A Class of Iterative FDE Techniques for Reduced-CP SC-Based Block Transmission

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Abstract - For conventional CP-assisted (Cyclic Prefix) block transmission systems, the CP length is selected on the basis of the expected maximum delay spread. With regard to SC-based (Single Carrier) block transmission implementations, a full-length CP is recommendable, since it allows good performances through the use of simple FDE (Frequency-Domain Equalization) techniques.

Recently, an algorithm for a "Decision-Directed Correction" (DDC) of the FDE inputs, under reduced-CP conditions, was shown to provide good DDC-FDE performances, without significant error propagation, when using specially designed SC-based frame structures.

In this paper, a low-complexity, SDDC-aided (Soft Decision Directed Correction), iterative FDE technique is proposed for reduced-CP, SC-based block transmission systems using conventional frame structures. A more sophisticated, SDDC-aided, Turbo FDE technique is also proposed to provide improved performances in such systems, through a moderately increased complexity. The relations with already known Turbo FDE and related iterative techniques, designed to operate under full-length CP, are established. A set of performance results concerning the proposed SDDC-aided techniques, with several complexity levels, as well as conventional iterative FDE techniques, is reported and discussed. The advantages of the SDDC-aided approach are emphasized, including the fact that it can operate in a satisfactory way even when, for the sake of implementation simplicity, no decoding operations are used to improve the iterative FDE process.

I. Introduction

Conventional SC (Single Carrier) modulations have been shown to be suitable for CP-assisted (Cyclic Prefix) block transmission within broadband wireless communication systems, similarly to the usually proposed OFDM [1]. With a CP long enough to cope with the maximum relative channel delay, a low-complexity FDE (Frequency-Domain Equalization) technique, involving simple FFT (Fast Fourier Transform) computations, can be employed to solve the severe ISI problem: this is due to the fact that, under full-length CP conditions, any interblock interference (IBI) is avoided, in what concerns the useful part of each received block (the CP-related part is discarded); moreover, the linear convolutions which are inherent to the time-dispersive channels, become equivalent to circular convolutions, corresponding to frequency-domain multiplications. In conventionally designed CP-assisted block transmission systems, after selecting a full-length CP according to the channel memory order, the data block size is selected to be small enough to ensure a negligible channel variation over the block, but large enough to avoid a significant degradation of both bandwidth and power efficiencies.

In recent years, the possibility of achieving improved FDE performances in SC-based systems, under full-length CP conditions, was considered by several authors. One approach, as presented in [2] and [3], is "turbo equalization in the frequency domain" (Turbo FDE), where the linear FDE procedures and the decoding procedures (assuming a coded data transmission) are jointly performed, in an iterative way. Another FDE approach, with lower complexity, is the so-called "Iterative Block Decision Feedback Equalization" (IB-DFE), which does not use decoding within the iterative process: this approach, introduced in [4], was later extended and shown to be easily compatible with space diversity and MIMO systems [5], [6], as well as selected CP-assisted "OFDMA-Type" and CDMA multiple access schemes [7], [8], [9]. Both FDE approaches can take advantage of state-of-the-art, low-cost, FFT-based technology for practical implementations.

Since the use of a full-length CP reduces the block transmission efficiency, the possibility of adopting a reduced CP (below the channel memory order), while keeping an essentially FFT-based implementation, deserves to be considered. In [10], [11], we presented a basic algorithm for a "decision-directed correction" (DDC) of the FDE inputs under reduced CP conditions. This algorithm was shown to provide good DDC-FDE performances, without significant error propagation, when using specially designed SC-based frame structures.

In this paper, in the reduced-CP context, we consider the use of a "soft-decision version" of the DDC algorithm, in an iterative way, as an aid to either the conventional FDE/MMSE (Minimum Mean Squared Error) technique (Sec. II) [1] or the improved, "turbo-type" FDE techniques reported above (Sec. III). Sec. IV provides numerical performance results, and Sec. V (conclusions) completes the paper.

II. Low-Complexity Iterative SDDC-FDE Technique

A. Basic DDC-FDE Scheme

For a length-L CIR (Channel Impulse Response), let us consider the transmission of length-N SC-based data blocks $\mathbf{s}(m) = [s_0(m), s_1(m), \ldots, s_{N-1}(m)]^T$ ($s_n(m)$ symbol coefficients taken from, e.g., a Quaternary Phase Shift Keying alphabet), with $N > L$. Whenever a length-L CP is appended to each data block, the length-N mth useful received block can be represented by

$$\mathbf{y}_{CP}(m) = \mathbf{H}\mathbf{s}(m) + \mathbf{n}(m), \quad (1)$$

where $\mathbf{n}(m) = [n_0(m), n_1(m), \ldots, n_{N-1}(m)]^T$ is the mth received noise vector and $\mathbf{H}$ is the $N \times N$ circulant matrix which describes the channel effects. The entries of this square matrix, given by $h_{j,k} = h_{(j-k)\ mod\ N}$, are related to the length-L CIR ($h_n = 0$ for $n = L+1, \ldots, N-1$).

