EFFICIENT DETECTION OF ZERO-PADDED OFDM SIGNALS WITH LARGE BLOCKS

Teresa Araújo and Rui Dinis
ISR - Instituto Superior Técnico
Lisboa, Portugal
email: rdinis@ist.utl.pt

ABSTRACT

In this paper, we present a frequency-domain receiver for ZP-OFDM schemes (Zero-Padded Orthogonal Frequency Division Multiplexing) where the duration of the channel impulse response is a significant fraction of the duration of the OFDM block. The proposed receiver has a relatively low complexity, allowing an FFT-based (Fast Fourier Transform) implementation. The proposed receiver is suitable to ZP-OFDM systems with very large blocks (hundreds or even thousands of subcarriers), since it does not requires the inversion or the multiplication of large matrixes.[1]

KEY WORDS

OFDM, Block Transmission, Zero-Padded.

1 Introduction

Recently, there is a great interest in multicarrier schemes, namely the OFDM modulation (Orthogonal Frequency Division Multiplexing) [1] and the DMT schemes (Discrete MultiTone) [2], since they are suitable for high rate transmission over severely time-dispersive channels. For this reason, OFDM schemes have been selected for digital audio and video broadcast systems and to wireless local area networks [3]; DMT schemes have been selected for DSL standards (Digital Subscriber Line) [2]. Presently, several OFDM modulations are also being considered for cellular systems [4] and for UWB (UltraWide Band) systems [5].

Within these block transmission techniques, a CP (Cyclic Prefix), longer than the maximum overall channel impulse response, is appended to each block to make the linear convolution associated to the transmission channel equivalent to a cyclic convolution. This means that, if we do not consider the CP for detection proposes, the different subcarriers remain orthogonal after a transmission over a time-dispersive channel (i.e., there is no interference between different subcarriers), allowing low-complexity, FFT-based (Fast Fourier Transform) receiver implementations.

However, several drawbacks are usually associated to OFDM schemes. Due to the multicarrier nature of the transmitted signals, they have high envelope fluctuations and a large PMEPR (Peak-to-Mean Envelope Power Ratio), which lead to amplification difficulties. This problem can be minimized by employing suitable PMEPR-reducing signal processing techniques (see [6] and references therein).

Another problem associated to OFDM schemes results from the fact that, since a frequency selective channel behaves as a flat fading channel at the subcarrier level, the subcarriers that are at a deep frequency notch will have a poor performance, compromising the overall performance. However, the performance improves substantially if the multicarrier schemes are combined with suitable channel coding schemes [7].

Finally, the CP leads to a decrease of the spectral efficiency of the modulation, since that interval is not effectively used for data transmission. It also leads to a decrease in the power efficency of the modulation, due to the power spent on the CP. For this reason, the length of the CP should be a small fraction of the length of the OFDM block.

ZP-OFDM schemes (Zero-Padded) have recently been proposed as an alternative to conventional CP-assisted OFDM schemes [8]. Contrarily to conventional OFDM, ZP-OFDM can have good uncoded performances, even in the presence of deep frequency notches in the inband region. However, to achieve this we need to employ complex receiver structures, involving the inversion and/or the multiplication of matrixes whose dimensions grow with the block length [9]. Therefore, these techniques are not suitable to ZP-OFDM schemes with large blocks. By using overlap-and-add techniques, the receiver complexity is similar to the one of conventional CP-assisted OFDM schemes, but the performances are also identical [9].

In this paper, we consider the use of ZP-OFDM in systems where the duration of the channel impulse response is not a small fraction of the duration of the OFDM block (e.g., for OFDM-based UWB systems or for the DSL), and we present a suitable frequency-domain receiver. The proposed receiver has good performance and relatively low complexity, allowing an FFT-based implementation. The proposed receiver is suitable to ZP-OFDM systems with very large blocks (hundreds or even thousands of subcarriers), since it does not require the inversion or the multiplication of large matrixes.

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2 Conventional OFDM vs ZP-OFDM

Let us consider a conventional, CP-assisted OFDM scheme. The transmitted signal associated to the mth data block is given by

$$s_m(t) = \sum_{n=-N_G}^{N-1} s_{n,m} h_T(t - nT_S) \qquad (1)$$

with T_S denoting the sampling interval, N_G denoting the number of samples at the cyclic prefix and $h_T(t)$ is the impulse response of the reconstruction filter. The block of time-domain samples $\{s_{n,m}; n = 0, 1, \ldots, N-1\}$ is the IDFT of the data block to be transmitted $\{S_{k,m}; k = 0, 1, \ldots, N-1\}$, where $S_{k,m}$ is the data symbol associated to the kth subcarrier, selected from a given constellation under an appropriate mapping rule[2]. It is assumed that the time-domain block is periodic, with period N, i.e., $s_{-n,m} = s_{N-n,m}$. The first N_G samples can be regarded as a CP for the OFDM block.

