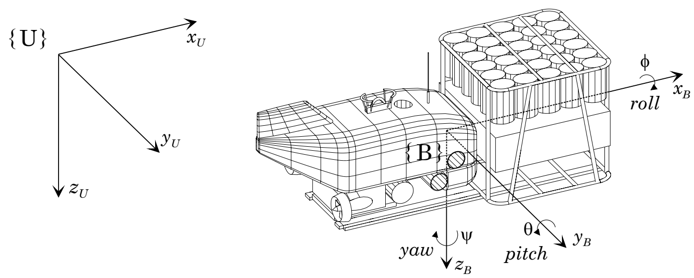

Figure 2. The *Sirene* autonomous underwater vehicle coupled to a benthic laboratory. Body-fixed $\{B\}$ and earth-fixed $\{U\}$ reference frames

In practice, an AUV must also be capable of operating in the presence of unknown ocean currents. Interestingly enough, even for the case where the current is constant, the problem of regulating an underactuated AUV to a desired point with an arbitrary desired orientation does not have a solution. In fact, if the desired orientation of the vehicle is such that its main x-axis is not aligned against the direction of the current, point stabilization controllers derived without taking the current into account will in general yield one of two possible behaviors: *i)* the vehicle will diverge from the desired target position, or *ii)* the controller will keep moving the vehicle around a neighborhood of the desired position, trying insistently to steer it to the given point, and consequently inducing an oscillatory behavior. There is therefore the need to explicitly address the presence of ocean currents.

Another practical problem that extends the previous ones is that of designing a combined guidance and control system to achieve way-point tracking of an AUV before it stops at the final goal position. The AUV can then be made to follow a predefined reference path (leading to the final target point) that is

[2]distance between the center of buoyancy and the center of mass.

specified by a sequence of way points. Way-point tracking can in principle be done in a number of ways. Most of them lend themselves to intuitive interpretation but lack a solid theoretical background. Perhaps the most widely known is so-called line-of-sight scheme [11]. In this case, vehicle guidance is simply done by issuing heading reference commands to the vehicle's steering system so as to align the main axis of the vehicle along the line of sight between the present position of the vehicle and the way-point to be reached. Tracking of the reference command is done via a properly designed autopilot. However, as is well known, separation of guidance and autopilot functions may not yield stability [15].

Motivated by the above considerations, this paper addresses the problem of dynamic positioning and way-point tracking of underactuated autonomous underwater vehicles in the horizontal plane in the presence of constant unknown ocean currents and parametric modeling uncertainty. The problem is posed and solved in a rigorous mathematical framework. A *nonlinear adaptive controller* is proposed that steers the AUV and moves it through a sequence of points consisting of desired positions (x, y) in a inertial reference frame, followed by vehicle positioning at the final target point. To tackle the positioning problem, the approach considered here is to drop the specification on the final desired orientation and to use this additional freedom to force the vehicle to converge to the desired point. Naturally, the vehicle will align itself against the direction of the current. The nonlinear adaptive controller proposed yields *convergence of the trajectories of the closed-loop system in the presence of a constant unknown ocean current disturbance and parametric model uncertainty.* Controller design relies on a non smooth coordinate transformation in the original state space, followed by the derivation of a Lyapunov-based, adaptive, control law in the new coordinates and an exponential observer for the ocean current disturbance. For clarity of presentation, the controller is first derived at the kinematic level, assuming that the ocean current disturbance is known. Then, an observer for the current is designed and convergence of the trajectories of the resulting closed-loop system is analyzed. Finally, resorting to integrator backstepping and Lyapunov techniques, a nonlinear adaptive controller is developed that extends the kinematic controller to the dynamic case and deals with model parameter uncertainties. Simulation results are presented and discussed.

The organization of the paper is as follows: Section 2 formulates the problem of vehicle dynamic positioning and way-point tracking in the presence of constant unknown ocean currents and parametric modeling uncertainty. In Section 3, a solution to the dynamic positioning problem is proposed in terms of a nonlinear adaptive control law. The convergence of the resulting closed-loop system is analyzed. Section 4 extends the strategy proposed in the previous section to force the AUV to track a sequence of points consisting of desired positions (x, y) in a inertial reference frame, before it converges to the finally desired point. Section 5 evaluates the performance of the control algorithms developed using computer simulations. Finally, Section 6 contains some concluding remarks and discusses problems that warrant further research.

2 The AUV. Control Problem Formulation

This section describes the kinematic and dynamic equations of motion of the AUV depicted in Figure 2 in the horizontal plane and formulates the problem of dynamic positioning and way-point tracking. The notation is standard [10]. The control inputs are the surge force and the yaw torque provided by the back thrusters. Recall that the AUV has no side thruster. See [1; 3] for model details.

2.1 *Vehicle Modeling*

Following standard practice, the general kinematic and dynamic equations of motion of the vehicle in the horizontal plane can be developed using a global coordinate frame $\{U\}$ and a body-fixed coordinate frame $\{B\}$, as depicted in Figure 2. In the case of a fixed current $(v_{c_x}, v_{c_x})' \neq 0$, the kinematic equations take the form

$$\dot{x} = u_r \cos\psi - v_r \sin\psi + v_{c_x}, \tag{1a}$$

$$\dot{y} = u_r \sin\psi + v_r \cos\psi + v_{c_y}, \tag{1b}$$

$$\dot{\psi} = r, \tag{1c}$$

where u_r (surge speed) and v_r (sway speed) are the body axis components of the vehicle's velocity with respect to the water, x and y are the cartesian coordinates of its center of mass, ψ defines its orientation (heading angle), and r is the vehicle's angular speed.
Neglecting the motions in heave, roll, and pitch the simplified equations of motion for surge, sway and heading yield [10]

$$m_u \dot{u}_r - m_v v_r r + d_{u_r} u_r = \tau_u, \tag{2a}$$

$$m_v \dot{v}_r + m_u u_r r + d_{v_r} v_r = 0, \tag{2b}$$

$$m_r \dot{r} - m_{uv} u_r v_r + d_r r = \tau_r, \tag{2c}$$

where $m_u := m - X_{\dot{u}}$, $m_v := m - Y_{\dot{v}}$, $m_r := I_z - N_{\dot{r}}$, and $m_{uv} := m_u - m_v$ are mass and hydrodynamic added mass terms and $d_{u_r} := -X_u - X_{|u|u}|u_r|$, $d_{v_r} := -Y_v - Y_{|v|v}|v_r|$, and $d_r := -N_r - N_{|r|r}|r|$ capture hydrodynamic damping effects. The symbols τ_u and τ_r denote the external force in surge and the external torque about the z axis of the vehicle, respectively. Notice that the equation for v_r in (2b) is not driven by any external input.

2.2 *Problem Formulation*

The general control problem that we consider in this paper can be formulated as follows (see Figure 3):

Consider the state model of an underactuated AUV, given by (1) *and* (2). *Let* $p = \{p_1, p_2, \ldots, p_n\}$*;* $p_i = (x_i, y_i) \in \mathbb{R}^2$*;* $i = 1, 2, \cdots, n$ *be a given sequence of way-points expressed in a inertial frame* $\{U\}$*. Associated with each* p_i*;* $i = 1, 2, \cdots, (n-1)$ *consider the closed ball* $B_{\epsilon_i}(p_i)$ *with center* p_i *and radius* $\epsilon_i > 0$, i.e., $B_{\epsilon_i}(p_i) := \{p \in \mathbb{R}^2 : \|p - p_i\| \leq \epsilon_i\}$*. Derive a feedback control law for surge force* τ_u *and torque* τ_r *so that the vehicle's center of mass* (x, y) *converges to* p_n *after visiting (that is, reaching) the ordered sequence of neighborhoods* $B_{\epsilon_i}(p_i)$*;* $i = 1, 2, \cdots, (n-1)$ *in the presence of a constant unknown ocean current disturbance and parametric model uncertainty.*

The requirement that the neighborhoods be visited temporarily only, applies to $i = 1, 2, \cdots, (n-1)$. As for the last way-point ($i = n$), the physics of the problem, namely the fact that the lateral dynamics are not actuated directly and the presence of an ocean current, impose restrictions on how the vehicle approaches it. In fact, the controller must recruit the control signals for the back thrusters in such a way as to counteract the effect of the ocean current disturbance and to ensure that the sway velocity relative to the water v_r is naturally driven to zero.

3 Nonlinear controller design for dynamic positioning

This section proposes a nonlinear adaptive control law to regulate the motion of an underactuated AUV to a given point in the presence of a constant unknown ocean current disturbance and parametric modeling uncertainty. The controller is first derived at the kinematic level by assuming that the control signals are the surge velocity u_r and the yaw angular velocity r. It is also assumed that the ocean current disturbance intensity V_c and its direction ϕ_c are known. These assumptions will be lifted latter.

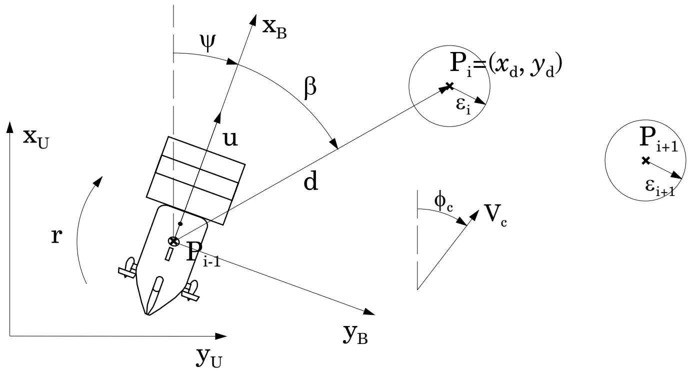

Figure 3. Coordinate Transformation.

3.1 *Coordinate Transformation*

In what follows, a coordinate transformation akin to that proposed in [4] is introduced. To this effect, let $(x_d, y_d)'$ denote a generic way-point, d the vector from the origin of frame $\{B\}$ to $(x_d, y_d)'$, and e its length. Denote by β the angle measured from x_B to d. Consider the coordinate transformation (see Figure 3)

$$e := \sqrt{(x - x_d)^2 + (y - y_d)^2}, \tag{3a}$$

$$x - x_d := -e\cos(\psi + \beta), \tag{3b}$$

$$y - y_d := -e\sin(\psi + \beta), \tag{3c}$$

$$\psi + \beta := \tan^{-1}\left(\frac{-(y - y_d)}{-(x - x_d)}\right). \tag{3d}$$

In (3d), care must be taken to select the proper quadrant for β. Let the ocean current disturbance be characterized by its intensity V_c and direction ϕ_c. The kinematics equations of motion of the AUV can be rewritten in the new coordinate system to yield

$$\dot{e} = -u_r\cos\beta - v_r\sin\beta - V_c\cos(\beta + \psi - \phi_c), \tag{4a}$$

$$\dot{\beta} = \frac{\sin\beta}{e}u_r - \frac{\cos\beta}{e}v_r - r + \frac{V_c}{e}\sin(\beta + \psi - \phi_c), \tag{4b}$$

$$\dot{\psi} = r. \tag{4c}$$

Note that the coordinate transformation (3) is only valid for non zero values of e, since for $e = 0$ the angle β is undefined. The above coordinate transformation is similar the one introduced in [4] as a means to include a discontinuity in the feedback control law that steers a wheeled robot to a point with a desired orientation. This was done to overcome the stringent condition imposed by Brockett's result [6]. In this paper, however the transformation is simply used to display the vehicle kinematics in a form that greatly helps motivate the structure of the controller derived.

