JOINT TURBO EQUALIZATION AND CANCELATION OF NONLINEAR DISTORTION EFFECTS IN MC-CDMA SIGNALS

Rui Dinis, Paulo Silva and Teresa Araújo
ISR - Instituto Superior Técnico
Lisboa, Portugal
email: rdinis@ist.utl.pt

ABSTRACT

In this paper, we consider low-PMEPR (Peak-to-Mean Envelope Power Ratio) MC-CDMA (Multicarrier Coded Division Multiple Access) schemes. We develop frequency-domain turbo equalizers combined with an iterative estimation and cancellation of nonlinear distortion effects. Our receivers have relatively low complexity, since they allow FFT-based (Fast Fourier Transform) implementations. The proposed turbo receivers allow significant performance improvements at low and moderate SNR (Signal-to-Noise Ratio), even when a low-PMEPR MC-CDMA transmission is intended.[1]

KEY WORDS

MC-CDMA, nonlinear distortion, turbo equalization, frequency-domain processing, iterative detection.

1 Introduction

MC-CDMA techniques [1] combine a CDMA scheme with an OFDM modulation (Orthogonal Frequency Division Multiplexing) [2], so as to allow high transmission rates over severely time-dispersive channels without the need of complex receiver implementations. The diversity effect inherent to the spreading allows good performances when high code rates are employed, or even for uncoded scenarios.

However, since the transmission over time-dispersive channels destroys the orthogonality between spreading codes, an FDE (Frequency-Domain Equalizer) is required before the despreading operation [3]. To avoid significant noise enhancement for channels with deep in-band notches, the FDE is usually optimized under an MMSE criterion (Minimum Mean-Squared Error) [3]. The performances can be further improved by employing the iterative receiver proposed in [4]. That receiver, which is based on the IB-DFE (Iterative Block Decision Feedback Equalizer) proposed for block transmission with single-carrier modulations [5, 6], allows significant performance improvements as we increase the number of iterations, especially for fully loaded systems and/or in the presence of high interference levels. The asymptotic performances can be close to the single-code performance, provided that we have severely

time-dispersive channels and the spreading factor is high. However, for moderate spreading factors (say a spreading factor of 16), the performances are rather poor, and far from the single-code performance. Moreover, the performance improvements as we increase the number of iterations are much lower for low and moderate SNR, the typical working regions when appropriate channel coding schemes are employed.

As with other multicarrier schemes, the MC-CDMA signals have strong envelope fluctuations and high PMEPR values, which makes them very prone to nonlinear effects. A promising approach to reduce the envelope fluctuations of the transmitted signals while maintaining the spectral occupation of conventional schemes is to employ clipping techniques, combined with a frequency-domain filtering [7, 8]. However, the nonlinear distortion effects can be severe when a low-PMEPR transmission is intended [7, 8]. To improve the performances, we can employ the receivers proposed in [9] where the nonlinear distortion effects are iteratively estimated and compensated.

In this paper, we consider MC-CDMA schemes with appropriate channel coding schemes. To reduce the envelope fluctuations of the transmitted signals while maintaining the spectral occupation of conventional MC-CDMA signals we employ the clipping and frequency-domain filtering techniques considered in [8]. We consider the use of iterative receiver structures based on the ones proposed in [4], combined with the estimation and cancelation of nonlinear distortion effects. To improve the performances at low and moderate SNRs we consider the use of turbo equalization schemes, where the equalization and channel decoding operations are repeated iteratively, sharing information between them [10]. Our frequency-domain turbo equalizers have relatively low complexity, since they allow FFT-based, frequency-domain implementations.