Let us consider the transmission of length-N SC-based blocks with a length-L_R, shortened CP: $0 \leq L_R < L, N + L_R \geq 2L$. Therefore, the initial portion, with $\Delta L = L - L_R$ samples, of each received block will differ from the corresponding initial portion under full-length CP, unless

$$\Delta s_p(m) = s_p(m) - s_{p+L_R}(m-1) \ \ (p = N-L, \ldots, N-L_R-1) \quad (2)$$

is equal to zero. The insufficient CP leads to some IBI and also to an imperfect circular convolution regarding the channel impact on the data block contents.

Obviously, when using $\mathbf{y}(m)$ to denote the new length-N received block, $\mathbf{y}_{CP}(m) - \mathbf{y}(m)$ will depend on $\Delta s_p(m), p = N-L, \ldots, N-L_R-1$. It can be shown that

$$\mathbf{y}_{CP}(m) - \mathbf{y}(m) = \mathbf{I}'_{\Delta L}\mathbf{H}\boldsymbol{\Delta}(m), \quad (3)$$

with $\mathbf{I}'_{\Delta L}$ and $\boldsymbol{\Delta}(m)$ as follows:

$$\mathbf{I}'_{\Delta L} = \mathbf{diag}[\underbrace{1, 1, \ldots, 1}_{\Delta L}, \underbrace{0, \ldots, 0}_{N-\Delta L}]; \quad (4)$$

$$\boldsymbol{\Delta}(m) = [\underbrace{0, \ldots, 0}_{N-L}, \underbrace{\Delta s_{N-L}(m), \ldots, \Delta s_{N-L_R-1}(m)}_{\Delta L}, \underbrace{0, \ldots, 0}_{L_R}]^T. \quad (5)$$

If a perfect a priori knowledge of the ΔL pairs $(s_p(m), s_{p+L_R}(m-1)), p = N-L, \ldots, N-L_R-1$, could be assumed, it should be possible, having in mind (3), to "correct" the received vector $\mathbf{y}(m)$, by replacing it by the appropriate vector $\mathbf{y}_{CP}(m)$ prior to frequency-domain equalization. When an estimate of those ΔL pairs (i.e., an estimate $\widehat{\boldsymbol{\Delta}}(m)$ of $\boldsymbol{\Delta}(m)$) is available, a Decision-Directed Correction (DDC) of $\mathbf{y}(m)$ can be carried out to obtain a suitable approximation to $\mathbf{y}_{CP}(m)$:

$$\begin{aligned}\tilde{\mathbf{y}}_{CP}(m) &= \mathbf{y}(m) + \mathbf{I}'_{\Delta L}\mathbf{H}\widehat{\boldsymbol{\Delta}}(m) = \\ &= \mathbf{y}(m) + \mathbf{I}'_{\Delta L}\mathbf{F}^{-1}\mathbf{diag}[H_0, H_1, \ldots, H_{N-1}]\mathbf{F}\widehat{\boldsymbol{\Delta}}(m) \quad (6)\end{aligned}$$

where $\mathbf{F}$ and $\mathbf{F}^{-1}$ denote a DFT matrix and an IDFT matrix, respectively, and $[H_0, H_1, \ldots, H_{N-1}]^T$ is the CFR (Channel Frequency Response) vector, DFT of the CIR vector $[h_0, h_1, \ldots, h_{N-1}]^T$.

Therefore, a low-complexity algorithm for obtaining $\tilde{\mathbf{y}}_{CP}(m)$ from the received block $\mathbf{y}(m)$, through the use of the symbol estimates $\hat{s}_p(m)$ and $\hat{s}_{p+L_R}(m-1)$, $p = N-L, \ldots, N-L_R-1$, is as follows:

a) By defining $\widehat{\Delta s}_p(m) = \hat{s}_p(m) - \hat{s}_{p+L_R}(m-1)$, obtain

$$\widehat{\boldsymbol{\Delta}}(m) = [\underbrace{0, \ldots, 0}_{N-L}, \underbrace{\widehat{\Delta s}_{N-L}(m), \ldots, \widehat{\Delta s}_{N-L_R-1}(m)}_{\Delta L}, \underbrace{0, \ldots, 0}_{L_R}]^T \quad (7)$$

b) Compute the DFT of the vector estimate $\widehat{\boldsymbol{\Delta}}(m)$, so as to obtain a frequency-domain vector $[\hat{\Delta}_0(m), \hat{\Delta}_1(m), \ldots, \hat{\Delta}_{N-1}(m)]^T$.

c) Obtain the corresponding frequency-domain vector at the channel output, $[\hat{\Delta}_0(m)\hat{H}_0, \hat{\Delta}_1(m)\hat{H}_1, \ldots, \hat{\Delta}_{N-1}(m)\hat{H}_{N-1}]^T$, using a CFR estimate.

d) Compute the IDFT of the vector obtained in c).

e) Retain the initial ΔL components of the vector computed in d), and then add them to the initial ΔL components of $\mathbf{y}(m)$, so as to obtain the vector $\tilde{\mathbf{y}}_{CP}(m)$ which plays the role of $\mathbf{y}_{CP}(m)$ in the subsequent FDE procedures.