3.2 *Kinematic Controller*

At the kinematic level, the control objective consists of recruiting the linear and angular velocities u_r and r, respectively, to regulate the position $p := (x, y)$ to a desired point which for the time-being, and without loss of generality, will be taken as the origin. The signals u_r and r are viewed as control inputs. At this stage, the relevant equations of motion of the AUV are (4) and (2b). It is important to stress that the dynamics of the sway velocity v must be explicitly taken into account, since the presence of this term in the kinematics equations (1) is not negligible (contrary to the case of wheeled mobile robots).

From (4), it is easy to verify that a possible strategy for controller design is to: *i)* manipulate r to regulate

β to zero (this will align x_B with vector d), and *ii)* actuate on u_r to force the position of the vehicle to approach to $p = 0$. However, to avoid problems concerning the boundedness of the control inputs, we will not drive formally the distance e to zero, but to some positive constant. To this effect, we re-define the coordinates (x_d, y_d) adequately. See the statement of the theorem below. At this stage, it is assumed that the intensity V_c and the direction ϕ_c of the ocean current disturbance are known. The following result applies.

THEOREM 3.1 *Consider the nonlinear invariant system Σ_{kin} described by the AUV model* (1), (2b) *in closed-loop with the control law*

$$u_r = k_1 e - k_1 \gamma - V_c \cos(\psi - \phi_c), \tag{5a}$$

$$r = k_1 \sin\beta - k_1 \frac{\gamma}{e} \sin\beta + \frac{V_c}{e} \sin(\psi - \phi_c)\cos\beta - \frac{v_r}{e}\cos\beta + k_2\beta, \tag{5b}$$

where e and β are defined in (3),

$$x_d := -\gamma \cos\phi_c, \tag{6a}$$

$$y_d := -\gamma \sin\phi_c, \tag{6b}$$

and k_1, k_2, $\gamma \in \mathbb{R}\backslash\{-\frac{x(t_0)}{\cos\phi_c}, -\frac{y(t_0)}{\sin\phi_c}\}$ are positive constants such that

$$\frac{d_{v_r}}{m_u} > k_1, \quad k_1 > 2\frac{V_c}{\gamma}, \quad \gamma > \delta, \tag{7a}$$

where δ is an arbitrary small positive constant and $x(t_0)$ and $y(t_0)$ are the initial conditions of x and y, respectively. Let $\mathbf{x}_{kin} : [t_0, \infty) \to \mathbb{R}^4$, $t_0 \geq 0$, $\mathbf{x}_{kin}(t) := (x, y, \psi, v_r)'$ be a solution of Σ_{kin} and $D \subset \mathbb{R}^4$ the domain in which the closed-loop system is forward complete[1] *with $e(t) \geq \delta > 0$. Then, for every initial condition $\mathbf{x}_{kin}(t_0) \in D$ the control signals and the solution $\mathbf{x}_{kin}(t)$ are bounded. Furthermore, for almost every initial condition $\mathbf{x}_{kin}(t_0) \in D$, the position $p := (x, y)$ converges to zero as $t \to \infty$.*

Proof The proof is organized as follows: First, it will be shown that β converges to zero as $t \to \infty$. The proof that e and v_r are bounded will follow. Using these results, it will be proved that $\lim_{t\to\infty}(e, v_r, \psi) = (\gamma, 0, \phi_c + k\pi)$, for some $k \in \mathbb{Z}$. It will also be shown that the equilibrium points $(e, \beta, v_r, \psi) = (\gamma, 0, 0, \phi_c + k\pi)$ with k odd are asymptotically stable and with k even are unstable. From (3), (6) it will then be concluded that for almost every initial condition $\mathbf{x}_{kin}(t_0) \in D$, $(x, y) \to 0$ as $t \to \infty$.

Convergence of β

Consider the positive definite function

$$V_{kin}(\beta) := \frac{1}{2}\beta^2. \tag{8}$$

Computing its time derivative along the trajectories of the system Σ_{kin} and using (5) yields

$$\begin{aligned} \dot{V}_{kin} &= \beta\Big[\frac{\sin\beta}{e}u_r - \frac{\cos\beta}{e}v_r - r + \frac{V_c}{e}\sin(\beta + \psi - \phi_c)\Big] \\ &= -k_2\beta^2. \end{aligned}$$

Thus, β is bounded and converges exponentially to zero as $t \to \infty$.

[1]A system is forward complete on the domain D if for all initial conditions in D and starting times t_0, all solutions are defined for all $t \geq t_0$.

Boundedness of e and v_r

In closed-loop, the (e, v_r)-dynamics can be written as

$$\dot{e} = -k_1 \cos\beta e + h_e(\beta, \psi) - v_r \sin\beta, \tag{9a}$$

$$\dot{v}_r = -d(e, \beta, \psi)v_r + g(\beta)e + h(e, \beta, \psi), \tag{9b}$$

where

$$\begin{aligned}
h_e(\beta, \psi) &:= k_1\gamma \cos\beta + V_c \left[\cos(\psi - \phi_c)\cos\beta - \cos(\beta + \psi - \phi_c)\right], \\
d(e, \beta, \psi) &:= \frac{d_{v_r}}{m_v} - \frac{m_u}{m_v}\Big[k_1\big(1 - \frac{\gamma}{e}\big)\cos\beta - \frac{V_c}{e}\cos(\psi - \phi_c)\cos\beta\Big], \\
g(\beta) &:= -\frac{m_u}{m_v}\left[k_1^2 \sin\beta + k_1 k_2 \beta\right], \\
h(e, \beta, \psi) &:= -\frac{m_u}{m_v}\Big[-2k_1^2\gamma \sin\beta + k_1 V_c\big(1 - \frac{\gamma}{e}\big)\sin(\psi - \phi_c)\cos\beta - k_1 k_2 \gamma\beta \\
&\quad - V_c\cos(\psi - \phi_c)k_1\big(1 - \frac{\gamma}{e}\big)\sin\beta - V_c\cos(\psi - \phi_c)k_2\beta \\
&\quad + k_1^2\frac{\gamma^2}{e}\sin\beta - \frac{V_c^2}{e}\cos(\psi - \phi_c)\sin(\psi - \phi_c)\cos\beta\Big].
\end{aligned}$$

Later, it will be necessary to analyze the dynamics of (9) with extra input signals $w_e(t)$, $w_{v_r}(t)$, that is, the dynamics of

$$\dot{e} = -k_1 \cos\beta e + h_e(\beta, \psi) - v_r \sin\beta + w_e, \tag{10a}$$

$$\dot{v}_r = -d(e, \beta, \psi)v_r + g(\beta)e + h(e, \beta, \psi) + w_{v_r}, \tag{10b}$$

where it will be assumed that

$$|w_e(t)| \leq c_1, \tag{11a}$$

$$|w_{v_r}(t)| \leq \gamma_1(t)|e| + \gamma_2(t)|v_r| + \gamma_3(t), \tag{11b}$$

for some $c_1 \geq 0$ and nonnegative uniformly bounded continuous functions $\gamma_i(t); i = 1, 2, 3$, that converge to zero as $t \to \infty$. At this stage, $w_e(t)$, $w_{v_r}(t)$ can obviously be taken as zero. However, to prepare the ground for Section 3.3, the result on the boundedness of (e, v_r) will be given for the general case where w_e and w_{v_r} are present.

Consider the positive definite function

$$V(e, v_r) := \frac{1}{2}e^2 + \frac{1}{2}v_r^2. \tag{12}$$

Evaluating its time derivative along the trajectories of Σ_{kin} yields

$$\begin{aligned}
\dot{V} &= -k_1 \cos\beta e^2 + h_e(\beta, \psi)e - e v_r \sin\beta + e w_e \\
&\quad - d(e, \beta, \psi)v_r^2 + g(\beta)v_r e + h(e, \beta, \psi)v_r + v_r w_{v_r}.
\end{aligned} \tag{13}$$

Since β is bounded, it follows that for all $t \in [t_0, T]$ with T finite $\dot{V}$ satisfies

$$\begin{aligned}
\dot{V} &\le k_1 e^2 + \bar{h}_{eT} e + e|v_r| + c_1 e \\
&\quad + \bar{d}_T v_r^2 + \bar{g}_T |v_r| e + \bar{h}_T |v_r| + \bar{\gamma}_{1T} e|v_r| + \bar{\gamma}_{2T} v_r^2 + \bar{\gamma}_{3T} v_r \\
&\le \Big(\frac{\bar{g}_T}{2} + k_1 + \frac{\bar{h}_{eT}}{2} + \frac{1}{2}\bar{\gamma}_{1T} + 1\Big) e^2 + \Big(\bar{d}_T + \frac{\bar{g}_T}{2} + \frac{\bar{h}_T}{2} + \frac{1}{2}\bar{\gamma}_{2T} + 1\Big) v_r^2 \\
&\quad + \frac{\bar{h}_T}{2} + \frac{\bar{h}_{eT}}{2} + \frac{c_1^2}{2} + \frac{\bar{\gamma}_{3T}^2}{2} \\
&\le \lambda V + c, \qquad (14)
\end{aligned}$$

where $\bar{d}_T := \sup_{t_0 \le t \le T} |d(t)|$, $\bar{g}_T := \sup_{t_0 \le t \le T} |g(t)|$, $\bar{h}_T := \sup_{t_0 \le t \le T} |h(t)|$, $\bar{h}_{eT} := \sup_{t_0 \le t \le T} |h_e(t)|$, $\bar{\gamma}_{iT} := \sup_{t_0 \le t \le T} |\gamma_i(t)|; i = 1, 2, 3$, $\lambda := 2\max\{\bar{d}_T + \frac{\bar{g}_T}{2} + \frac{\bar{h}_T}{2} + \frac{1}{2}\bar{\gamma}_{1T} + 1, \frac{\bar{g}_T}{2} + k_1 + \frac{\bar{h}_{eT}}{2} + \frac{1}{2}\bar{\gamma}_{2T} + 1\}$, and $c := \frac{\bar{h}_T}{2} + \frac{\bar{h}_{eT}}{2} + \frac{c_1^2}{2} + \frac{\bar{\gamma}_{3T}^2}{2}$ are finite constants. The second inequality in (14) follows from Young's inequality in the form $ab \le \frac{\lambda}{2}a^2 + \frac{1}{2\lambda}b^2$, for all $a, b, \lambda \ge 0$. We can now conclude from (12), (14) that for every finite time $T \ge t_0$, e and v_r are bounded for all $t \in [t_0, T]$. To prove that the same is true for all $t \ge T$, we use the fact that $(\beta, \gamma_1, \gamma_2, \gamma_3)$ is bounded, converges to zero as $t \to \infty$ and choose T sufficiently large such that the inequalities

$$\begin{aligned}
k_1 \cos\beta(T) &> \frac{\bar{g}_\infty}{2} + \frac{\lambda_1}{2}\bar{h}_{e\infty} + \frac{\sin\beta(T)}{2} + \frac{\gamma_1(T)}{2} + \frac{\lambda_2}{2}c_1 + a_1, \\
\underline{d}_\infty &> \frac{\bar{g}_\infty}{2} + \frac{\lambda_3}{2}\bar{h}_\infty + \frac{\sin\beta(T)}{2} + \frac{\gamma_1(T)}{2} + \gamma_2(T) + \frac{\lambda_4}{2}\gamma_3(T) + a_2
\end{aligned}$$

hold, where $\underline{d}_\infty := \inf_{t \ge T} d(t)$, $\bar{g}_\infty := \sup_{t \ge T} |g(t)|$, $\bar{h}_\infty := \sup_{t \ge T} |h(t)|$, $\bar{h}_{e\infty} := \sup_{t \ge T} |h_{e\infty}(t)|$ are positive and finite constants and $a_1, a_2, \lambda_i; i = 1, \ldots, 4$, are sufficiently small positive constants. From (13) and using Young's inequality, it follows that for all $t \ge T$

$$\dot{V} \le -2\min\{a_1, a_2\} V + \frac{\bar{h}_{e\infty}}{2\lambda_1} + \frac{\bar{h}_\infty}{2\lambda_3} + \frac{c_1}{2\lambda_3} + \frac{\bar{\gamma}_{3\infty}}{2\lambda_4},$$

which clearly shows that e and v_r are uniformly bounded.