2 MC-CDMA Schemes

2.1 Conventional MC-CDMA

In this paper we consider MC-CDMA systems employing frequency-domain spreading. The frequency-domain block to be transmitted is $\{S_k; k = 0, 1, \ldots, N-1\}$, where $N = KM$, with K denoting the spreading factor and M the number of data symbols for each spread-

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ing code. The frequency-domain symbols are given by $S_k = \sum_{p=1}^{P} \xi_p S_{k,p}$, where ξ_p is an appropriate weighting coefficient that accounts for the different powers assigned to different spreading codes (the power associated to the pth spreading code is proportional to $|\xi_p|^2$) and $S_{k,p} = C_{k,p}A_{k \bmod M,p}$ [2] is the kth chip for the pth spreading code, where $\{A_{k,p}; k = 0, 1, \ldots, M-1\}$ is the block of data symbols associated to the pth spreading code and $\{C_{k,p}; k = 0, 1, \ldots, N-1\}$ is the corresponding spreading sequence. An orthogonal spreading, combined with pseudo-random scrambling, is assumed throughout this paper, with $C_{k,p}$ belonging to a QPSK constellation. Without loss of generality, it is assumed that $|C_{k,p}| = 1$.

As with conventional OFDM, an appropriate cyclic extension is appended to each block transmitted by the BS (Base Station). At the receiver, the cyclic extension is removed and the received samples are passed to the frequency domain, leading to the block $\{Y_k; k = 0, 1, \ldots, N-1\}$. When the cyclic extension is longer than the overall channel impulse response, the samples Y_k can be written as $Y_k = H_k S_k + N_k$ (a perfect synchronization is assumed), where H_k and N_k denote the channel frequency response and the noise term for the kth frequency, respectively.

Since the orthogonality between users is lost in frequency selective channels, an FDE (Frequency-Domain Equalizer) is required before the despreading operation [3]. Under perfect channel estimation, the FDE coefficients are given by $F_k = H_k^*/(\beta + |H_k|^2)$, with $\beta = E[|N_k|^2]/E[|S_k|^2]$, which corresponds to optimize the FDE under an MMSE criterion. The data symbols associated to the pth spreading code can be estimated by despreading the samples at the FDE output $\tilde{S}_k = Y_k F_k$, i.e., from

$$\tilde{A}_{k,p} = \sum_{k' \in \Psi_k} \tilde{S}_{k'} C_{k',p}^*, \tag{1}$$

with Ψ_k denoting the set of frequencies employed to transmit the kth data symbol of each spreading code (for a $K \times M$ interleaving, the set Ψ_k is given by $\Psi_k = \{k, k+M, \ldots, k+(K-1)M\}$).

2.2 Low-PMEPR MC-CDMA

To reduce the envelope fluctuations of the transmitted signals, we employ the transmitter structure depicted in fig. 1, which is based on the nonlinear signal processing schemes proposed in [7] for reducing the PMEPR of OFDM signals while maintaining the spectral efficiency of conventional OFDM schemes. Within that transmitter, $N'-N$ zeros are added to the original frequency-domain block (i.e., $N'-N$ idle subcarriers), followed by an IDFT operation so as to generate a sampled version of the time-domain MC-CDMA

This corresponds to employing a rectangular interleaver with dimensions $K \times M$, i.e., the different chips associated to a different data symbol are uniformly spread within the transmission band.

signal, with an oversampling factor $M_{Tx} = N'/N$. Each time-domain sample is submitted to a nonlinear device corresponding to an ideal envelope clipping, so as to reduce the envelope fluctuations on the transmitted signal. The clipped signal is then submitted to a frequency-domain filtering procedure, through the set of multiplying coefficients $G_k, k = 0, 1, \ldots, N'-1$, in order to reduce the out-of-band radiation levels inherent to the nonlinear operation.
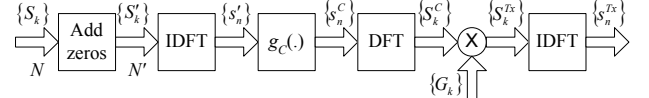

Figure 1. Low-PMEPR MC-CDMA signals.

It is shown in [8] that the frequency-domain block to be transmitted $\{S_k^{Tx} = S_k^C G_k; k = 0, 1, \ldots, N'-1\}$ can be decomposed into useful and nonlinear self-interference components: $S_k^{Tx} = \alpha S_k G_k + D_k G_k$, with α defined in [7, 8]. Throughout this paper we assume that $G_k = 1$ for the N in-band subcarriers and 0 for the $N'-N$ out-of-band subcarriers. In this case, $S_k^{Tx} = \alpha S_k + D_k$ for k in-band and 0 for k out-of-band. It can be shown that D_k is approximately Gaussian-distributed, with zero mean; moreover, $E[D_k D_{k'}^*]$ can be computed analytically, as described in [7, 8].