B. Iterative FDE Technique Using an SDDC Aid

In the following, we assume an SC-based block transmission, with length-N useful symbol blocks (corresponding to blocks of coded data) and a length-L_R cyclic prefix for every block. A continuous symbol stream is then transmitted, since the length-P ($P = N + L_R$) CP-assisted blocks are assumed to be contiguous, as shown in fig. 1. This is a conventional frame for block transmission; however, we will consider that the CP length (L_R) can be chosen to be smaller than the channel memory order (L), i.e., $L_R \leq L$.

In this context, a very simple iterative FDE technique using a "DDC aid" could easily be devised: for a given block m and a given FDE iteration $i \in \{1, 2, \ldots, I\}$, the length-$N$ time-domain input block $\tilde{\mathbf{y}}^{(i)}_{CP}(m)$ for FDE purposes could be obtained from the received vector $\mathbf{y}(m)$ simply by using the DDC algorithm described in sec. II.A, with

$$\hat{\Delta s}^{(i)}_p(m) = \hat{s}^{(i-1)}_p(m) - \hat{s}^{(I)}_{p+L_R}(m-1), \quad (8)$$

taking advantage of the appropriate ΔL current decisions on symbols of block m (iteration $i-1$) and the final decisions regarding the last ΔL symbols of block $m-1$ (iteration I). Obviously, for $i = 1$ no previous decision regarding block m are available; therefore, we should assume $\hat{s}^{(0)}_p(m) = 0$ when using (8) for $i = 1$. Certainly, there is a more efficient way of using the DDC idea: it consists of replacing the hard decisions by some kind of soft decisions, derived from the soft information that a SISO (Soft-In, Soft-Out) channel decoder can provide. An appropriate choice is a Soft Decision Directed Correction (SDDC) scheme, as an alternative to the DDC scheme described in Sec. II.A, where the $\hat{\boldsymbol{\Delta}}^{(i)}(m)$ input vector is replaced by a vector $\overline{\boldsymbol{\Delta}}^{(i)}(m)$ defined as follows:

$$\begin{aligned}&\overline{\boldsymbol{\Delta}}^{(i)}(m) = \\ &= [\underbrace{0, \ldots, 0}_{N-L}, \underbrace{\overline{\Delta s}^{(i)}_{N-L}(m), \ldots, \overline{\Delta s}^{(i)}_{N-L_R-1}(m)}_{\Delta L}, \underbrace{0, \ldots, 0}_{L_R}]^T, \quad (9)\end{aligned}$$

where

$$\overline{\Delta s}^{(i)}_p(m) = \overline{s}^{(i)}_p(m) - \overline{s}^{(I)}_{p+L_R}(m-1), \quad (10)$$

with $\overline{s}_p^{(i)}(m)$ and $\overline{s}_{p+L_R}^{(I)}(m-1)$ being mean symbol values (in the statistical sense) rather than hard decisions. In Appendix A, we indicate a way to compute these values, based on the LLRs of the coded bits, provided by the channel decoder, for a QPSK (Quaternary Phase Shift Keying) modulation. It should be noted that, when assuming $|s_n|^2 = \sigma_s^2$ in this case,

$$\overline{s}_p^{(i-1)}(m) = \frac{\sigma_s}{\sqrt{2}}\left(\tanh\left(\frac{L_{p,I}^{(i-1)}(m)}{2}\right) + j\tanh\left(\frac{L_{p,Q}^{(i-1)}(m)}{2}\right)\right) \tag{11}$$

where $L_{p,I}^{(i-1)}(m)$ and $L_{p,Q}^{(i-1)}(m)$ are the LLRs of the "in-phase coded bit" and "quadrature coded bit", respectively. Of course, the mean symbol value can be expressed as a function of the I/Q correlation coefficients $(\rho_{p,I}^{(i-1)}(m), \rho_{p,Q}^{(i-1)}(m))$ and the I/Q "decisions" $(\hat{s}_{p,I}^{(i-1)}(m), \hat{s}_{p,Q}^{(i-1)}(m))$:

$$\overline{s}_p^{(i-1)}(m) = \rho_{p,I}^{(i-1)}(m)\hat{s}_{p,I}^{(i-1)}(m) + j\rho_{p,Q}^{(i-1)}(m)\hat{s}_{p,Q}^{(i-1)}(m), \tag{12}$$

(and similarly for $\overline{s}_p^{(I)}(m-1)$), where $0 \leq \rho_{p,I}^{(i-1)}(m) \leq 1$ and $0 \leq \rho_{p,Q}^{(i-1)}(m) \leq 1$. For $i = 1$, $\rho_{p,I}^{(i-1)}(m) = \rho_{p,Q}^{(i-1)}(m) = 0$, hence $\overline{s}_p^{(i-1)}(m) = 0$; after some iterations and/or when the SNR is high, typically $\rho_{p,I}^{(i-1)}(m) \approx 1$ and $\rho_{p,Q}^{(i-1)}(m) \approx 1$, leading to $\overline{s}_p^{(i-1)}(m) \approx \hat{s}_p^{(i-1)}(m)$.

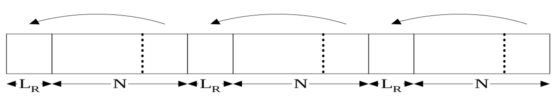

Fig. 1. Conventional frame structure for CP-assisted block transmission.