Convergence of e to γ

Let $\tilde{e} := e - \gamma$. In closed-loop, the dynamics of $\tilde{e}$ satisfy

$$\dot{\tilde{e}} = -k_1 \cos\beta \tilde{e} + w_e,$$

where

$$w_e := V_c \left[\cos(\psi - \phi_c)\cos\beta - \cos(\beta + \psi - \phi_c)\right] - v_r \sin\beta.$$

Since $\beta \to 0$ as $t \to \infty$ and v_r is bounded, we conclude that $w_e(\cdot) \to 0$ as $t \to \infty$. Moreover, there exist finite constants $T \ge t_0 \ge 0$ and $\bar{k}_1 > 0$ such that the time derivative of $V(\tilde{e}) := \frac{1}{2}\tilde{e}^2$ satisfies

$$\begin{aligned}
\dot{V} &\le -\bar{k}_1 \tilde{e}^2 + \tilde{e} w_e, \quad \forall t \ge T \\
&= -\bar{k}_1 (1 - \theta)\tilde{e}^2 - \theta \tilde{e}^2 + \tilde{e} w_e, \quad 0 < \theta < 1 \\
&\le -\bar{k}_1 (1 - \theta)\tilde{e}^2, \quad \forall |\tilde{e}| \ge \frac{|w_e|}{\bar{k}_1 \theta}.
\end{aligned}$$

Thus, $\tilde{e}$ is input-to-state stable (ISS) (cf. e.g. [12]) with w_e as input. Moreover, since w_e converges to zero as $t \to \infty$, then $\tilde{e}$ also converges to zero.

Convergence of (x, y) *to zero*

To prove that (x, y) converges to zero, it is necessary to analyze the evolution of (ψ, v_r) given by

$$\dot{\psi} = \frac{V_c}{\gamma}\sin(\psi - \phi_c) - \frac{v_r}{\gamma} + h_\psi(e, \beta, v_r) + w_\psi(t), \tag{15a}$$

$$\begin{aligned} \dot{v}_r = &-\Big[\frac{d_{v_r}}{m_v} + \frac{V_c}{\gamma}\frac{m_u}{m_v}\cos(\psi - \phi_c)\Big]v_r + \frac{m_u}{m_v}\frac{V_c^2}{\gamma}\cos(\psi - \phi_c)\sin(\psi - \phi_c) \\ &+ h_{v_r}(e, \beta, v_r) + w_{v_r}(t), \end{aligned} \tag{15b}$$

where

$$\begin{aligned} h_\psi(e, \beta, v_r) := {} & k_1\big(1 - \frac{\gamma}{e}\big)\sin\beta + k_2\beta \\ & + \frac{v_r}{\gamma}(1 - \cos\beta) + v_r\gamma\cos\beta\big(1 - \frac{\gamma}{e}\big) \\ & - \frac{V_c}{\gamma}\sin(\psi - \phi_c)(1 - \cos\beta) - \frac{V_c}{\gamma}\big(1 - \frac{\gamma}{e}\big)\sin(\psi - \phi_c)\cos\beta, \\ h_{v_r}(e, \beta, v_r) := {} & -\frac{m_u}{m_v}k_1\big(1 - \frac{\gamma}{e}\big)v_r\cos\beta - \frac{V_c}{\gamma}v_r\cos(\psi - \phi_c)(1 - \cos\beta) \\ & - \frac{V_c}{\gamma}v_r\cos(\psi - \phi_c)\cos\beta\big(1 - \frac{V_c}{e}\big) + g(t) + h_1(t) \\ & - \frac{m_u}{m_v}\Big[k_1^2\frac{\gamma^2}{e}\sin\beta - k_1\gamma\frac{V_c}{e}\sin(\psi - \phi_c)\cos\beta \\ & + \frac{V_c}{e}\cos(\psi - \phi_c)k_1\gamma\sin\beta \\ & - \frac{m_u}{m_v}\frac{V_c^2}{\gamma}\cos(\psi - \phi_c)\sin(\psi - \phi_c)(1 - \cos\beta) \\ & - \frac{m_u}{m_v}\frac{V_c^2}{\gamma}\big(1 - \frac{\gamma}{e}\big)\cos(\psi - \phi_c)\sin(\psi - \phi_c)\cos\beta, \end{aligned}$$

and $w_\psi(t)$ denotes a continuous, bounded, and vanishing input signal, that is, $\lim_{t\to\infty} w_\psi(t) = 0$. Again, this general case will be exploited later in the paper. At this stage, the result of interest is simply obtained with $w_\psi = 0$. The functions $h_\psi(\cdot)$ and $h_{v_r}(\cdot)$ are bounded in D and converge to zero as $t \to \infty$ because v_r is bounded and $(e, \beta) \to (\gamma, 0)$ as $t \to \infty$.

Consider now the positive definite function

$$V(\psi, v_r) := V_c^2\frac{m_u}{m_v}\big[1 + \cos(\psi - \phi_c)\big] + \frac{1}{2}v_r^2. \tag{16}$$

Its time derivative is given by

$$\dot{V} = -\begin{bmatrix} V_c\sqrt{\frac{m_u}{m_v}}\sin(\psi - \phi_c) \\ v_r \end{bmatrix}' Q \begin{bmatrix} V_c\sqrt{\frac{m_u}{m_v}}\sin(\psi - \phi_c) \\ v_r \end{bmatrix} + h_{\psi v_r}(e, \beta, v_r, \psi),$$

where

$$Q := \begin{bmatrix} \frac{V_c}{\gamma} & -\frac{V_c}{2\gamma}\sqrt{\frac{m_u}{m_v}}\big[1+\cos(\psi-\phi_c)\big] \\ -\frac{V_c}{2\gamma}\sqrt{\frac{m_u}{m_v}}\big[1+\cos(\psi-\phi_c)\big] & \frac{d_{v_r}}{m_v}+\frac{V_c}{\gamma}\frac{m_u}{m_v}\cos(\psi-\phi_c) \end{bmatrix}$$

and

$$\begin{aligned} h_{\psi v_r}(e,\beta,v_r,\psi) &:= -V_c^2\frac{m_u}{m_v}\sin(\psi-\phi_c)\big[h_\psi(e,\beta,v_r)+w_\psi(t)\big] \\ &\quad + v_r h_{v_r}(e,\beta,v_r)+v_r w_{v_r}(t). \end{aligned}$$

Notice that $h_{\psi v_r}(\cdot)$ is bounded and converges to zero as $t\to\infty$, and the symmetric matrix Q is positive definite if the inequalities

$$\frac{V_c}{\gamma} > 0, \quad d_{v_r} > 2\frac{V_c}{\gamma}m_u, \tag{17a}$$

hold. This is clearly true in view of conditions (7). See Remark 7.4 for the case $V_c = 0$.

Therefore,

$$\begin{aligned} \dot V &\le -\lambda_{min}(Q)\left[V_c^2\frac{m_u}{m_v}\sin^2(\psi-\phi_c)+v_r^2\right] + |h_{\psi v_r}(\cdot)| \\ &\le -\lambda_{min}(Q)\left[V_c^2\frac{m_u}{m_v}\big[1+\cos(\psi-\phi_c)\big]\big[1-\cos(\psi-\phi_c)\big]+v_r^2\right] + |h_{\psi v_r}(\cdot)| \\ &\le -\lambda_{min}(Q)\Big[V_c^2\frac{m_u}{m_v}\big[1+\cos(\psi-\phi_c)\big]\big[1-\cos(\psi-\phi_c)\big] \\ &\quad +\frac{1}{2}\big[1-\cos(\psi-\phi_c)\big]v_r^2\Big] - \lambda_{min}(Q)\left[1-\frac{1}{2}\big[1-\cos(\psi-\phi_c)\big]\right]v_r^2 + |h_{\psi v_r}(\cdot)| \\ &\le -\lambda_{min}(Q)\big[1-\cos(\psi-\phi_c)\big]\left[V_c^2\frac{m_u}{m_v}\big[1+\cos(\psi-\phi_c)\big]+\frac{1}{2}v_r^2\right] + |h_{\psi v_r}(\cdot)| \\ &\le -\lambda_{min}(Q)\big[1-\cos(\psi-\phi_c)\big]V + |h_{\psi v_r}(\cdot)|, \end{aligned}$$

where $\lambda_{min}(Q)$ denotes the minimum eigenvalue of the positive definite matrix Q. Hence, it can be concluded that $\lim_{t\to\infty}\dot V(t) = 0$ which implies that $(\sin(\psi-\phi_c), v_r)'$ converges to zero as $t\to\infty$. Consequently, $\psi(t)$ approaches the set $E := \{\psi\in\mathbb{R} : \psi = \phi_c + k\pi,\ k\in\mathbb{Z}\}$. However, only the equilibrium points $\psi = \phi_c + k\pi$ with k odd are asymptotically stable. This can be verified by analysing the dynamic equation of ψ on the manifold $(e,\beta,v_r) = (\gamma,0,0)$, which satisfies

$$\dot\psi = \frac{V_c}{\gamma}\sin(\psi-\phi_c).$$

Thus, defining $\tilde\psi_k := \psi-\phi_c-k\pi$, yields

$$\dot{\tilde\psi}_k = \begin{cases} \frac{V_c}{\gamma}\sin(\tilde\psi_k), & k \text{ even} \\ -\frac{V_c}{\gamma}\sin(\tilde\psi_k), & k \text{ odd} \end{cases}$$

which by linearization clearly shows that the equilibrium points in $E_s := \{\psi\in\mathbb{R} : \psi = \phi_c + k\pi,\ k \text{ odd}\}$ are asymptotically stable and those in $E_u := \{\psi\in\mathbb{R} : \psi = \phi_c + k\pi,\ k \text{ even}\}$ are unstable.

It can now be concluded from (3b)–(3c) and (6) that

$$\lim_{t\to\infty} x(t) = -2\gamma \sin\left(\frac{\phi_c+\psi}{2}\right)\cos\left(\frac{\phi_c-\psi}{2}\right),$$
$$\lim_{t\to\infty} y(t) = -2\gamma \cos\left(\frac{\phi_c+\psi}{2}\right)\cos\left(\frac{\phi_c-\psi}{2}\right),$$

and consequently, when ψ approaches E_s, the position $(x, y) \to 0$ as $t \to \infty$. This concludes the proof of Theorem 3.1. □

Remark 1 For simplicity, we derived the feedback laws without taking into account the problem of input saturations. However, it is straightforward to conclude that for every input constraint $|u| \le u_{sat}$ and $|r| \le r_{sat}$ with $u_{sat} > V_c$, if the initial condition of $v(t_0)$ is sufficiently close to zero, than Theorem 3.1 holds with

$$u_r = k_1 \tanh(e) - k_1\gamma - V_c \cos(\psi - \phi_c),$$
$$r = k_1 \sin\beta - k_1\frac{\gamma}{e}\sin\beta + \frac{V_c}{e}\sin(\psi-\phi_c)\cos\beta - \frac{v_r}{e}\cos\beta + k_2\tanh(\beta),$$

by selecting the gains such that $k_1(1+\gamma)+V_c < u_{sat}$ and $k_1(1+\gamma/\delta)+V_c/\delta + k_2 < r_{sat}$.