3 IB-DFE Receivers

3.1 Conventional MC-CDMA

Fig. 2 presents the IB-DFE receiver structure proposed in [4] for MC-CDMA signals. For a given iteration, the output samples are given by

$$\tilde{S}_k = F_k Y_k - B_k \hat{S}_k, \tag{2}$$

where $\{F_k; k = 0, 1, \ldots, N-1\}$ and $\{B_k; k = 0, 1, \ldots, N-1\}$ denote the feedforward and the feedback coefficients, respectively. $\{\hat{S}_k = \sum_{p=1}^{P} \xi_p \hat{S}_{k,p}; k = 0, 1, \ldots, N-1\}$, with $\hat{S}_{k,p} = \hat{A}_{k \bmod M,p} C_{k,p}$, where $\hat{A}_{k,p}$ denotes the estimate of $A_{k,p}$ from the previous iteration, obtained by despreading $\tilde{S}_k$, i.e., from (1).
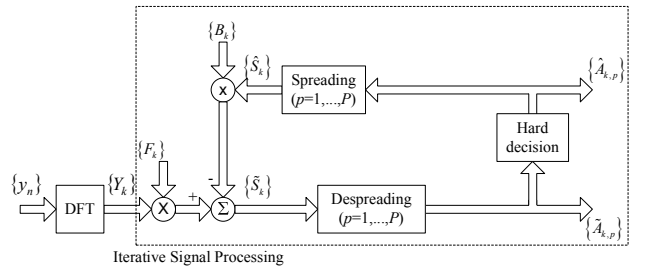

Figure 2. IB-DFE receivers for MC-CDMA schemes.

The optimum coefficients F_k and B_k that maximize

the overall SNR in the samples $\tilde{S}_k$ are given by [4][3]

$$F_k = \frac{\kappa H_k^*}{\beta + (1 - \rho^2)|H_k|^2}, \qquad (3)$$

and $B_k = \rho(F_k H_k - 1)$, respectively, where $\beta = E[|N_k|^2]/E[|S_k|^2]$ and κ is selected so as to ensure that $\sum_{k=0}^{N-1} F_k H_k / N = 1$.

The correlation coefficient ρ, which can be regarded as the blockwise reliability of the decisions used in the feedback loop (from the previous iteration), is given by

$$\rho = \frac{E[\hat{S}_k S_k^*]}{E[|S_k|^2]} = \sum_{p=1}^{P} \xi_p^2 \frac{E[\hat{A}_{k,p} A_{k,p}^*]}{E[|A_{k,p}|^2]} = \sum_{p=1}^{P} \xi_p^2 \rho_p, \quad (4)$$

with $\rho_p = E[\hat{A}_{k,p} A_{k,p}^*]/E[|A_{k,p}|^2]$.

On the first iteration we do not have any information about S_k and the correlation coefficient is zero. This means that the receiver reduces to a linear FDE optimized under the MMSE criterion [3]. After the first iteration, and if the residual BER is not too high (at least for the spreading codes to which a higher transmit power is associated), we have $\hat{A}_{k,p} = A_{k,p}$ for most of the data symbols, leading to $\hat{S}_k \approx S_k$. Consequently, we can use the feedback coefficients to eliminate a significant part of the residual interference.