When the proposed SDDC scheme is employed as an aid to a conventional, linear FDE, in an iterative way, the appropriate receiver configuration is as depicted in Fig. 2. By assuming the MMSE criterion, the entries of $\mathbf{F}(m)$ can be expressed as

$$F_k(m) = K_F(m)\frac{\hat{H}_k^*}{\hat{\alpha} + |\hat{H}_k|^2}, k = 0, 1, \ldots, N-1, \tag{13}$$

when $\hat{\mathbf{H}} = [\hat{H}_0, \hat{H}_1, \ldots, \hat{H}_{N-1}]^T$ corresponds to the channel frequency response (CFR) and $\alpha = \sigma_n^2/\sigma_s^2$, with σ_n^2 denoting the noise variance. $K_F(m)$ can be adopted as a normalization factor (e.g., so that $\frac{1}{N}\sum_{k=0}^{N-1} F_k(m)H_k = 1$). In Fig. 2, the "$\Delta$ unit" operates according to (9), Π and Π^{-1} stand for "interleaver" and "deinterleaver", respectively, and $\odot$ denotes "element-by-element multiplication". An alternative way of writing a matrix equation for this single-tap (per subchannel) FDE procedure is $\tilde{\mathbf{S}}^{(i)}(m) = \mathbf{diag}[F_0, F_1, \ldots, F_{N-1}] \cdot \tilde{\mathbf{Y}}_{CP}^{(i)}(m)$.

The soft demapper in Fig. 2 provides the inputs to the SISO decoder (LLRs of the several coded bits). The decoder outputs must correspond to the full soft information, not the extrinsic one (this is also true for the receivers of Figs. 3 and 4).

A simplified version of this receiver can be adopted, where the decoding procedures are removed from the iterative process. In this case, Fig. 2 may still be considered for receiver characterization, but with "decoder" outputs identical to the corresponding inputs (there is no extrinsic information at all).

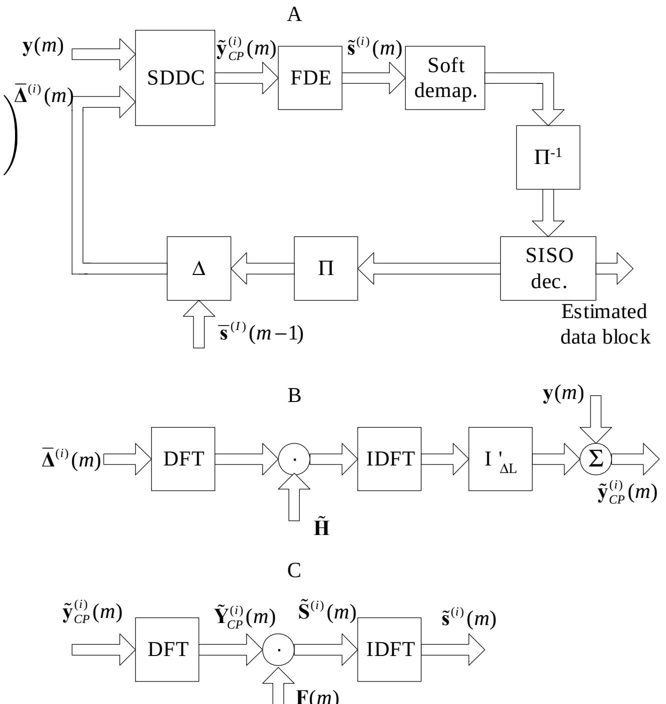

Fig. 2. SDDC-FDE receiver structure (A), with characterization of the SDDC unit (B) and the FDE unit (C)..

III. Improved Turbo FDE Technique for Reduced-CP Block Transmission

A. A New Look into Conventional Turbo FDE Techniques

When a full-length CP is employed, turbo FDE techniques such as that proposed in [2], [3] can provide a strongly improved FDE performance, while avoiding a high complexity of implementation. Such techniques, which resort to a conventional, linear, single-tap FDE scheme, take advantage of the SISO decoder outputs in order to carry out a turbo soft-cancellation of residual ISI through the use of the soft information on the coded bits. According to the "switched APPLE/MF approach" (APProximate Linear (MMSE) Equalization/Matched Filtering) proposed in [2], [3], the FDE coefficients for each iteration are selected on the basis of an "average SNR" concerning the length-N block of equalizer outputs, which plays the role of an extrinsic information on the coded data block, delivered by the equalizer to the channel SISO decoder. This means that the receiver uses the algorithm (APPLE (MMSE) or MF) with largest estimated SNR.

In the following, we describe a new "conventional" (for full-length CP conditions) turbo FDE technique which is strongly related to that proposed in [2], [3], but replaces the selection principle (APPLE/MF) regarding the linear FDE parameters by an appropriate "compromise choice": in fact, the values of the linear FDE parameters are adaptively adjusted, iteration by iteration, according to the available block of SISO decoder outputs. The proposed receiver structure is as depicted in Fig.