Remark 2 The domain D is not $\mathbb{R}^4$ because it may happen that for some $t = T$ $e(t)$ will cross the singular position $e(T) = 0$. However, the dynamic equation

$$\dot{e} = -k_1 \cos\beta e - v_r \sin\beta + h_e(\beta, \psi)$$

suggests that if e starts sufficiently large and the lateral velocity v_r is small, then e will not cross the origin. From a practical point of view, a reasonable solution is to detect the proximity of the singular position $e = 0$ and in this case to modify the control law by replacing the terms with $\frac{1}{e}$ by $\frac{1}{\delta}$.

Remark 3 Theoretically, with a conveniently chosen initial condition, the yaw angle ψ could converge to one of the unstable equilibrium points in E. In practice this is not a problem since any small perturbation/noise will make the system converge to the stable equilibrium points.

Remark 4 If $V_c = 0$, (15b) reduces to

$$\dot{v}_r = -\frac{d_{v_r}}{m_v}v_r + h_{v_r}(e,\beta,v_r) + w_{v_r},$$

where $h_{v_r}(\cdot)$ and w_{v_r} can be viewed as vanishing perturbations. Since $\frac{d_{v_r}}{m_v} > 0$, then $v_r \to 0$ as $t \to \infty$. This, together with the fact that $\lim_{t\to\infty}(e,\beta) = (\gamma, 0)$, imply using (4c) and (5b) that $\dot{\psi}$ converges to zero and ψ to an equilibrium value that depends on the initial conditions. In this case, to force (x, y) to converge exactly to zero, the control parameter γ must be zero. One way to solve this problem is to choose γ as a function of V_c, *i.e.*, $\gamma = f(V_c)$, with $f(0) = 0$.

3.3 *Observer Design*

In this section, an observer is proposed to estimate the ocean current disturbance. We assume a position system is available that provides measurements of the position (x, y) of the vehicle. The structure of the observer is simple because it is based on the vehicle kinematics only. However, care must be taken to ensure boundedness and convergence of the trajectories of the system that arises from putting together the observer and the nonlinear control laws derived, because no general separation principle exist in this case.

Let v_{c_x} and v_{c_y} denote the components of the ocean current disturbance expressed in $\{U\}$. From (1), the kinematic equations for position are

$$\dot{x} = u_r \cos\psi - v_r \sin\psi + v_{c_x},$$
$$\dot{y} = u_r \sin\psi + v_r \cos\psi + v_{c_y}.$$

A simple observer for the component v_{c_x} of the current is

$$\dot{\hat{x}} = u_r \cos\psi - v_r \sin\psi + \hat{v}_{c_x} + k_{x_1}\tilde{x}, \tag{18a}$$
$$\dot{\hat{v}}_{c_x} = k_{x_2}\tilde{x}, \tag{18b}$$

where $\tilde{x} := x - \hat{x}$. Clearly, the estimate errors $\tilde{x}$ and $\tilde{v}_{c_x} := v_{c_x} - \hat{v}_{c_x}$ are asymptotically exponentially stable if all roots of the characteristic polynomial $p(s) = s^2 + k_{x_1}s + k_{x_2}$ associated with the system

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{v}}_{c_x} \end{bmatrix} = \begin{bmatrix} -k_{x_1} & 1 \\ -k_{x_2} & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{v}_{c_x} \end{bmatrix}$$

have strictly negative real parts.

The observer for the component v_{c_y} can be written in an analogous manner, to yield

$$\dot{\hat{y}} = u_r \sin\psi + v_r \cos\psi + \hat{v}_{c_y} + k_{y_1}\tilde{y}, \tag{19a}$$
$$\dot{\hat{v}}_{c_y} = k_{y_2}\tilde{y}, \tag{19b}$$

where $\tilde{y} := y - \hat{y}$ and $\hat{v}_{c_y}$ is the estimation of v_{c_y}.

It is now straightforward to conclude the following lemma which will be useful for establishing the convergence of the closed-loop system.

LEMMA 3.2 *Suppose that the ocean current disturbance is constant. Consider the observer system* (18)-(19), *where the gains* k_{x_1}, k_{x_2}, k_{y_1}, *and* k_{y_2} *are chosen such that the observer system is asymptotically stable. Define the variables* $\hat{V}_c$ *and* $\hat{\phi}_c$ *as the length and argument of vector* $(\hat{v}_{c_x}, \hat{v}_{c_y})'$, *respectively. Consider also that* $\hat{\phi}_c$ *is always computed so as to be a continuous signal with respect to time. Then, the variables* $\tilde{v}_{c_x}$, $\tilde{v}_{c_y}$, $\hat{v}_{c_x}$, $\hat{v}_{c_y}$, $\hat{V}_c$, $\hat{\phi}_c$, $\tilde{V}_c = V_c - \hat{V}_c$, $\tilde{\phi}_c = \phi_c - \hat{\phi}_c$, $\dot{\tilde{V}}_c = -\dot{\hat{V}}_c$, *and* $\dot{\tilde{\phi}}_c = -\dot{\hat{\phi}}_c$ *are bounded. Moreover, the errors* $\tilde{v}_{c_x}$, $\tilde{v}_{c_y}$, $\tilde{V}_c$, $\tilde{\phi}_c$, $\dot{\tilde{V}}_c$, *and* $\dot{\tilde{\phi}}_c$ *converge to zero as* t *goes to infinity.*

The following result shows that the control law in Theorem 3.1 with V_c and ϕ_c replaced by their estimates achieves closed-loop stability and convergence of (x, y) to zero.

THEOREM 3.3 *Consider the nonlinear invariant system* $\Sigma_{kin+Obs}$ *composed by the AUV model* (1), (2b), *the current observer* (18)-(19), *and the control law*

$$u_r = k_1 e - k_1\gamma - \hat{V}_c \cos(\psi - \hat{\phi}_c), \tag{20a}$$
$$r = k_1 \sin\beta - k_1\frac{\gamma}{e}\sin\beta + \frac{\hat{V}_c}{e}\sin(\psi - \hat{\phi}_c)\cos\beta - \frac{v_r}{e}\cos\beta + k_2\beta, \tag{20b}$$

where k_1, k_2, *and* γ *are positive constants that satisfy conditions* (7). *Let* β *and* e *be given as in* (3), *where* x_d *and* y_d *are now redefined to be*

$$x_d := -\gamma\cos\hat{\phi}_c, \tag{21a}$$
$$y_d := -\gamma\sin\hat{\phi}_c. \tag{21b}$$

Let $\mathbf{x}_{kin+Obs} : [t_0, \infty) \to \mathbb{R}^6$, $t_0 \geq 0$, $\mathbf{x}_{kin+Obs}(t) := (x, y, \psi, v, \tilde{v}_{c_x}, \tilde{v}_{c_y})'$ be a solution of $\Sigma_{kin+Obs}$ and $D \subset \mathbb{R}^6$ the domain in which the closed-loop system is forward complete with $e(t) \geq \delta$, for some $\delta > 0$. Then, for every initial condition $\mathbf{x}_{kin+Obs}(t_0) \in D$ the control signals and the solution $\mathbf{x}_{kin+Obs}(t)$ are bounded. Furthermore, for almost every initial condition $\mathbf{x}_{kin+Obs}(t_0) \in D$, the position (x, y) converges to zero as $t \to \infty$.

Proof The proof exploits LaSalle's invariance principle. From Theorem 3.1 it follows that on the manifold $E := \{\mathbf{x}_{kin+Obs} \in D : \tilde{v}_{c_x} = 0, \tilde{v}_{c_y} = 0\}$, for every initial condition $\mathbf{x}_{kin+Obs}(t_0) \in E$ the solution $\mathbf{x}_{kin+Obs}(t)$ is bounded and (e, β, v_r, ψ) converges to $(\gamma, 0, 0, \phi_c + k\pi)$ as $t \to \infty$, for some $k \in \mathbb{Z}$. From Lemma 3.2, $(\tilde{v}_{c_x}, \tilde{v}_{c_y})$ is bounded and $(\tilde{v}_{c_x}, \tilde{v}_{c_y}) \to 0$ as $t \to \infty$. Thus, to apply the invariance principle and thus conclude the proof, it remains to show that all off-manifold solutions are bounded.

Since x_d and y_d redefined in (21) are not constant, the state equations (4) change to

$$\begin{aligned}
\dot{e} &= -u_r \cos\beta - v_r \sin\beta - V_c \cos(\beta + \psi - \phi_c) - \gamma\,\dot{\hat{\phi}}_c \sin(\beta + \psi - \hat{\phi}_c), \\
\dot{\beta} &= \frac{\sin\beta}{e} u_r - \frac{\cos\beta}{e} v_r - r + \frac{V_c}{e}\sin(\beta + \psi - \phi_c) - \dot{\hat{\phi}}_c \frac{\gamma}{e}\cos(\beta + \psi - \hat{\phi}_c).
\end{aligned}$$

In closed-loop,

$$\dot{\beta} = -k_2\beta + w_\beta,$$

where

$$\begin{aligned}
w_\beta := &-\Big[\frac{\hat{V}_c}{e}\sin(\psi - \hat{\phi}_c) - \frac{V_c}{e}\sin(\psi - \phi_c)\Big]\cos\beta \\
&-\Big[\frac{\hat{V}_c}{e}\cos(\psi - \hat{\phi}_c) - \frac{V_c}{e}\cos(\psi - \phi_c)\Big]\sin\beta,
\end{aligned} \tag{22}$$

which shows that β is ISS with w_β as input. Since w_β is bounded, it follows that β is also bounded.

To prove that e and v_r are bounded, observe that the closed-loop dynamics of (e, v_r) are given by (10), where in this case the signals $w_e(t)$ and $w_{v_r}(t)$ take the form

$$\begin{aligned}
w_e &= w_{eobs}, \\
w_{v_r} &= w_{v_r obs},
\end{aligned}$$

with

$$\begin{aligned}
w_{eobs} := &\,\hat{V}_c\big[\cos(\psi - \hat{\phi}_c)\cos\beta - \cos(\beta + \psi - \hat{\phi}_c)\big] \\
&- V_c\big[\cos(\psi - \phi_c)\cos\beta - \cos(\beta + \psi - \phi_c)\big] \\
&- \gamma\dot{\hat{\phi}}_c \sin(\beta + \psi - \hat{\phi}_c),
\end{aligned}$$

$$\begin{aligned}
w_{v_r obs} := &-\frac{m_u}{m_v}\Big[-\frac{\hat{V}_c}{e}\cos(\psi - \hat{\phi}_c) + \frac{V_c}{e}\cos(\psi - \phi_c)\Big] v_r \cos\beta \\
&- \frac{m_u}{m_v} k_1\big(1 - \frac{\gamma}{e}\big)\Big[\hat{V}_c \sin(\psi - \hat{\phi}_c) - V_c \sin(\psi - \phi_c)\Big]\cos\beta \\
&- \frac{m_u}{m_v} k_1\big(1 - \frac{\gamma}{e}\big)\Big[-\hat{V}_c \cos(\psi - \hat{\phi}_c) + V_c \cos(\psi - \phi_c)\Big]\sin\beta \\
&- \frac{m_u}{m_v}\Big[-\hat{V}_c \cos(\psi - \hat{\phi}_c) + V_c \cos(\psi - \phi_c)\Big] k_2\beta
\end{aligned}$$

$$
\begin{aligned}
&- \frac{m_u}{m_v}\Big[-\frac{\hat{V}_c^2}{e}\cos(\psi-\hat{\phi}_c)\sin(\psi-\hat{\phi}_c) \\
&+ \frac{\hat{V}_c^2}{e}\cos(\psi-\hat{\phi}_c)\sin(\psi-\hat{\phi}_c)\Big]\cos\beta.
\end{aligned}
$$

From Lemma 3.2 and since β is bounded and converges to zero, it can be concluded that w_e and w_{v_r} satisfy (11). Thus, with the positive definite function $V(e, v_r)$ defined in (12), it follows that e and v_r are bounded.