If we discard the samples associated to the CP at the receiver then there is no interference between OFDM blocks, provided that the length of the CP is higher than the length of the overall channel impulse response. Moreover, the linear convolution associated to the channel can be regarded as a cyclic convolution relatively to the N-length, useful part of the received block, $\{y_n; n = 0, 1, \ldots, N-1\}$ (for the sake of notation simplicity, we will ignore the dependence with the block index m in the following). This means that the corresponding frequency-domain block (i.e., the length-N DFT (Discrete Fourier Transform) of the block $\{y_n; n = 0, 1, \ldots, N-1\}$) is $\{Y_k; k = 0, 1, \ldots, N-1\}$, where

$$Y_k = S_k H_k + N_k \qquad (2)$$

with H_k denoting the channel frequency response for the kth subcarrier and N_k the corresponding channel noise.

Clearly, the impact of a time-dispersive channel reduces to a scaling factor for each subcarrier. Therefore, the receiver just has to invert the channel frequency response for each subcarrier. If $E[|H_k|^2] = 1$ the BER (Bit Error Rate) when a QPSK constellation (Quaternary Phase Shift Keying) with Gray mapping is assumed at each subcarrier is given by

$$P_b = \frac{1}{N} E\left[\sum_{k=0}^{N-1} Q\left(\sqrt{2\eta_G \frac{E_b}{N_0}|H_k|^2}\right)\right] \qquad (3)$$

where E_b is the average bit energy and N_0 is the one-sided PSD (Power Spectral Density) of the channel noise and the expectation is over the channel realizations. In (3), $\eta_G = N/(N+N_G)$, accounts for the performance degradation due to the power spent on the CP. Clearly the BER can be very poor for the subcarriers at a deep frequency notch (i.e., when $|H_k| << 1$). If $|H_k|$ has a Rayleigh distribution (a reasonable assumption for severely time-dispersive channels [10]), the average BER is given by

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\eta_G E_b/N_0}{1 + \eta_G E_b/N_0}}\right), \qquad (4)$$

(see [11]).

From (2), the OFDM transmission over a time-dispersive fading channel can be regarded as the dual version of a conventional, single-carrier transmission over a time-varying, flat fading channel. Therefore, by using cannel coding and interleaving schemes we can obtain significant coding gains (naturally, for the coded case, the direct inversion of the channel effects is not required since it is implicitly performed by the channel decoder). Moreover, for severely time-dispersive channels (i.e., highly frequency-selective channels) an intrablock interleaving is enough.

Clearly, there is a degradation η_G due to the power spent on the CP. Whenever the duration of the CP is a significant fraction of the block duration this degradation is not negligible. To avoid this degradation we will replace the CP by N_G zero-valued samples, i.e., we will consider a ZP-OFDM scheme [8].

Unfortunately, the linear convolution associated to the channel is no longer equivalent to a cyclic convolution and (2) does not hold for ZP-OFDM. Therefore, the low-complexity receiver employed with conventional, CP-assisted OFDM can not be employed. For this reason, a reduced complexity receiver was proposed for ZP-OFDM where an overlap-and-add technique is employed [9], where the received samples at the "zero-padded region" are added to the beginning of the useful part of the received block. It can be shown that the signal component generated this way is equivalent to the signal component of the corresponding conventional OFDM scheme. Therefore, the rest of the receiver could be similar to a conventional OFDM receiver.

However, if the noise variance of each sample is σ_n^2, the noise variance for the N_G samples is $2\sigma_n^2$ due to the "overlap-and-add" procedure. This means that the "average" noise variance[3] is $\sigma_{n,eq}^2 = \sigma_n^2(N+N_G)/N = \sigma_n^2/\eta_G$, i.e., although no power is spent with the CP, the performance of ZP-OFDM with and "overlap-and-add" receiver is similar to the performance of conventional OFDM[4].

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[2] Due to implementation reasons, usually $S_{k,m} = 0$ for the subcarriers at the edge of the band.

[3] It should be noted that the variance of the noise component at the subcarrier level is proportional to the "average" noise variance in the time domain.

[4] To be exact, the noise components at the subcarrier level are no longer uncorrelated. However, this correlation has no impact in the uncoded performance and a very small impact in the coded performance due to the interleaving after the channel encoder.