3. At the equalizer output, the time-domain vector $\tilde{\mathbf{s}}^{(i)}(m)$ is the IDFT of $\tilde{\mathbf{S}}^{(i)}(m)$ given by

$$\tilde{\mathbf{S}}^{(i)}(m) = \mathbf{F}^{(i)}(m) \odot \mathbf{Y}(m) + \mathbf{G}^{(i)}(m). \quad (14)$$

This means that the N entries of $\tilde{\mathbf{S}}^{(i)}(m)$ can be expressed as

$$\tilde{S}_k^{(i)}(m) = F_k^{(i)}(m) Y_k(m) + G_k^{(i)}(m), k = 0, \ldots, N-1, \quad (15)$$

where $F_k^{(i)}(m), k = 0, 1, \ldots, N-1$, are the multiplicative FDE parameters for iteration i, and

$$G_k^{(i)}(m) = (\gamma^{(i)}(m) - F_k^{(i)}(m)\hat{H}_k)\overline{S}_k^{(i-1)}(m), \quad (16)$$

with

$$\gamma^{(i)}(m) = \frac{1}{N}\sum_{k=0}^{N-1} F_k^{(i)}(m)\hat{H}_k, \quad (17)$$

are complementary FDE parameters for ISI soft-cancellation purposes. $\overline{\mathbf{S}}^{(i-1)}(m) = [\overline{S}_0^{(i-1)}(m), \overline{S}_1^{(i-1)}(m), \ldots, \overline{S}_{N-1}^{(i-1)}(m)]^T$ is the DFT of $\overline{\mathbf{s}}^{(i-1)}(m) = [\overline{s}_0^{(i-1)}(m), \overline{s}_1^{(i-1)}(m), \ldots, \overline{s}_{N-1}^{(i-1)}(m)]^T$, with $\overline{s}_n^{(i-1)}(m)$ given by (11) (or (12), equivalently) resulting from the soft information provided by the SISO decoder as explained in Appendix A, when assuming a QPSK modulation. As to the multiplicative FDE parameters, alternatively to $F_k^{(i)}(m) = \hat{H}_k^*/(\hat{\alpha} + |\hat{H}_k|^2)$ (APPLE) or $F_k^{(i)}(m) = \hat{H}_k^*$ (MF) [11], [12], we propose to adopt

$$F_k^{(i)}(m) = \frac{K_F^{(i)}(m)\hat{H}_k^*}{\hat{\alpha} + (1 - (\hat{\rho}^{(i-1)}(m))^2)|\hat{H}_k|^2} \quad (18)$$

where

$$\hat{\rho}^{(i-1)}(m) = \frac{1}{N}\sum_{n=0}^{N-1} \frac{E[s_n^*(m)\hat{s}_n^{(i-1)}(m)]}{E[|s_n(m)|^2]} = $$
$$= \frac{1}{2N}\sum_{n=0}^{N-1}(\rho_{n,I}^{(i-1)}(m) + \rho_{n,Q}^{(i-1)}(m)), \quad (19)$$

is an "overall correlation coefficient". It can be obtained as an average value of the $2N$ correlation coefficients per bit, derived from the SISO decoder outputs. $K_F^{(i)}(m)$ is a normalization factor: using $\gamma^{(i)}(m) = 1$, $\tilde{s}_n^{(i)}(m) = s_n(m) + \xi_n^{(i)}(m)$, where $\xi_n^{(i)}(m)$ is the zero-mean "error" (assumed to be approximately complex Gaussian) concerning symbol $s_n(m)$ at the FDE output. Under the "Gaussian assumption", the LLRs of the "in-phase bit" and the "quadrature bit", at the SISO decoder input, are given by $L_{n,I}^{(i)}(m) = \frac{\sqrt{8}}{\sigma_{eq}^{2(i)}(m)}\sigma_s \mathrm{Re}\{\tilde{s}_n^{(i)}(m)\}$ and $L_{n,Q}^{(i)}(m) = \frac{\sqrt{8}}{\sigma_{eq}^{2(i)}(m)}\sigma_s \mathrm{Im}\{\tilde{s}_n^{(i)}(m)\}$, respectively, where $\sigma_{eq}^{2(i)}(m)$ is the mean-squared error.

We point out that, for $i = 1$, the $F_k^{(i)}(m)$ parameters meet the MMSE criterion, since $\hat{\rho}^{(i-1)}(m) = 0$ in (18). After a number of iterations and/or for high SNRs, typically $\hat{\rho}_{n,I}^{(i-1)}(m) \approx 1$ and $\hat{\rho}_{n,Q}^{(i-1)}(m) \approx 1$, leading to $\hat{\rho}^{(i-1)}(m) \approx 1$ in (18), and, therefore, to $F_k^{(i)}(m)$ parameters approximately in accordance with the MF criterion. It should also be noted that the proposed multiplicative FDE coefficients can be written as $F_k^{(i)}(m) = K_F^{'(i)}(m)\hat{H}_k^*/(\hat{\alpha}^{'(i)}(m) + |\hat{H}_k|^2)$, where $K_F^{'(i)}(m) = K_F^{(i)}(m)/(1 - (\hat{\rho}^{(i-1)}(m))^2)$ and $\hat{\alpha}^{'(i)}(m) = \hat{\sigma}_n^2/\hat{\sigma}_s^{2(i)}(m)$, with $\hat{\sigma}_s^{2(i)}(m) = \sigma_s^2(1 - (\hat{\rho}^{(i-1)}(m))^2)$. Having in mind the symbol variance parameter given by (25), the proposed $F_k^{(i)}(m)$ coefficients can be regarded as roughly approximating the MMSE criterion for all iterations, taking into account the available information from the SISO decoder.