To prove that ψ is bounded, observe that in closed-loop the (ψ, v_r)-dynamics satisfy (15) with $w_\psi = w_{\psi obs}$, where

$$
w_{\psi obs} := -\frac{\tilde{V}_c}{\gamma}\sin(\psi-\phi_c) + 2\frac{\hat{V}_c}{\gamma}\cos(\psi-\frac{\phi+\phi_c}{2})\sin\frac{\tilde{\phi}_c}{2},
$$

and

$$
\begin{aligned}
w_{v_r} = &-\frac{\tilde{V}_c}{\gamma}\frac{m_u}{m_v}\cos(\psi-\phi_c)v_r \\
&- 2\frac{\hat{V}_c}{\gamma}\frac{m_u}{m_v}v_r\sin(\psi-\frac{\phi_c+\hat{\phi}_c}{2})\sin\tilde{\phi}_c \\
&- \frac{m_u}{m_v}\frac{V_c+\hat{V}_c}{\gamma}\tilde{V}_c\cos(\psi-\phi_c)\sin(\psi-\phi_c) \\
&- 2\frac{m_u}{m_v}\frac{\hat{V}_c^2}{\gamma}\sin(\psi-\frac{\phi_c+\hat{\phi}_c}{2})\sin\tilde{\phi}_c\sin(\psi-\phi_c) \\
&+ 2\frac{m_u}{m_v}\frac{\hat{V}_c^2}{\gamma}\cos(\psi-\hat{\phi}_c)\cos(\psi-\frac{\phi_c+\hat{\phi}_c}{2})\sin\tilde{\phi}_c.
\end{aligned}
$$

Thus, since w_ψ is bounded and converges to zero as $t \to \infty$, we conclude that ψ is bounded.

It has thus been shown that all off-manifold solutions are bounded and that $(\tilde{v}_{c_x}, \tilde{v}_{c_y})$ converge to zero. Theorem 3.3 now follows from a straightforward application of LaSalle's invariance principle. □

3.4 *Nonlinear dynamic controller design*

This section indicates how the kinematic controller is extended to the dynamic case. This is done by resorting to backstepping techniques [13]. Following this methodology, let u_r and r in (20a) and (20b), respectively, be virtual control inputs and α_1 and α_2 the corresponding virtual control laws. Introduce the error variables

$$
\begin{aligned}
z_1 &:= u_r - \alpha_1, \\
z_2 &:= r - \alpha_2,
\end{aligned}
$$

and consider the function V_{kin} defined in (8) augmented with the quadratic terms z_1 and z_2, that is,

$$
V_{dyn}(\beta, z_1, z_2) := V_{kin} + \frac{1}{2}m_u z_1^2 + \frac{1}{2}m_r z_2^2. \tag{23}
$$

The time derivative of V_{dyn} can be written as

$$
\begin{aligned}
\dot{V}_{dyn} &= -k_2\beta^2 - \beta z_2 + \beta\frac{z_1}{e}\sin\beta + m_u z_1\dot{z}_1 + m_r z_2\dot{z}_2 \\
&= -k_2\beta^2 + z_1\Big[\tau_u + m_v v_r r - d_{u_r}u_r - m_u\dot{\alpha}_1 + \frac{\sin\beta}{e}\beta\Big] \\
&\quad + z_2\Big[\tau_r + m_{uv}u_r v_r - d_r r - m_r\dot{\alpha}_2 - \beta\Big] + \beta w_\beta.
\end{aligned}
$$

For simplicity we did not expand the derivative of α_1 and α_2. Let the control law for τ_u and τ_r be chosen as

$$
\tau_u = -m_v v_r r + d_{u_r}u_r + m_u\dot{\alpha}_1 - \frac{\sin\beta}{e}\beta - k_3 z_1, \tag{24a}
$$

$$
\tau_r = -m_{uv}u_r v_r + d_r r + m_r\dot{\alpha}_2 + \beta - k_4 z_2, \tag{24b}
$$

where k_3 and k_4 are positive constants. Then,

$$
\begin{aligned}
\dot{V}_{dyn} &= -k_2\beta^2 - k_3 z_1^2 - k_4 z_2^2 - \beta w_\beta \\
&= -k_2(1-\theta)\beta^2 - k_3 z_1^2 - k_4 z_2^2 - k_2\theta\beta^2 + \beta w_\beta, \quad 0 < \theta < 1 \\
&\leq -k_2(1-\theta)\beta^2 - k_3 z_1^2 - k_4 z_2^2, \quad \forall|\beta| \geq \frac{|w_\beta|}{k_2\theta}
\end{aligned}
$$

which shows that the subsystem (β, z_1, z_2) is ISS with w_β as input.

The extension of Theorem 3.3 to the dynamic case follows.

THEOREM 3.4 *Consider the nonlinear invariant system $\Sigma_{Dyn+Obs}$ described by the AUV model* (1), (2), *the observer* (18)–(19), *and the control law* (24). *Assume the control gains k_i, $i = 1,2,3,4$ and γ are positive constants and satisfy conditions* (7). *Let β and e be given as* (3) *with x_d and y_d defined in* (21).

Let $\mathbf{x}_{Dyn+Obs} : [t_0,\infty) \to \mathbb{R}^8$, $t_0 \geq 0$, $\mathbf{x}_{Dyn+Obs}(t) := (x, y, \psi, u, v, r, \tilde{v}_{c_x}, \tilde{v}_{c_y})'$ be a solution of $\Sigma_{Dyn+Obs}$ and $D \subset \mathbb{R}^8$ the domain in which the closed-loop system is forward complete with $e(t) \geq \delta$, for some $\delta > 0$. Then, for every initial condition $\mathbf{x}_{Dyn+Obs}(t_0) \in D$ the control signals and the solution $\mathbf{x}_{Dyn+Obs}(t)$ are bounded. Furthermore, for almost every initial condition $\mathbf{x}_{Dyn+Obs}(t_0) \in D$ the position (x, y) converges to zero as $t \to \infty$.

Proof Since w_β defined in (22) converges to zero because of Lemma 3.2 and (β, z_1, z_2) is ISS with w_β as input, it follows that β, z_1, and z_2 are bounded and converge to zero as $t \to \infty$. Thus, the present Theorem can be proved in a way similar to Theorem 3.3 by showing that all the states are bounded and then applying LaSalle's invariance principle. To this effect, notice that

$$
\begin{aligned}
w_e &= w_{eobs} - z_1\cos\beta, \\
w_{v_r} &= w_{v_r obs} - \frac{m_u}{m_v}\big[z_1 z_2 + z_1\alpha_2 + \alpha_1 z_2\big], \\
w_\psi &= w_{\psi obs} + z_2.
\end{aligned}
$$

Therefore, by noticing that α_1, α_2 can be bounded by

$$
\begin{aligned}
|\alpha_1| &\leq \bar{\gamma}_1|e| + \bar{c}_1, \\
|\alpha_2| &\leq \bar{\gamma}_2|v_r| + \bar{c}_2,
\end{aligned}
$$

for some nonnegative constants γ_i, c_i; $i = 1, 2$, and that (β, z_1, z_2) is bounded and converges to zero, it follows that w_e, w_{v_r}, and w_ψ satisfy the boundedness conditions imposed in the proof of Theorem 3.1. Thus, all off-manifold solutions are bounded and Theorem 3.4 follows. □

3.5 *Adaptive nonlinear controller design*

So far, it was assumed that the AUV model parameters are known precisely. This assumption is unrealistic. In fact, the methods used to estimate the parameters of an underwater vehicle rely on different techniques that include semi-analytic and empirical methods and tests in hydrodynamic tanks. The first are specially suited for streamlined, symmetric bodies and cannot be simply applied to bluff bodies like that of the *Sirene* AUV. The latter are expensive to execute and can in many cases be applied to estimate only a few parameters. Even so, the uncertainty in the estimated parameters can be quite large.

In this section, the control law developed is extended to ensure robustness against uncertainties in the model parameters.

Consider the set of all parameters of the AUV model (2) concatenated in the vector

$$\Theta := \left(m_u, m_v, m_{uv}, m_r, X_u, X_{|u|u}, N_r, N_{|r|r}, m_r\frac{m_u}{m_v}, m_r\frac{Y_v}{m_v}, m_r\frac{Y_{|v|v}}{m_v}\right)',$$

and define the parameter estimation error $\tilde{\Theta} := \Theta - \hat{\Theta}$, where $\hat{\Theta}$ denotes a nominal value of Θ. Consider the augmented quadratic function

$$V_{adp}(\beta, z_1, z_2, \tilde{\Theta}) := V_{dyn} + \frac{1}{2}\tilde{\Theta}'\Gamma^{-1}\tilde{\Theta}, \tag{25}$$

where $\Gamma := \text{diag}\{\gamma_1, \gamma_2, ..., \gamma_{11}\}$, $\gamma_i > 0$, $i = 1, 2, ...11$ are adaptation gains and V_{dyn} is defined in (23).

The time derivative of $\dot{V}_{adp}$ is given by

$$\begin{aligned}\dot{V}_{adp} &= -k_2\beta^2 + z_1\left[\tau_u + m_v v_r r - d_{u_r}u_r - m_u\dot{\alpha}_1 + \frac{sin\beta}{\beta}\beta\right] \\ &+ z_2\left[\tau_r + m_{uv}u_r v_r - d_r r - m_r(\dot{\alpha}_{2_a} + \dot{\alpha}_{2_b}) - \beta\right] \\ &- \tilde{\Theta}'\Gamma^{-1}\dot{\hat{\Theta}} + \beta w_\beta,\end{aligned}$$

where $\alpha_{2_b} := -\frac{v_r}{e}\cos\beta$ and $\alpha_{2_a} := \alpha_2 - \alpha_{2_b}$. Motivated by the choices in the previous sections, select the control laws

$$\tau_u = -\hat{\theta}_2 v_r r - \hat{\theta}_5 u_r - \hat{\theta}_6|u_r|u_r + \hat{\theta}_1\dot{\alpha}_1 - \frac{\sin\beta}{e}\beta - k_3 z_1, \tag{26a}$$

$$\begin{aligned}\tau_r &= -\hat{\theta}_3 u_r v_r - \hat{\theta}_7 r - \hat{\theta}_8|r|r + \hat{\theta}_4\dot{\alpha}_{2_a} + \hat{\theta}_9\frac{u_r}{e}r\cos\beta \\ &+ \hat{\theta}_{10}\frac{v_r}{e}\cos\beta + \hat{\theta}_{11}|v_r|\frac{v_r}{e}\cos\beta + \hat{\theta}_4\frac{v_r}{e}\left(\frac{\dot{\hat{e}}}{e}\cos\beta + \dot{\hat{\beta}}\sin\beta\right) \\ &+ \beta - k_4 z_2,\end{aligned} \tag{26b}$$

where $\hat{\theta}_i$ denotes the i-th element of vector $\hat{\Theta}$ and

$$\begin{aligned}\dot{\hat{e}} &= -u_r\cos\beta - v_r\sin\beta - \hat{V}_c\cos(\beta + \psi - \hat{\phi}_c) - \gamma\,\dot{\hat{\phi}}_c\sin(\beta + \psi - \hat{\phi}_c), \\ \dot{\hat{\beta}} &= \frac{\sin\beta}{e}u_r - \frac{\cos\beta}{e}v_r - r + \frac{\hat{V}_c}{e}\sin(\beta + \psi - \hat{\phi}_c) - \dot{\hat{\phi}}_c\frac{\gamma}{e}\cos(\beta + \psi - \hat{\phi}_c).\end{aligned}$$

