A 2D HOMING STRATEGY FOR AUTONOMOUS UNDERWATER VEHICLES

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Abstract: This paper presents a new sensor based integrated guidance and control law to solve the homing problem of an underactuated autonomous underwater vehicle (AUV), in the horizontal plane, using the information provided by an Ultra-Short Base Line (USBL) positioning system. Global asymptotic stability (GAS) is achieved in the presence (and absence) of constant known ocean currents and simulation results are provided to illustrate the performance and behavior of the overall closed loop system.

Keywords: Marine systems, autonomous vehicles, nonlinear control.

1. INTRODUCTION

The need to know and explore the ocean and its frontiers has driven the scientific community, in the recent past, to develop not only advanced but also cost-efficient research tools such as Autonomous Underwater Vehicles (AUVs) and Remotely Operated Vehicles (ROVs) (Sarradin *et al.*, 2002; Silvestre and Pascoal, 2004; Silvestre *et al.*, 1998). The control of these vehicles has naturally been subject of growing interest, and while the control of fully actuated vehicles is nowadays fairly well understood, as evidenced by the large body of publications, e.g. (Isidori, 1995; Nijmeijer and van der Schaft, 1990; Sastry, 1999) and the references therein, the control of underactuated vehicles is still an open field of research. To solve the problems of stabilization and trajectory tracking of an underactuated vehicle several solutions have been proposed in the literature, see (Wichlund *et al.*, 1995; Reyhanoglu, 1996; Pettersen and Nijmeijer, 1998; Mazenc *et al.*, 2002) and (Aguiar and Hespanha, 2003; Aguiar *et al.*, 2003), respectively. In (Indiveri *et al.*, 2000) a solution to the problem of following a straight line is presented and in (Aguiar and Pascoal, 2002) a way point tracking controller for an underactuated AUV is introduced. It turns out that in all the aforementioned references the vehicle's position is computed in the inertial coordinate frame and the control laws are developed in the body frame, disregarding onboard sensors. Sensor-based control has been a hot topic in the field of computer vision where the so-called visual servoing techniques have been subject of an intensive research effort during the last years, see (Cowan *et al.*, 2002; Malis and Chaumette, 2002) for further information.

This paper addresses the design of an integrated guidance and control law to drive an underactuated AUV to a fixed target, in 2D. The solution of this problem, usually denominated as homing in the literature, is central to drive the vehicle to the

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vicinity of a base station or support vessel. Once the vehicle is close enough to the base station a different control strategy should be adopted. In the paper it is assumed that an acoustic transponder is installed on a predefined fixed position in the mission scenario, denominated as target in the sequel, and an Ultra-Short Baseline (USBL) sensor, composed by an array of hydrophones and an acoustic emitter, is rigidly mounted on the vehicle's nose. During the homing phase the USBL sensor interrogates the transponder and synchronizes, detects and records the time of arrival measured by each receiver. The implementation of the control law also requires the vehicle's linear velocities, relative to the water and to the ground, as provided by a Doppler velocity log, and the vehicle attitude and angular velocities measured by an Attitude and Heading Reference System (AHRS).

The paper is organized as follows. In Section 2 the homing problem is introduced and the dynamics of the horizontal plane of the AUV are briefly described, whereas the USBL model is presented in Section 3. A Lyapunov based guidance and control law is derived, in Section 4, using the vehicle's kinematics directly expressed in terms of the time differences of arrival (TDOAs) and range to the target obtained from the USBL data. This control law is then extended to include the dynamics of the vehicle resorting to backstepping techniques. Afterwards, this strategy is further extended, in Section 5, to the case where known constant ocean currents affect the vehicle's dynamics. Global asymptotic stability (GAS) is achieved in both cases. Simulation' results are presented and discussed in Section 6, and finally Section 7 summarizes the main results of the paper.

2. PROBLEM STATEMENT

Let $\{I\}$ be an inertial coordinate frame, and $\{B\}$ a body-fixed coordinate frame, whose origin is located at the center of mass of the vehicle. Consider $\mathbf{p} = [x,\ y]^T$ as the position of the origin of $\{B\}$, described in $\{I\}$, ψ the orientation of the vehicle relative to $\{I\}$, $\mathbf{v} = [u,\ v]^T$ the linear velocity of the vehicle relative to $\{I\}$, expressed in body-fixed coordinates, and ω the angular velocity. The vehicle kinematics can be written as

$$\dot{\mathbf{p}} = {}^{I}_{B}\mathbf{R}\,(\psi)\mathbf{v} \quad \dot{\psi} = \omega \tag{1}$$