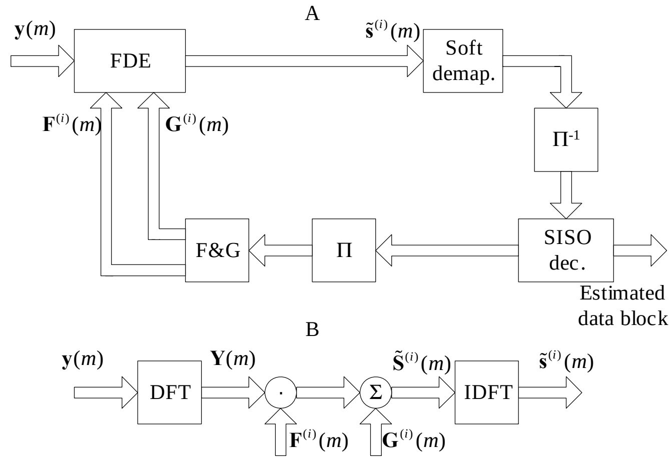

Fig. 3. Turbo FDE receiver structure (A) and characterization of the FDE unit (B).

To conclude, let us consider a simplified iterative FDE implementation, based on the ideas above, where no decoding effort is really involved in the FDE process. The corresponding receiver derives from that shown in Fig. 3, by suppressing the SISO decoder; therefore, no extrinsic information is provided to help the iterative FDE process, and the soft FDE outputs in a given iteration $(i-1)$ are directly used to compute $F_k^{(i)}(m)$ and $G_k^{(i)}(m)$ for next iteration. An additional simplification is to replace the correlation coefficient concerning every bit by $\hat{\rho}^{(i-1)}(m)$, as given by (19), in the computation of $\overline{s}_n^{(i-1)}(m)$, leading to $\overline{S}_k^{(i-1)}(m) = \hat{\rho}^{(i-1)}(m)\hat{S}_k^{(i-1)}(m)$, ($[\hat{S}_0^{(i-1)}(m), \hat{S}_0^{(i-1)}(m), \ldots, \hat{S}_{N-1}^{(i-1)}(m)]^T$ denotes the DFT of $[\hat{s}_0^{(i-1)}(m), \hat{s}_0^{(i-1)}(m), \ldots, \hat{s}_{N-1}^{(i-1)}(m)]^T$). In this case,

$$\tilde{S}_k^{(i)}(m) = F_k^{(i)}(m)Y_k(m) - B_k^{(i)}(m)\hat{S}_k^{(i-1)}(m) \quad (20)$$

where

$$B_k^{(i)}(m) = \hat{\rho}^{(i-1)}(m)(F_k^{(i)}(m)\hat{H}_k - \gamma^{(i)}(m)), \quad (21)$$

with eqns. (18), (19), (20) and (21) jointly defining an IB-DFE (Iterative Block Decision Feedback Equalization) FDE receiver, as proposed in [4] and extended in [5]-[9].

B. Generalized SDDC-Aided Turbo FDE Technique

When the CP length (L_R) is smaller than the channel memory order (L), good performances through the iterative receiver technique of Fig. 3 cannot be ensured. In this case, Fig. 4 shows a suitable receiver technique, which somehow combines the capabilities of the techniques in Figs. 2 and 3.

This technique actually uses an SDDC aid, as proposed in Sec. III.B, to the Turbo FDE technique described in Sec. III.A. This means that, for each iteration of the Turbo FDE scheme, the time-domain channel input to the FDE unit is updated, not only the FDE parameters. Of course, as in Sec. III.A, one may consider to suppress the contribution of SISO decoding, in the context of Fig. 4, for the sake of implementation simplicity.

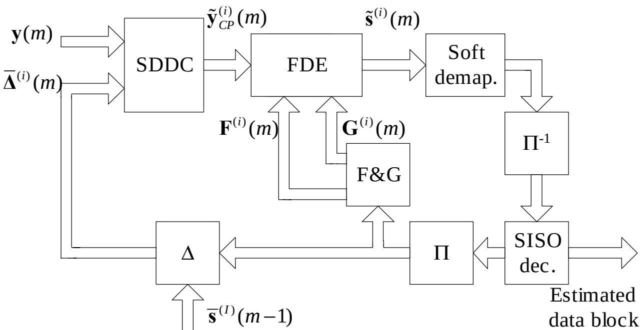

Fig. 4. Turbo SDDC-FDE receiver structure.

IV. Performance Results

A set of numerical results is presented below, with regards to broadband transmission over a strongly frequency-selective Rayleigh fading channel, when the iterative techniques described in Secs. II and III are employed, under perfect channel estimation. We adopt the power delay profile type C within HIPERLAN/2, with uncorrelated Rayleigh fading on the different paths. A CP-assisted block transmission scheme in accordance with Fig. 1 is assumed, with $N = 256$ data symbols per block, each one selected from a QPSK constellation. The duration of the useful part of the block is 5μs and we consider either a full-length CP ($L_R = L = 64$) or a reduced CP ($L_R = L/8 = 8$); when considering the latter alternative, about 1/3 of the CIR (Channel Impulse Response) energy falls outside the time interval of the reduced CP.