Then,

$$\dot{V}_{adp} = -k_2\beta^2 - k_3 z_1^2 - k_4 z_2^2 + \tilde{\Theta}'\left[Q - \Gamma^{-1}\dot{\hat{\Theta}}\right] + \beta w_\beta, \tag{27}$$

where Q is a diagonal matrix given by

$$Q := \text{diag}\Big\{-\dot{\alpha}_1 z_1, z_1 v_r r, z_2 u_r v_r, -z_2\dot{\alpha}_{2_a} - z_2\frac{v_r}{e}\Big(\frac{\dot{e}}{e}\cos\beta + \dot{\beta}\sin\beta\Big), z_1 u_r,$$
$$z_1|u_r|u_r, z_2 r, z_2|r|r, -u_r r\frac{z_2}{e}\cos\beta, \frac{v_r}{e}z_2\cos\beta, \frac{v_r}{e}|v_r|z_2\cos\beta\Big\}.$$

Notice in (27) how the terms containing $\tilde{\Theta}_i$ have been grouped together. To eliminate them, choose the parameter adaptation law as

$$\dot{\hat{\Theta}} = \Gamma Q, \tag{28}$$

to yield

$$\begin{aligned}\dot{V}_{adp} &= -k_2\beta^2 - k_3 z_1^2 - k_4 z_2^2 + \beta w_\beta \\ &\leq -k_2(1-\theta)\beta^2 - k_3 z_1^2 - k_4 z_2^2, \quad \forall|\beta| \geq \frac{|w_\beta|}{k_2\theta}, \quad 0 < \theta < 1\end{aligned} \tag{29}$$

In the manifold $\{\tilde{v}_{c_x} = \tilde{v}_{c_y} = 0\}$, $w_\beta = 0$, and therefore from (29) it can be concluded that (β, z_1, z_2) converges to zero as $t \to \infty$.

The above results play an important role in the proof of the following theorem that extends Theorem 3.3 to deal with vehicle dynamics and model parameter uncertainty.

Theorem 3.5 *Consider the nonlinear invariant system Σ_{adp} consisting of the AUV model* (1), (2)*, the current observer* (18)–(19)*, and the adaptive control law* (26), (28)*, where the adaptation gain Γ is a* (11×11) *diagonal positive definite matrix. Assume the control gains k_i, $i = 1, 2, 3, 4$ and γ are positive constants and satisfy conditions* (7)*. Let β and e be given as in* (3) *with x_d and y_d defined in* (21)*.*

Let $\mathbf{x}_{adp} : [t_0, \infty) \to \mathbb{R}^{19}$, $t_0 \geq 0$, $\mathbf{x}_{adp}(t) := (x, y, \psi, u, v, r, \tilde{v}_{c_x}, \tilde{v}_{c_y}, \tilde{\Theta}')'$ be a solution to Σ_{adp} and $D \subset \mathbb{R}^{19}$ the domain in which the closed-loop system is forward complete with $e(t) \geq \delta$, for some $\delta > 0$. Then, for every initial condition $\mathbf{x}_{adp}(t_0) \in D$ the control signals and the solution $\mathbf{x}_{adp}(t)$ are bounded. Furthermore, for almost every initial condition $\mathbf{x}_{adp}(t_0) \in D$, the position (x, y) converges to zero as $t \to \infty$.

Proof From (25), (29) and by resorting to the LaSalle's invariance principle, it can be shown that $\tilde{\Theta}$ is bounded and converges to a finite constant. The rest of the proof is a simple application of the arguments used in the previous theorems. □

4 Tracking a sequence of points

This section proposes a nonlinear adaptive control law to steer the underactuated AUV through a sequence of neighborhoods $B_{\epsilon_i}(p_i) := \{p \in \mathbb{R}^2 : \|p - p_i\| \leq \epsilon_i\}$; $i = 1, 2, \cdots, n$ in the presence of a constant but unknown ocean current disturbance and parametric modeling uncertainty. In the end, the AUV should converge to p_n.

Let $\mathbb{Z}_n = \{1, 2, \ldots, n\}$. Consider the piecewise constant signal $\sigma : [t_0, \infty) \to \mathbb{Z}_n$ that is continuous from the right at every point and defined recursively by

$$\sigma = \eta\big((x, y), \sigma^-\big), \quad t \geq t_0 \tag{30}$$

where $\sigma^-(\tau)$ is equal to the limit from the left of $\sigma(\tau)$ as $\tau \to t$. Define the transition function $\eta : \mathbb{R}^2 \times \mathbb{Z}_n \to \mathbb{Z}_n$ as

$$\eta\big((x,y),i\big) = \begin{cases} i, & \|p-p_i\|_2 > \epsilon_i \\ i+1, & \|p-p_i\|_2 \le \epsilon_i;\ i \ne n \\ n, & i = n. \end{cases} \tag{31}$$

In this case, the position error e is defined as in (3a) with

$$(x_d, y_d) = \begin{cases} p_\sigma & \text{if } \sigma < n, \\ p_\sigma - \gamma(\cos\phi_c, \sin\phi_c) & \text{if } \sigma = n. \end{cases} \tag{32}$$

where the signal σ indexes the current way-point to be reached. Note that in order to single out the last way-point as a desired target towards which the AUV should converge, for $i = n$, vector $(x_d, y_d)'$ is defined using (6).

4.1 *Kinematic controller*

At the kinematic level, it is assumed that u_r and r are the control inputs and the relevant equations of motion of the AUV are (4) and (2b). The controller design strategy for $i = 1, 2, \cdots (n-1)$ consists of keeping the surge velocity at a constant positive value U_d, and manipulating r to regulate β to zero (this will align x_B with vector d). For $i = n$ (the final target), the strategy is to actuate on u_r to force the vehicle to converge to the position p_n using the controller developed in the Section 3. At this stage, it is assumed that the intensity V_c and the direction ϕ_c of the ocean current disturbance are known. The following result applies for the case where $i < n$.

THEOREM 4.1 *Consider the sequence of points $\{p_1, p_2, \ldots, p_{n-1}\}$ and the associated neighborhoods $\{B_{\epsilon_1}(p_1), B_{\epsilon_2}(p_2), \ldots, B_{\epsilon_{n-1}}(p_{n-1})\}$. Let $\underline{\epsilon} := \min_{1 \le i < n} \epsilon_i$, U_d, k_2, and $\underline{k}_2 > 0$ be positive constants. Consider the nonlinear system Σ_{kin} described by the AUV model (1), (2b) and assume that*

$$k_2 \ge \frac{U_d + V_c}{\underline{\epsilon}} + \underline{k}_2, \quad U_d > V_c, \quad \frac{d_{v_r}}{m_u} > \frac{U_d}{\underline{\epsilon}}. \tag{33}$$

Let the control law $u_r = \alpha_1$ and $r = \alpha_2$ be given by

$$\alpha_1 := U_d, \tag{34a}$$

$$\alpha_2 := k_2\beta + \frac{V_c}{e}\sin(\psi - \phi_c)\cos\beta - \frac{v_r}{e}\cos\beta \tag{34b}$$

with β and e as given in (3), and $(x_d, y_d)'$ computed using (30)-(32). Let $\mathbf{x}_{kin} : [t_0, \infty) \to \mathbb{R}^4$, $t_0 \ge 0$, $\mathbf{x}_{kin}(t) := (x, y, \psi, v_r)'$ be a solution to Σ_{kin}. Then, for every initial condition $\mathbf{x}_{kin}(t_0) \in \mathbb{R}^4$, there exist finite instants of time $t_0 \le t_1^m \le t_1^M \le t_2^m \le t_2^M, \ldots, \le t_{n-1}^m \le t_{n-1}^M$ such that the control signals and the solution $\mathbf{x}_{kin}(t)$ are bounded for all $t \in [t_0, t_{n-1}^M]$ and the position $p(t) := \big(x(t), y(t)\big)'$ stays in $B_{\epsilon_i}(p_i)$ for $t_i^m \le t \le t_i^M$, $i = 1, 2, \ldots, n-1$.

Proof Consider the candidate Lyapunov function

$$V_{kin}(\beta) = \frac{1}{2}\beta^2. \tag{35}$$

Computing its time derivative along the trajectories of system Σ_{kin} yields

$$\dot{V}_{kin} = -\beta^2\Big[k_2 - \frac{U_d}{e}\frac{\sin\beta}{\beta} - \frac{V_c}{e}\frac{\sin\beta}{\beta}\cos(\psi-\phi_c)\Big]$$

which is negative definite if k_2 satisfies condition (33). Thus, $\beta \to 0$ as $t \to \infty$. To prove that v_r is bounded, consider its dynamic motion in closed-loop as

$$\dot{v}_r = -\Big[\frac{d_{v_r}}{m_v} - \frac{m_u}{m_v}\frac{U_d}{e}\cos\beta\Big]v_r + w_{v_r}, \tag{36}$$

where

$$w_{v_r} := -\frac{m_u}{m_v}U_d\Big[k_2\beta + \frac{V_c}{e}\cos\beta\sin(\psi-\phi_c)\Big].$$

Clearly, if condition (33) holds, then v_r is bounded and ISS with w_{v_r} as input. To conclude the proof, observe that in closed-loop e satisfies

$$\dot{e} = -U_d\cos\beta - v_r\sin\beta - V_c\cos(\beta+\psi-\phi_c).$$

Thus, since $\beta \to 0$, v_r is bounded and $U_d > V_c$ it follows that there exist a time $T \geq t_0$ and a finite positive constant δ such that $\dot{e} < -\delta$ for all $t > T$. Consequently, for any $i \in \{1, \ldots, n-1\}$, the vehicle position (x, y) reaches the neighborhood $B_{\epsilon_i}(p_i)$ of p_i in finite time. □

Notice that Theorem 4.1 only deals with the first $n-1$ way-points. Steering the AUV to the last way-point can be done using the control structure proposed in Section 3.

4.2 *Adaptive nonlinear controller design*

Following the methodology described in Section 3.4–3.5, one arrives at the same control structure expressed in (26). However, in this case α_1 and α_2 are given by (34). The next result extends Theorem 4.1 to deal with vehicle dynamics, model parameter uncertainty, and a constant unknown ocean current disturbance. The proof is omitted since it follows the same steps of the proofs given in the previous section.