3.2 Low-PMEPR MC-CDMA

It was shown in [9] that we can improve significantly the performance of OFDM schemes submitted to nonlinear devices by employing a receiver with iterative cancelation of nonlinear distortion effects. This concept can be extended to MC-CDMA, leading to the receiver structure of fig. 3. The basic idea behind this receiver is to use an estimate of the nonlinear self-distortion $\{\hat{D}_k; k = 0, 1, \ldots, N - 1\}$ provided by the preceding iteration to remove the nonlinear distortion effects in the received samples. Therefore, the received frequency-domain block $\{Y_k; k = 0, 1, \ldots, N - 1\}$ is replaced by the corrected block $\{Y_k^{Corr}; k = 0, 1, \ldots, N - 1\}$, where $Y_k^{Corr} = \frac{1}{\alpha}\left(Y_k - H_k \hat{D}_k\right)$. The remaining of the receiver is similar, but with β given by

$$\beta = \frac{E[|N_k|^2]}{E[|\alpha S_k|^2] + E[|D_k|^2]}. \qquad (5)$$

For the first iteration, the expectations of (5) can easily be obtained by using the analytical approach of [7, 8]; for the remaining iterations they have to be obtained by simulation. However, in most cases of interest we cancel a significant part of the nonlinear self-distortion at the first iteration; therefore, after the first iteration, we can ignore $E[|D_k|^2]$ in (5).

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[3]It should be noted that, contrarily to [4], we are considering a normalized feedforward filter.

For a given iteration, $\{\hat{D}_k; k = 0, 1, \ldots, N - 1\}$ can be estimated from the blocks $\{\hat{A}_{k,p}; k = 0, 1, \ldots, M - 1\}$ ($p = 1, 2, \ldots, P$) as follows: each block $\{\hat{A}_{k,p}; k = 0, 1, \ldots, M - 1\}$ is re-spread to generate the "average chip block" $\{\hat{S}_{k,p}; k = 0, 1, \ldots, N - 1\}$; the "average transmitted block" $\{\hat{S}_k; k = 0, 1, \ldots, N - 1\}$ is formed, with $\hat{S}_k = \sum_{p=1}^{P} \xi_p \hat{S}_{k,p}$; $\{\hat{S}_k; k = 0, 1, \ldots, N - 1\}$ is submitted to a replica of the nonlinear signal processing scheme employed in the transmitter so as to form the "average transmitted block" $\{\hat{S}_k^{Tx}; k = 0, 1, \ldots, N - 1\}$; $\hat{D}_k$ is given by $\hat{D}_k = \hat{S}_k^{Tx} - \alpha \hat{S}_k$ (naturally, for the first iteration, $\hat{D}_k = 0$).

4 Turbo Equalizer for MC-CDMA

Clearly, (2) could be written as $\tilde{S}_k = F_k Y_k - B_k' \overline{S}_k$, with $\rho B_k' = B_k$ and $\overline{S}_k$ denoting the average of the block of overall time-domain chips associated to a given iteration, given by $\overline{S}_k = \rho \hat{S}_k$, and ρ can be regarded as the blockwise reliability of the estimates $\{\hat{S}_k; k = 0, 1, \ldots, N - 1\}$.

To improve the performances, we could replace the "blockwise averages" by "symbol averages", which can be done as described in the following. Let us assume that the transmitted symbols are selected from a QPSK constellation under a Gray mapping rule (the generalization to other cases is straightforward). We will define $A_{k,p} = \pm 1 \pm j = A_{k,p}^I + j A_{k,p}^Q$, with $A_{k,p}^I = \text{Re}\{A_{k,p}\}$ and $A_{k,p}^Q = \text{Im}\{A_{k,p}\}$, $k = 0, 1, \ldots, M - 1$; $p = 1, 2, \ldots, P$, (similar definitions can be made for $\tilde{A}_{k,p}$, $\hat{A}_{k,p}$ and $\overline{A}_{k,p}$).