Figs. 5, 6 and 7 are concerned to simplified iterative FDE techniques with no channel decoding involved. Fig. 5 shows uncoded BER results, when $L_R = L = 64$, with both the Turbo FDE technique of Sec. III.A, and, for the sake of comparisons, the related IB-DFE technique [4]: the improved performance of the former technique is shown to be asymptotically very close to the MF bound. Fig. 6 shows the uncoded BER performance for both the SDDC-FDE receiver of Sec. II.B and the conventional Turbo FDE receiver of Sec. III.A: clearly, the Turbo FDE performance is seriously affected by the shortened CP, exhibiting an error floor at about 3.5×10^{-3}; the receiver technique of Sec. II.B, which has a worse performance for low SNR, achieves an improved performance for high SNR, obviously due to the soft cancellation of the reduced-CP effects. The uncoded BER results of Fig. 7 for $L_R = L/8 = 8$ correspond to the SDDC-aided techniques of Secs. II.B and III.A (Turbo FDE results for $L_R = L = 64$ are also shown for the sake of comparisons): the benefits of jointly using a Turbo FDE approach and an SDDC aid are evident, allowing BER values below 10^{-3}; however, the lack of a SISO decoding contribution for the iterative FDE process still is a serious limitation.

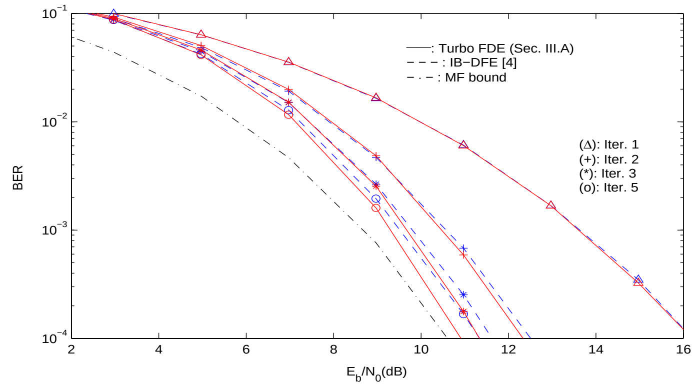

Fig. 5. Uncoded BER performances for $L_R = L$ (the first iteration leads to the conventional FDE (MMSE) performance).

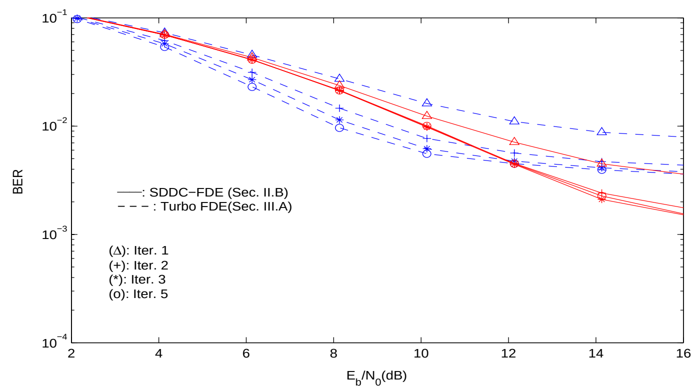

Fig. 6. Uncoded BER performances for $L_R < L$.

For the last performance results, in Fig. 8, we assumed a rate-1/2 convolutional code with $G(D) = [1 \quad (1 + D^2)/(1 + D + D^2)]$, a low-complexity SISO decoding through the use of the Max-Log-MAP algorithm, and the close equalization/decoding cooperation which is allowed by the receiver structures of Figs. 2, 3 and 4, as described in Secs. II and III. A special attention should be paid to the following pairs of performance curves: the best solid line, as compared to the best dashed line (Turbo FDE performances); the best dash-dotted line as compared to the worst dashed line (FDE (MMSE) performances). We must keep in mind that, when reducing the CP length from $L_R = L = 64$ to $L_R = L/8 = 8$ (e.g., to increase the bandwidth efficiency by about

$$\left(\frac{N + L}{N + L_R} - 1 \right) \times 100\%,$$

which, in our case, gives 21.2%), the maximum achievable power efficiency gain is $10 \log_{10}((N + L)/(N + L_R)) \approx$ 0.84dB. Therefore, it is clear that the SDDC-aided FDE techniques can practically ensure that there is no degradation of the power efficiency as a downside of that reduction, and, on the contrary, there is a gain close to the maximum.

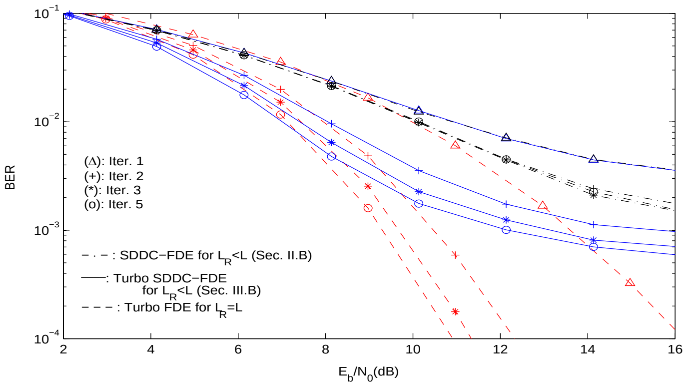

Fig. 7. Uncoded BER performances for the SDDC-aided techniques ($L_R < L$) and the Turbo FDE technique of Sec. III.A ($L_R = L$).