Theorem 4.2 *Consider the nonlinear invariant system Σ_{tsp} described by the AUV model* (1), (2), *the current observer* (18)–(19), *and the adaptive control law* (34), (26), *and* (28) *with V_c and ϕ_c replaced by their estimates $\hat{V}_c$ and $\hat{\phi}_c$, respectively. Assume that the control gains k_i, $i = 2, 3, 4$, and U_d are positive constants and satisfy conditions* (33). *Let the adaptation gain Γ be a* (11×11) *diagonal positive definite matrix, and let β and e given as in* (3) *with $(x_d, y_d)'$ computed using* (30)–(32). *Consider the sequence of points $\{p_1, p_2, \ldots, p_{n-1}\}$ and the associated neighborhoods $\{B_{\epsilon_1}(p_1), B_{\epsilon_2}(p_2), \ldots, B_{\epsilon_{n-1}}(p_{n-1})\}$. Let $\mathbf{x}_{tsp} : [t_0, \infty) \to \mathbb{R}^{19}$, $t_0 \geq 0$, $\mathbf{x}_{tsp}(t) := (x, y, \psi, u, v, r, \tilde{v}_{c_x}, \tilde{v}_{c_y}, \tilde{\Theta}')'$ be a solution to Σ_{tsp}. Then, for every initial condition $\mathbf{x}_{tsp}(t_0) \in \mathbb{R}^6$ there exist finite instants of time $t_0 \leq t_1^m \leq t_1^M \leq t_2^m \leq t_2^M, \ldots, \leq t_{n-1}^m \leq t_{n-1}^M$ such that the control signals and the solution $\mathbf{x}_{kin}(t)$ are bounded for all $t \in [t_0, t_{n-1}^M]$, and the position $p(t) := \big(x(t), y(t)\big)'$ stays in $B_{\epsilon_i}(p_i)$ for $t_i^m \leq t \leq t_i^M$, $i = 1, 2, \ldots, n-1$.*

5 Simulation results

To illustrate the performance of the control schemes proposed for vehicle positioning in the presence of parametric uncertainty and constant ocean current disturbance, computer simulations were carried out with a model of the *Sirene* AUV. The vehicle dynamic model is briefly described in Section 2. The reader is refereed to [1; 3] for complete details, including a list of the AUV hydrodynamic parameters.

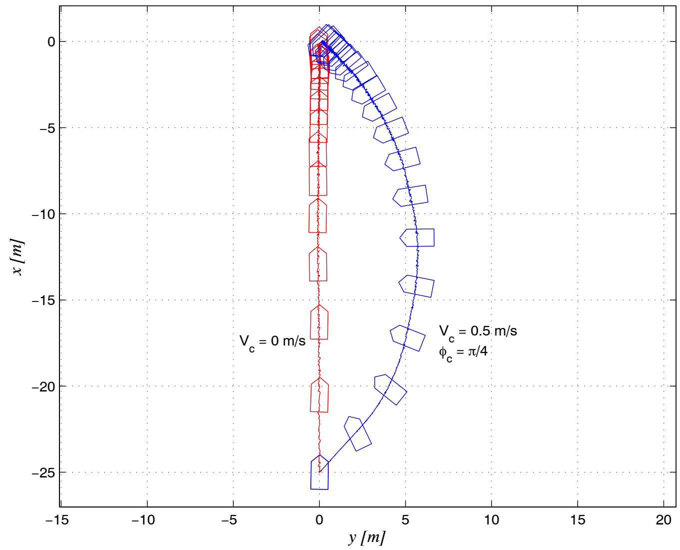

Figure 4. Simulation paths for the *Sirene* AUV. Initial condition $(x, y, \psi, u, v, r) = (-25\,m, 0, 0, 0, 0, 0)$.

Figure 4 shows the results of two simulations where the AUV starts from the initial condition $(x, y, \psi, u, v, r) = (-25\,m, 0, 0, 0, 0, 0)$ and is requested to reach and stay at position $(x, y) = (0, 0)$. In one simulation there is no current, that is, $V_c = 0\,m/s$. The other simulation captures the situation where the ocean current (which is unknown from the point of view of the controller) has intensity and direction $V_c = 0.5\,m/s$ and $\phi_c = \frac{\pi}{4}$ rad, respectively. In the simulations, the initial estimates for the vehicle parameters were disturbed by 50% from their true values.

To test the robustness of the proposed control algorithm with respect to sensor noise, zero mean uniform random noise was introduced in every sensed signal: the x and y positions, the orientation angle ψ, the linear velocities u, v, and the angular velocity r. The amplitudes of the noise signals were set to roughly 5% of the measurements. The dynamic positioning adaptive control law is that described in (26), (28). The control parameters were selected as follows: $k_1 = 0.04$, $k_2 = 0.8$, $k_3 = 2 \times 10^3$, $k_4 = 500$, $k_{x_1} = 1.0$, $k_{x_2} = 1.0$, $k_{y_1} = 1.0$, $k_{y_2} = 1.0$, $\gamma = 15$, and $\Gamma = \mathrm{diag}(10, 10, 10, 1, 1, 2, 2, 2, 1, 0.1, .1) \times 10^3$. The parameters satisfy the constraints (7). The criterium used to select them was based on the following procedure: *i)* use k_3 and k_4 to tune the performance of u_r and r in tracking the corresponding virtual signals generated by the kinematic feedback laws (5); *ii)* use k_1 and k_2 to tune the convergence behavior of e to γ and β to zero, respectively. To obtain good performance, the dynamics of the tracking errors of the velocities should be faster than the kinematic errors, and β should converge to zero faster than e to γ, that is, the vehicle should align itself properly before it maneuvers to reach the final position. The gains $k_{x_1}, k_{x_2}, k_{y_1}, k_{y_2}$ tune the response of the current observer. The parameter adaptation gain Γ determines how fast or slow the dynamics of the adaptive algorithm are.

We can see in Figure 4 that the vehicle converges to the desired position. Notice how in the presence of the ocean current the vehicle automatically recruits the yaw angle that is required to counteract the current at the target point. Thus, at the end of the maneuver the vehicle is at the goal position and faces the current with surge velocity u_r equal to V_c. This is clearly illustrated in Figures 5–8 that show the time responses of some representative signals (e.g., velocities, error signals, and control actuations) for the case where the ocean current is different from zero. Figure 8 shows good performance of the observer.

To illustrate the efficacy of the way-point tracking control algorithm, a new simulation experiment was performed. Figures 9–11 display the resulting vehicle trajectory in the xy-plane for three different simulations scenarios using the way-point guidance adaptive control law described in Section 4 for $i < n$ and the controller described in Section 3 for $i = n$ (the last way-point). The control parameters (for $i < n$) were selected as following: $k_2 = 1.8$, $k_3 = 1 \times 10^3$, $k_4 = 500$, $k_{x_1} = 1.0$, $k_{x_2} = 0.25$, $k_{y_1} = 1.0$, $k_{y_2} = 0.25$, and $\Gamma = \mathrm{diag}(10, 10, 10, 1, 1, 2, 2, 2, 1, 0.1, .1) \times 10^3$. The parameters satisfy the constraints (33). Zero mean uniform random noise was also included in every sensed signal with intensity identical to the value adopted in the first experiment. The initial estimates for

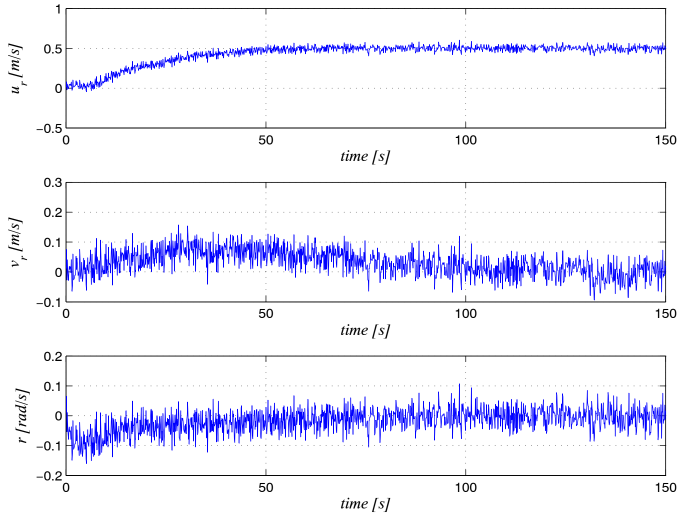

Figure 5. Time evolution of relative linear velocity in x-direction (surge) $u_r(t)$, relative linear velocity in y-direction (sway) $v_r(t)$, and angular velocity $r(t)$.

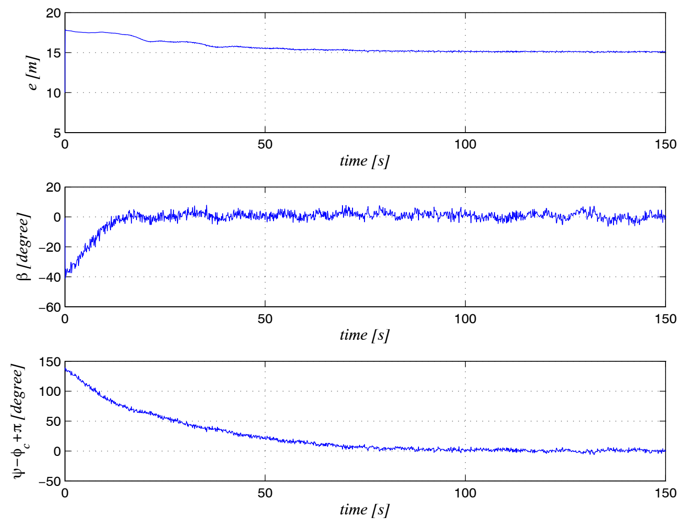

Figure 6. Time evolution of $e(t)$, $\beta(t)$, and $\psi(t) - \phi_c + \pi$.

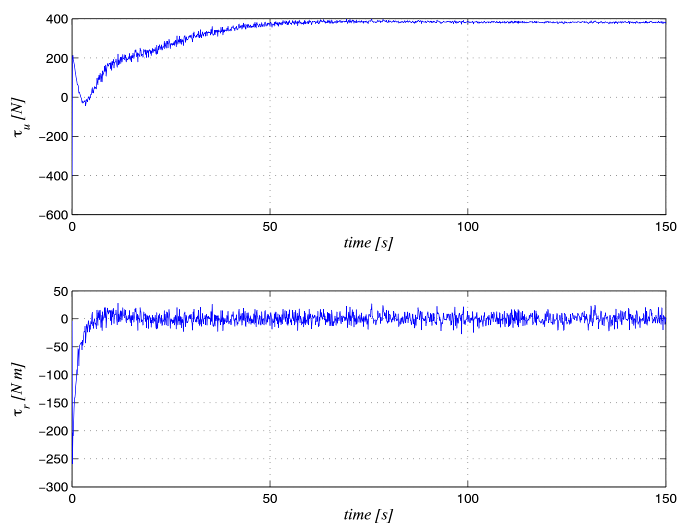

Figure 7. Time evolution of control signals $\tau_u(t)$ and $\tau_r(t)$.

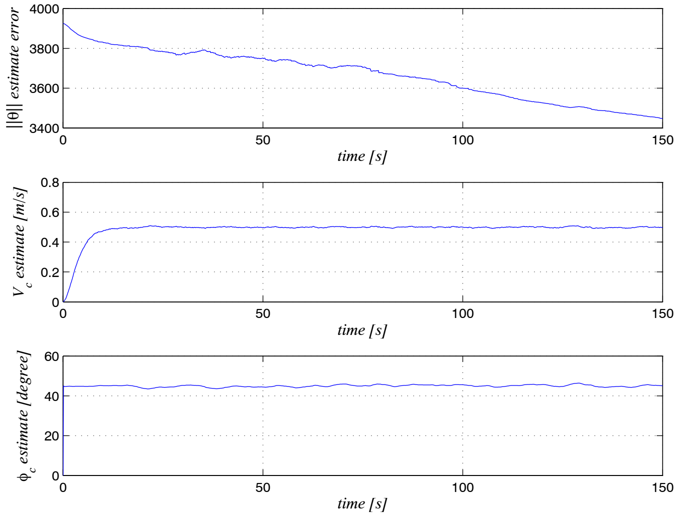

Figure 8. Time evolution of signals $\|\tilde{\theta}\|$, $\hat{V}_c$, and $\hat{\phi}_c$.