The LLRs (LogLikelihood Ratios) of the "in-phase bit" and the "quadrature bit", associated to $A_{k,p}^I$ and $A_{k,p}^Q$, respectively, are given by $L_{k,p}^I = 2\tilde{A}_{k,p}^I/\sigma_p^2$ and $L_{k,p}^Q = 2\tilde{A}_{k,p}^Q/\sigma_p^2$, respectively, where $\sigma_p^2 = \frac{1}{2} E[|A_{k,p} - \tilde{A}_{k,p}|^2] \approx \frac{1}{2M} \sum_{k=0}^{M-1} E[|\hat{A}_{k,p} - \tilde{A}_{k,p}|^2]$.
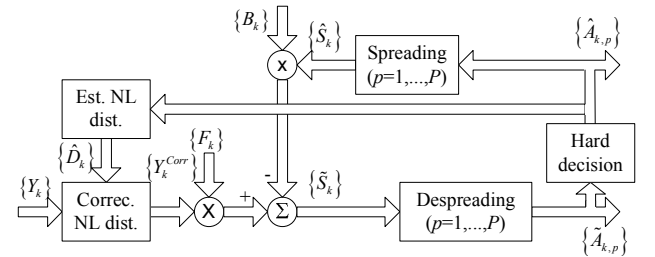

Figure 3. IB-DFE receivers with cancelation of nonlinear distortion effects.

Under a Gaussian assumption, it can be shown that the mean value of $A_{k,p}$, $\overline{A}_{k,p} = \rho_{k,p}^I \hat{A}_{k,p}^I + j \rho_{k,p}^Q \hat{A}_{k,p}^Q$, is

$$\overline{A}_{k,p} = \tanh\left(\frac{L_{k,p}^I}{2}\right) + j \tanh\left(\frac{L_{k,p}^Q}{2}\right), \qquad (6)$$

where the hard decisions $\hat{A}_{k,p}^I = \pm 1$ and $\hat{A}_{k,p}^Q = \pm 1$ are defined according to the signs of $L_{k,p}^I$ and $L_{k,p}^Q$, re-

spectively and $\rho_{k,p}^I$ and $\rho_{k,p}^Q$ can be regarded as the reliabilities associated to the "in-phase" and "quadrature" bits of the kth symbol of the pth spreading code, given by $\rho_{k,p}^I = E[A_{k,p}^I \hat{A}_{k,p}^I]/E[|A_{k,p}^I|^2] = \tanh\left(|L_{k,p}^I|/2\right)$ and $\rho_{k,p}^Q = E[A_{k,p}^Q \hat{A}_{k,p}^Q]/E[|A_{k,p}^Q|^2] = \tanh\left(|L_{k,p}^Q|/2\right)$ (for the first iteration, $\rho_{k,p}^I = \rho_{k,p}^Q = 0$ and $\overline{A}_{k,p} = 0$).

The feedforward coefficients are still obtained from (3), with the blockwise reliability given by (4) and ρ_p given by

$$\rho_p = \frac{1}{M}\sum_{k=0}^{M-1}\frac{E[A_{k,p}^* \hat{A}_{k,p}]}{E[|A_{k,p}|^2]} = \frac{1}{2M}\sum_{k=0}^{M-1}(\rho_{k,p}^I + \rho_{k,p}^Q). \quad (7)$$

The "overall chip averages" are then given by $\overline{S}_k = \sum_{p=1}^P \xi_p C_{k,p} \overline{A}_{k \bmod M,p}$.

The soft decisions based on the "symbol averages", $\overline{A}_{k,p}$, can also be used in the estimation of the "average nonlinear distortion" components, $\overline{D}_k$. The $\overline{D}_k$ are obtained as $\hat{D}_k$, as described in subsec. 3.2, but with $\overline{A}_{k,p}$ replacing $\hat{A}_{k,p}$.

Therefore, the receiver with "blockwise reliabilities", denoted in the following as IB-DFE-HD (IB-DFE with Hard Decisions), and the receiver with "symbol reliabilities", denoted in the following as IB-DFE-SD (IB-DFE with Soft Decisions), employ the same feedforward coefficients; however, in the first the feedback loop uses the "hard-decisions" on each data block, weighted by a common reliability factor, while in the second the reliability factor changes from symbol to symbol (in fact, the reliability factor is different in the real and imaginary component of each symbol).

We can also define turbo receivers that employ the channel decoder outputs instead of the uncoded "soft decisions" in the feedback loop. The receiver structure, that will be denoted as Turbo FDE, is similar to the IB-DFE-SD, but with a SISO channel decoder (Soft-In, Soft-Out) employed in the feedback loop. The SISO block, that can be implemented as defined in [11], provides the LLRs of both the "information bits" and the "coded bits". The input of the SISO block are LLRs of the "coded bits" at the deinterleaver output[4]. Once again, the feedforward coefficients are obtained from (3), with the blockwise reliability given by (7).