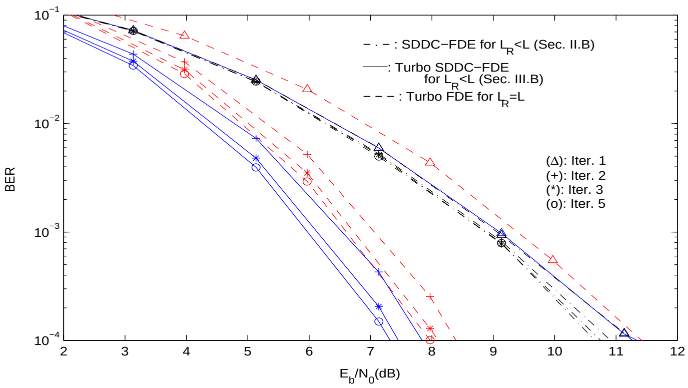

Fig. 8. Coded BER performances for the SDDC-aided techniques ($L_R < L$) and the Turbo FDE technique of Sec. III.A ($L_R = L$), when SISO channel decoding is involved in the iterative FDE process.

V. Conclusions

A low-complexity, SDDC-aided iterative FDE technique, based on the "DDC algorithm" [10], [11], was proposed for reduced-CP, SC-based block transmission systems using conventional frame structures. A more sophisticated, SDDC-aided, Turbo FDE technique was also proposed for improved performances in such systems, through a moderately increased complexity. The relations with already known Turbo FDE [2], [3] and IB-FDE [4]-[9] FDE techniques were established. The advantages of the SDDC-aided approach were emphasized, namely the following: the fact that it can operate in a satisfactory way even when, for the sake of implementation simplicity, no decoding operations are used to improve the iterative FDE process; the possibility of achieving the maximum power efficiency gains that a strong CP reduction allows, when adopting those techniques. They seem to be especially well-suited for the uplink of future broadband wireless systems.

Appendix A. Computation of QPSK Symbol Statistics Using Soft Decoder Outputs

Let us assume QPSK symbol coefficients $s_n = s_{n,I} + js_{n,Q}$, with $s_{n,I} = \pm\sigma_s/\sqrt{2}$ and $s_{n,Q} = \pm\sigma_s/\sqrt{2}$ ($n = 0, 1, \ldots, N-1$), according to the coded data block. When the LLRs concerning the nth in-phase bit and the nth quadrature bit, as provided by the channel decoder, are $L_{n,I}$ and $L_{n,Q}$, respectively, the resulting expected value $\overline{s}_n$ can be expressed as $\overline{s}_n = \overline{s}_{n,I} + j\overline{s}_{n,Q}$, with $\overline{s}_{n,I}$ and $\overline{s}_{n,Q}$ as follows:

$$\overline{s}_{n,I} = \frac{\sigma_s}{\sqrt{2}} \tanh\left(\frac{L_{n,I}}{2}\right); \quad \overline{s}_{n,Q} = \frac{\sigma_s}{\sqrt{2}} \tanh\left(\frac{L_{n,Q}}{2}\right) \tag{22}$$

Let us define the "coded bit decisions", $\hat{s}_{n,I} = \pm\sigma_s/\sqrt{2}$ and $\hat{s}_{n,Q} = \pm\sigma_s/\sqrt{2}$, according to the signs of $L_{n,I}$ and $L_{n,Q}$, respectively, and the following correlation coefficients:

$$\rho_{n,I} = \frac{E[s_{n,I}\hat{s}_{n,I}]}{E[|s_{n,I}|^2]}; \quad \rho_{n,Q} = \frac{E[s_{n,Q}\hat{s}_{n,Q}]}{E[|s_{n,Q}|^2]}. \tag{23}$$

Since $\rho_{n,I} = 1 - 2\ \mathrm{Prob}(\hat{s}_{n,I} = -s_{n,I}|L_{n,I}) = \tanh(|L_{n,I}|/2)$ and $\rho_{n,Q} = 1 - 2\ \mathrm{Prob}(\hat{s}_{n,Q} = -s_{n,Q}|L_{n,Q}) = \tanh(|L_{n,Q}|/2)$ (leading to $0 \leq \rho_{n,I} \leq 1$ and $0 \leq \rho_{n,Q} \leq 1$), the average values $\overline{s}_{n,I}$ and $\overline{s}_{n,Q}$ can be written as follows:

$$\overline{s}_{n,I} = \rho_{n,I}\hat{s}_{n,I}; \quad \overline{s}_{n,Q} = \rho_{n,Q}\hat{s}_{n,Q} \tag{24}$$

Regarding the nth QPSK symbol, a variance parameter can also be derived from the pair of decoder outputs and expressed as a function of the correlation coefficients $\rho_{n,I}$ and $\rho_{n,Q}$. By defining $\rho_n^2 = (\rho_{n,I}^2 + \rho_{n,Q}^2)/2$,

$$\sigma_{s_n}^2 = \sigma_s^2 - |\overline{s}_n|^2 = \sigma_s^2(1 - \rho_n^2). \tag{25}$$

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