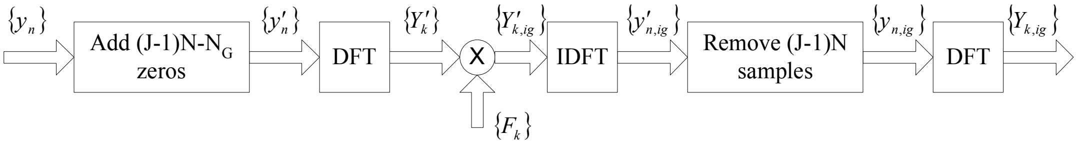

Figure 1. Frequency-domain receiver for ZP-OFDM.

3 Frequency-Domain Detection of ZP-OFDM Signals

Fig. 1 shows the receiver structure that we are considering for ZP-OFDM schemes. Within this receiver, an augmented block $\{y'_n; n = 0, 1, \ldots, JN-1\}$ is generated by adding $JN - N_G - N$ zeros to the received time-domain block $\{y_n; n = 0, 1, \ldots, N + N_G - 1\}$ that includes the N_G samples associated to the "zero-padded region" immediately after the "OFDM block".

The DFT of the augmented time-domain block is computed, leading to the frequency-domain block $\{Y'_k; k = 0, 1, \ldots, JN-1\}$. If the duration of the "zero-padded region" is longer than the duration of the overall channel impulse response, the N_G zeros before the "OFDM block" can be regarded as a CP for the "augmented ZP-OFDM block" that includes the subsequent N_G zeros. Therefore, a relation similar to (2) could be derived for the augmented frequency-domain block: $Y'_k = S'_k H'_k + N'_k$, with H'_k and N'_k replacing H_k and N_k of (2), respectively. The equivalent transmitted frequency-domain block $\{S'_k; k = 0, 1, \ldots, JN-1\}$ is the DFT of the block $\{s'_n = s_n r_n; n = 0, 1, \ldots, JN-1\}$, where $r_n = 1$ for $0 \le n \le N-1$ and 0 otherwise (once again, it is assumed that the samples s_n are periodic with period N). Since the DFT of the block $\{s_n; n = 0, 1, \ldots, JN-1\}$ is the block $\{S_k^{(J)}; k = 0, 1, \ldots, JN-1\}$, with

$$S_k^{(J)} = \begin{cases} JS_{k/J}, & \text{k/J integer} \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

then

$$S'_k = \sum_{k''=0}^{N-1} JS_{k''} R_{k-Jk''}, \tag{6}$$

with $\{R_k; k = 0, 1, \ldots, JN-1\}$ = DFT $\{r_n; n = 0, 1, \ldots, JN-1\}$.

Let us assume that $S_k = 0$ for $k \neq k_0$ (i.e., there is only one active subcarrier). In that case, $S'_k = JS_{k_0} R_{k-Jk_0}$ and $Y'_k = H'_k JS_{k_0} R_{k-Jk_0} + N'_k$. Therefore, that symbol is not restricted to a single subcarrier but spreaded over the entire transmission bandwidth. This has two main consequences: on the one hand, we are able to transmit even for the subcarriers that are in deep frequency notches, since, due to the inherent diversity effect, the equivalent channel associated to the kth data symbol is

$$|H_k^{eq}|^2 = \sum_{k'=0}^{JN-1} |H'_{k'} J R_{k'-Jk}|^2; \tag{7}$$

on the other hand, the intercarrier interference inherent to (6) might lead to significant performance degradation.

To cope with this intercarrier interference, the augmented block is submitted to a linear FDE (Frequency-Domain Equalization), leading to the frequency-domain block $\{Y'_{k,ig} = Y'_k F_k; k = 0, 1, \ldots, JN-1\}$, where F_k denotes the kth FDE coefficient. To avoid noise enhancement, these coefficients are optimized under the MMSE criterion (Minimum Mean Squared Error), i.e., $F_k = H_k^{'*}/(\alpha_k + |H'_k|^2)$, with $\alpha_k = E[|H'_k|^2]/E[|S'_k|^2]$.

The signal component associated to the corresponding time-domain block $\{y'_{n,ig}; n = 0, 1, \ldots, JN-1\}$, is almost restricted to the first N samples (it would be exactly restricted to these N samples if the FDE coefficients were optimized under the ZF criterion (Zero Forcing)), and the remaining $(J-1)N$ samples are composed almost entirely of noise. For this reason, the last $(J-1)N$ are removed, leading to the final time-domain block at the FDE output $\{y_{n,ig} = y'_{n,ig}; n = 0, 1, \ldots, N-1\}$. The detection is based on the corresponding frequency-domain block $\{Y_{k,ig}; k = 0, 1, \ldots, N-1\}$ = DFT $\{y_{n,ig}; n = 0, 1, \ldots, N-1\}$.