5   Performance Results

In this section, we present a set of performance results concerning the proposed receiver structure. We consider MC-CDMA schemes with $N = 256$ subcarriers and QPSK constellations. The channel is characterized by the power delay profile type C for the HIPERLAN/2 (HIgh PERformance Local Area Network) [12], with uncorrelated Rayleigh fading for the different paths (similar results were obtained for

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[4]As usual, it is assumed that the bits at the channel encoder output are interleaved before being mapped into the adopted constellation.

other severely time-dispersive channels). The duration of the useful part of the block is 4μs and the CP has duration 1.25μs. We consider perfect synchronization and channel estimation conditions. We use $P = K$ spreading codes, corresponding to a fully loaded scenario and $\xi_p = 1$ for all spreading codes (our simulations showed that the iterative receivers considered here are still suitable for scenarios where different powers are assigned to different spreading codes). We consider the PMEPR-reducing technique of fig. 1, with the oversampling factor $N'/N = 2$, the normalized clipping level $s_M/\sigma = 1$ and $G_k = 1$ for the N in-band subcarriers and 0 for the remaining subcarriers. With this technique, the PMEPR of the transmitted signals is 4.1dB (for conventional MC-CDMA signals PMEPR≈ 8.4dB [8]).

In figs. 4 and 5 we present the uncoded BER performances (averaged over all spreading codes) of IB-DFE receivers, for $K = P = 16$ and $K = P = 256$, respectively (naturally, the first iteration corresponds to a linear receiver). Both hard decisions (IB-DFE-HD) and soft decisions (IB-DFE-SD) are considered. From these figures it is clear that the use of soft decisions allows a slight improvement of the performances relatively to hard decisions, that is higher for $K = 16$. As we increase the number of iterations we improve significantly the performances when $K = 256$; for $K = 16$ the performances are significantly worse. The iterative receivers are especially interesting in the presence of nonlinear distortion effects, although there is still a significant degradation relatively to the case with linear transmitters, especially for $K = 256$.
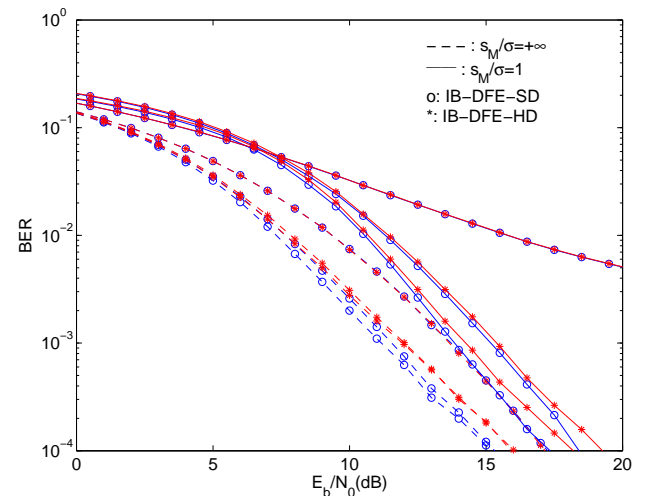

Figure 4.   Uncoded BER performances for IB-DFE receivers with 1, 2 and 3 iterations (improved performances as we increase the iteration order), when $K = 16$.