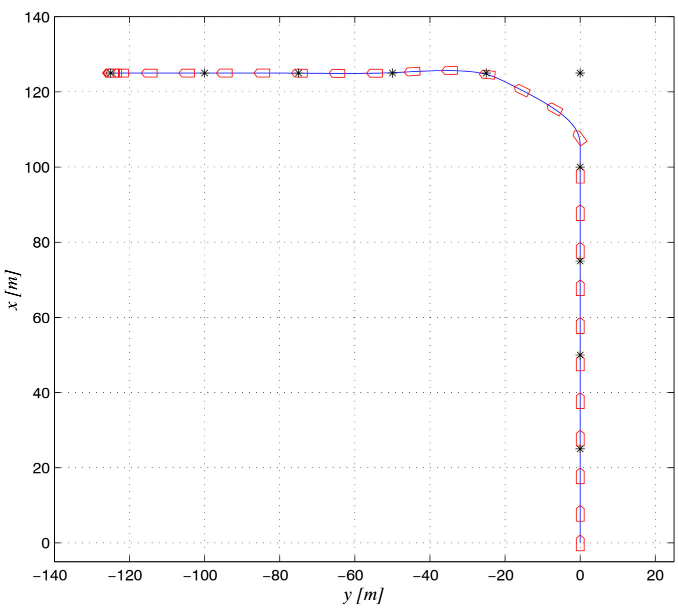

Figure 9. Way-point tracking with the *Sirene* AUV. $U_d = 0.5\,m/s$, $V_c = \phi_c = 0$.

the vehicle parameters were disturbed by 50% from their true values. The sequence of points are $\{(25.0, 0.0), (50.0, 0.0), (75.0, 0.0), (100.0, 0.0), (125.0, 0.0), (125.0, -25.0), (125.0, -50.0), (125.0, -75.0), (125.0, -100.0), (125.0, -125.0), (125.0, -125.0)\}$. The maximum admissible deviations from $p_i; i = 1, 2, \cdots, 10$ were fixed to $\epsilon_i = 5\,m$, except for $i = 5$, where $\epsilon_5 = 20\,m$. In both simulations, the initial condition for the vehicle was $(x, y, \psi, u, v, r) = 0$. In the first simulation (see Figure 9) there is no ocean current. The other two simulations capture the situation where the ocean current (which is unknown from the point of view of the controller) has intensity $V_c = 0.2\,m/s$ and direction $\phi_c = \frac{\pi}{4}$ rad, but with different values of the controller parameter U_d. Compare Figure 10 ($U_d = 0.5$) with Figure 11 ($U_d = 1.0$) that show the influence of the ocean current on the resulting xy-trajectory. Clearly, the influence is stronger for slow forward speeds u_r. In spite of this, notice that the vehicle always reaches the sequence of neighborhoods of points $p_1, p_2, \ldots, p_{10}$ until it finally converges to the desired position $p_{11} = (125, 125)\,m$. Figures 12–14 summarize the time responses of the relevant variables for the simulation with ocean current $V_c = 0.2\,m/s$, $\phi_c = \frac{\pi}{4}\,rad$, and $U_d = 0.5$.

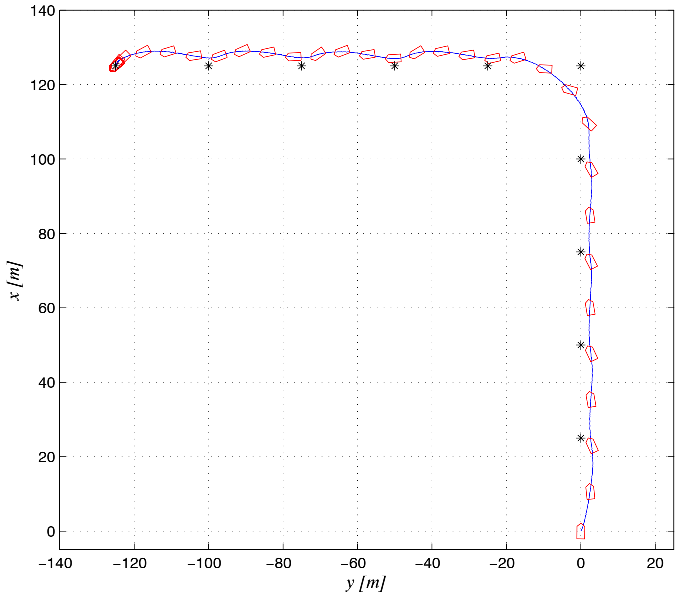

Figure 10. Way-point tracking with the *Sirene* AUV. $U_d = 0.5\,m/s$, $V_c = 0.2\,m/s$, $\phi_c = \frac{\pi}{4}\,rad$.

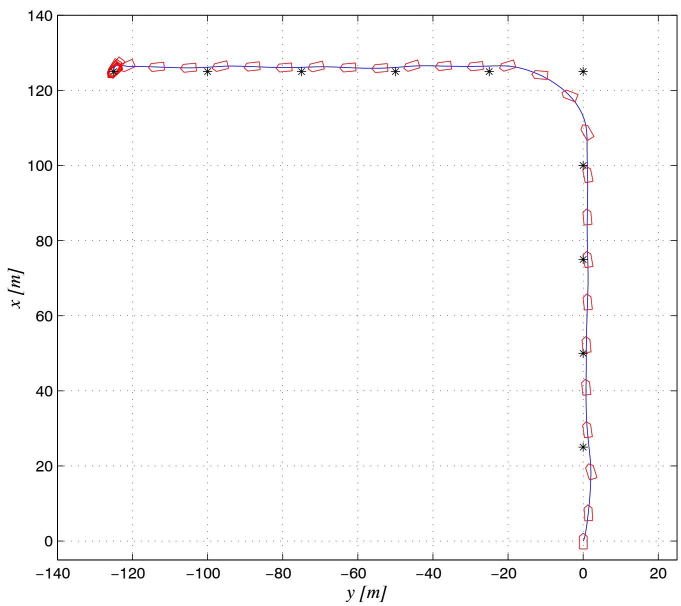

Figure 11. Way-point tracking with the *Sirene* AUV. $U_d = 1.0\,m/s$, $V_c = 0.2\,m/s$, $\phi_c = \frac{\pi}{4}\,rad$.

6 Conclusions

The paper addressed the problems of dynamic positioning and way-point tracking of underactuated autonomous underwater vehicles (AUVs) in the horizontal plane, in the presence of constant unknown ocean current disturbances and parametric modelling uncertainty. The solutions proposed borrow from nonlinear adaptive control theory and lead to controllers that embody in themselves an observer for the current.

The paper provided conditions under which the control laws yield convergence of the closed-loop systems trajectories. Simulations with a nonlinear model of a representative AUV showed the efficacy of the control laws proposed. The simulations also indicate, even though this was not proved formally, that the control laws yield good performance in the presence of measurement noise. In practice, the impact of sensor noise on system performance can be further alleviated by using state filter estimators. A rigorous analysis of this issue is certainly a topic for future research.

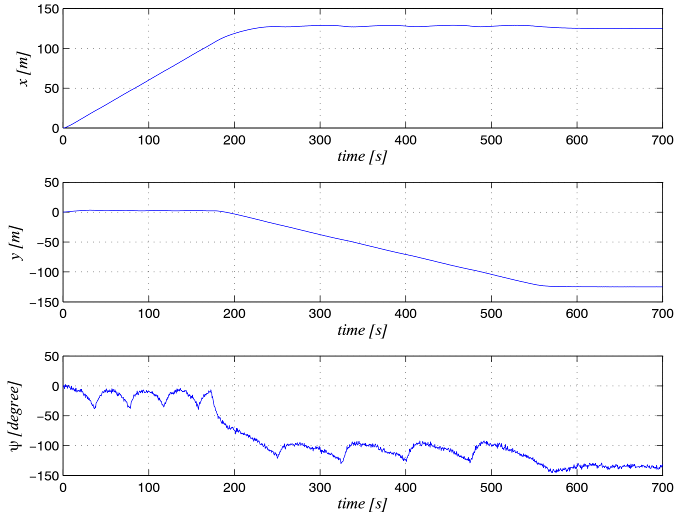

Figure 12. Time evolution of position $x(t)$, $y(t)$, and orientation $\psi(t)$.

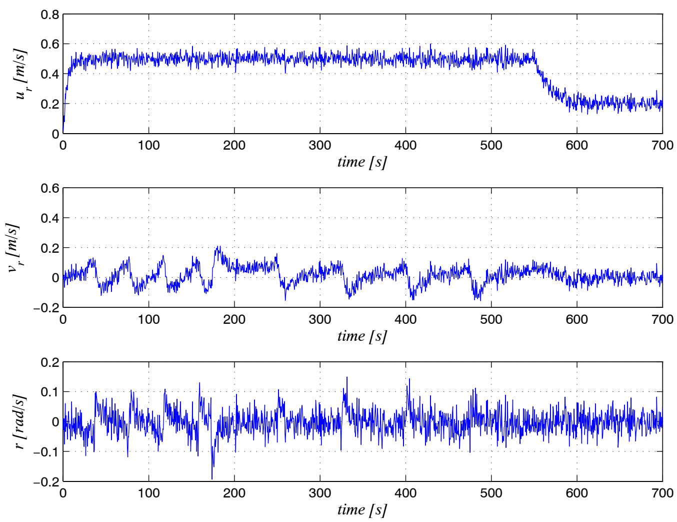

Figure 13. Time evolution of relative linear velocity in x-direction (surge) $u_r(t)$, relative linear velocity in y-direction (sway) $v_r(t)$, and angular velocity $r(t)$.

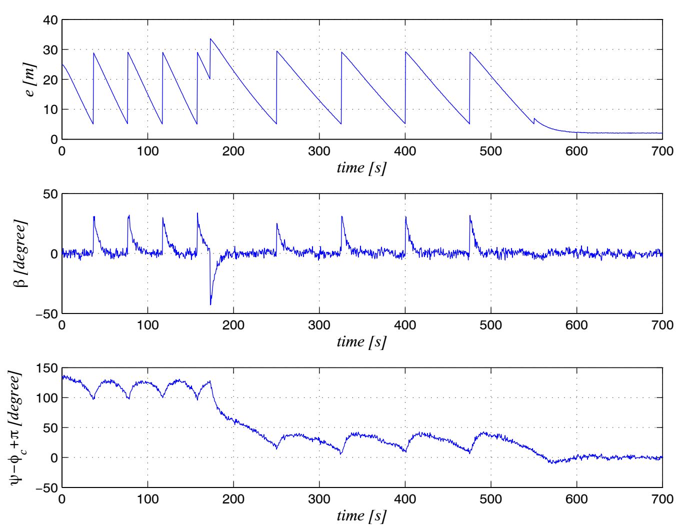

Figure 14. Time evolution of $e(t)$, $\beta(t)$, and $\psi(t) - \phi_c + \pi$.

The paper eschewed the study of the problems that may arise due to input saturation and actuator dynamics. In fact, it was only shown how input saturations are easily dealt with when the surge and yaw speeds are taken as controls. We recall that the control design methodology adopted resorts to Lyapunov-based and backstepping techniques that can in principle be extended to deal actuator dynamics, at the cost of obtaining more complex control laws. An alternative approach, and one that may prove more suitable in practice, is to adopt an inner-outer structure for the control laws adopted. In this framework, the strategy would be to design an inner-loop to deal with actuator and vehicle dynamics, and to use the kinematic control laws proposed in the paper in the outer-loop. The challenging task is to prove convergence of the trajectories of the resulting feedback control system. These and related problems warrant further research.

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