4 Performance Results

In this section, we present a set of performance results concerning the proposed receiver frequency-domain receiver structure for ZP-OFDM schemes. For the sake of comparisons, we also considered the corresponding conventional, CP-assisted OFDM schemes, which have the same performance of a ZP-OFDM scheme employing an overlap-and-add receiver. The OFDM blocks have $N = 256$ data symbols, selected from a QPSK constellation under a Gray mapping rule. We consider severely time-dispersive channels with rich multipath propagation. The duration of the overall channel impulse response can take three values: $\tau_{\max} = 0.25T$, $\tau_{\max} = 0.5T$ and $\tau_{\max} = T$, with $T = NT_S$ denoting the duration of the useful part of the OFDM block. The duration of the "zero-padded region" (or the duration of the CP, in the case of conventional OFDM schemes), is supposed to be $\tau_{\max}$. It should be noted that the duration of the overall channel impulse response is typically $\tau_{\max} \le 0.25T$ for conventional OFDM schemes. Therefore, the second and third cases correspond to scenarios where the channel impulse response is very long. We consider linear power amplification and perfect synchronization and channel estimation conditions.

Fig. 2 shows the uncoded BER performances for both ZP-OFDM schemes and conventional CP-assisted OFDM

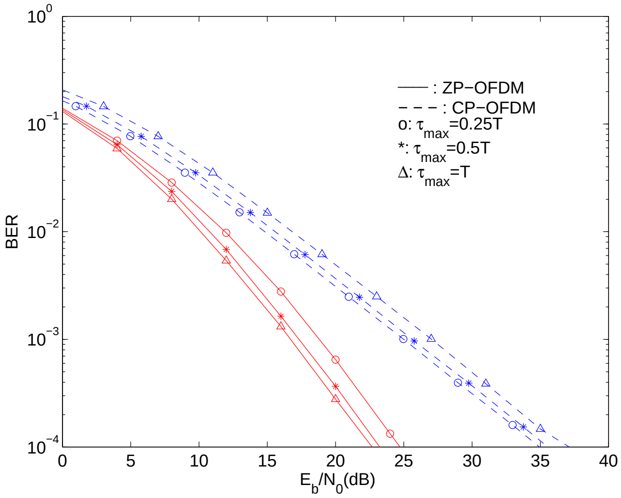

Figure 2. Uncoded BER performance.

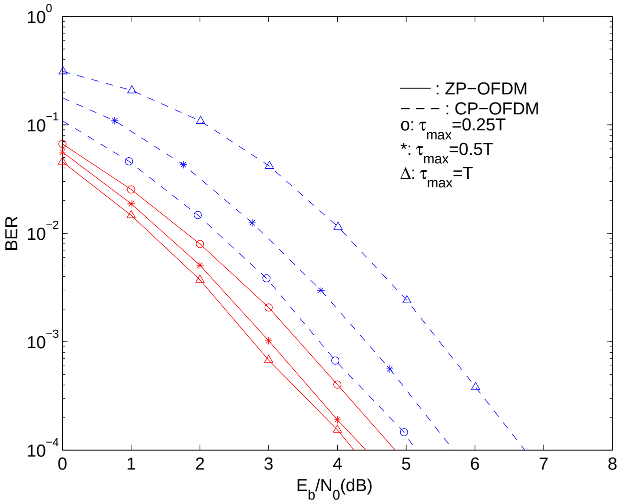

Figure 3. BER performance after the channel decoder.

schemes. Since the duration of the useful part of the OFDM is the same regardless of the channel impulse response duration, the BER performance of conventional OFDM schemes decreases with $\tau_{\max}$ (these results were expectable, due to the increased power spent on the CP). However, for ZP-OFDM schemes, the BER performances improve slightly with $\tau_{\max}$. This can be explained as follows: a larger value of $\tau_{\max}$ means a more time-dispersive channel, and, consequently, a higher diversity effect inherent to the ZP-OFDM signals (see (7)). From this figure it is also clear that ZP-OFDM has always better performance than conventional OFDM. Since the BER with ZP-OFDM schemes decreases at a higher rate than with conventional OFDM, the difference between ZP-OFDM and conventional OFDM is higher for lower BERs.

Let us consider now the impact of the channel coding. We consider the standard rate-1/2, 64-state convolutional code. The coded bits are interleaved, before being mapped on the different subcarriers (an intraburst interleaving is assumed). Fig. 3 shows the BER performances at the decoder output. Although the difference between ZP-OFDM and conventional OFDM decreases, the ZP-OFDM has always better performance, especially for larger $\tau_{\max}$.

5 Conclusions

In this paper, we considered a low-complexity, FFT-based receiver for ZP-OFDM schemes with large blocks. Our performance results showed that the proposed receiver has good performance, making ZP-OFDM a good alternative to conventional OFDM schemes, especially for channels with a very long impulse response.

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