Let us consider now the impact of channel coding. We adopted a 64-state, rate-1/2 convolutional code with generators $1 + D^2 + D^3 + D^5 + D^6$ and $1 + D + D^2 + D^3 + D^6$. For IB-DFE schemes, a soft-decision Viterbi decoder was employed; for turbo schemes, we adopted a SISO decoder implemented using the "Max-log-MAP approach" [11]. Figs. 6 and 7 concern the cases with $K = P = 16$ and
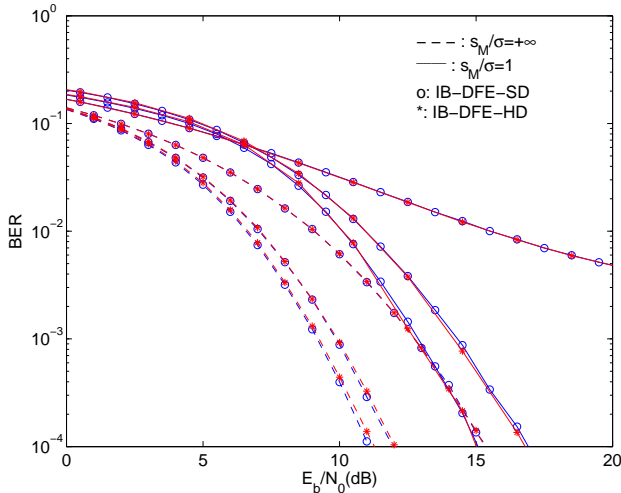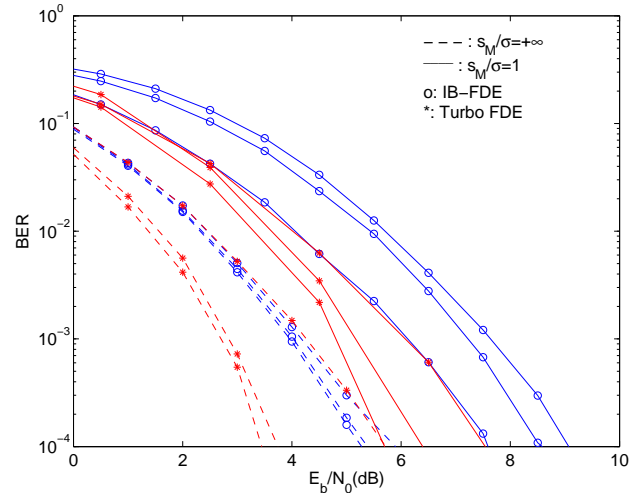
Figure 5. As in fig. 4, but for $K = 256$.

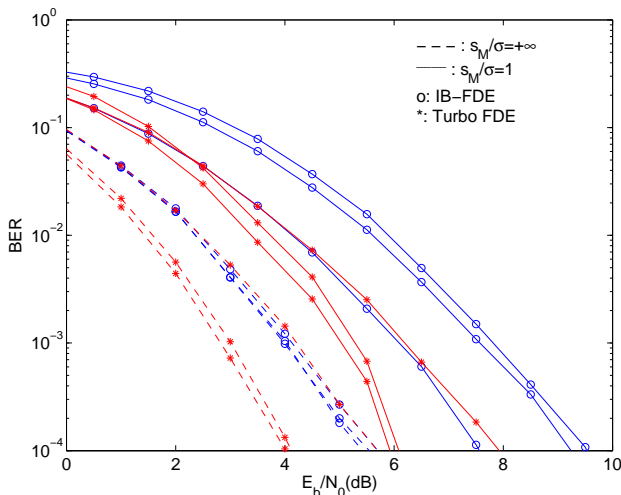Figure 7. As in fig. 6, but for $K = 256$.

Figure 6. BER performances for IB-DFE-SD and Turbo FDE receivers with 1, 2 and 3 iterations (improved performances as we increase the iteration order), when $K = 16$.

$K = P = 256$, respectively. Clearly, the use of Turbo FDE receivers leads to significant performance improvements relatively to the IB-DFE receivers, which increase as we increase the number of iterations. Contrarily to the uncoded case, the performances with $K = 16$ and $K = 256$ are identical. The degradation due to nonlinear distortion effects is about 2dB after 3 iterations.

6   Conclusions

In this paper, we considered low-PMEPR MC-CDMA schemes, with appropriate channel coding. We developed frequency-domain turbo equalizers combined with an iterative estimation and cancellation of nonlinear distortion effects. Our receivers have a relatively low complexity, since they allow FFT-based implementations. Moreover, the proposed turbo receivers allow significant performance improvements at low and moderate SNR, even when a low-PMEPR MC-CDMA transmission is intended.

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