where $\mathbf{R} = {}^{I}_{B}\mathbf{R} = ({}^{B}_{I}\mathbf{R})^T$ is the rotation matrix from $\{B\}$ to $\{I\}$, verifying $\dot{\mathbf{R}} = \mathbf{R}\mathbf{S}(\omega)$, and $\mathbf{S}(x)$ is the skew-symmetric matrix

$$\mathbf{S}(x) = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$$

The vehicle's dynamic equations of motion can be written in a compact form as

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}} = -\mathbf{S}(\omega)\mathbf{M}\mathbf{v} - \mathbf{D_v}(\mathbf{v})\mathbf{v} + \mathbf{l}u_{\mathbf{v}} \\ J\dot{\omega} = -d_\omega(\omega)\omega + u_\omega \end{cases} \tag{2}$$

where $\mathbf{M} = \mathrm{diag}\{m_u, m_v\}$ is the positive definite diagonal mass matrix, $\mathbf{D_v}(\mathbf{v}) = \mathrm{diag}\{d_u + d_{|u|u}|u|, d_v + d_{|v|v}|v|\}$ captures the hydrodynamic damping effects on the linear velocity, $d_\omega(\omega) = d_\omega + d_{|\omega|\omega}|\omega|$ captures the hydrodynamic damping effects on the angular velocity, and $\mathbf{l} = [1,\ 0]^T$. The control inputs $u_{\mathbf{v}} = \tau_u$ and $u_\omega = \tau_\omega$ are the surge force and the yaw torque, respectively.

The homing problem considered in this paper can be stated as follows:

<u>Problem Statement.</u> *Consider an underactuated AUV with kinematics and dynamics given by* (1) *and* (2)*, respectively. Assume that the vehicle is moving in a horizontal plane where a target equipped with an acoustic transponder is placed in a fixed position. Design a sensor based integrated guidance and control law to drive the vehicle towards a well defined neighborhood of the target using the time differences of arrival and range to the target as measured by an USBL sensor installed on the AUV.*

3. USBL MODEL

During the homing approach phase the vehicle is far away from the acoustic emitter, that is, the distance from the vehicle to the target is much larger than the distance between any pair of receivers. Therefore, the plane-wave assumption is valid. Let $\mathbf{r}_i = [x_i,\ y_i]^T \in \mathbb{R}^2,\ \ i = 1,\ 2,\ \ldots,\ N$, denote the positions of the N acoustic receivers installed on the USBL sensor and consider a plane-wave traveling along the opposite direction of the unit vector $\mathbf{d} = [d_x,\ d_y]^T$, as shown in Figure 1. Notice both $\mathbf{r}_i$ and $\mathbf{d}$ are expressed in the body frame.

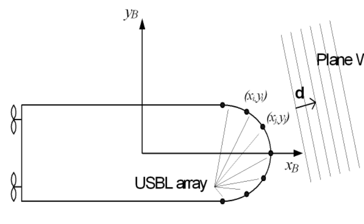

Fig. 1. Plane Wave and the USBL system

Let t_i be the instant of time of arrival of the plane-wave at i^{th} receiver and V_S the velocity of propagation of the sound in water, assumed to be constant and known. Then, assuming that the medium is homogeneous and neglecting the velocity of the vehicle, which is a reasonable assumption since $\|\mathbf{v}\| << V_S$, the time difference of arrival between receivers i and j satisfies

$$V_S\,(t_i - t_j) = -\left[d_x\,(x_i - x_j) + d_y\,(y_i - y_j)\right] \tag{3}$$

Denote by $\Delta_1 = t_1 - t_2$, $\Delta_2 = t_1 - t_3$, $\ldots$, $\Delta_M = t_{N-1} - t_N$, where $M = N\,(N-1)\,/2$, all the possible combinations of TDOA, and let $\boldsymbol{\Delta} = [\Delta_1,\ \Delta_2,\ \cdots,\ \Delta_M]^T$. Define

$$\mathbf{r}_x = [x_1 - x_2,\ x_1 - x_3,\ \cdots,\ x_{N-1} - x_N]^T$$

$$\mathbf{r}_y = [y_1 - y_2,\ y_1 - y_3,\ \cdots,\ y_{N-1} - y_N]^T$$

and $\mathbf{H}_R \in \mathbb{R}^{M\times 2}$ as $\mathbf{H}_R = [\mathbf{r}_x,\ \mathbf{r}_y]$. Then, the generalization of (3) for all TDOAs yields

$$\boldsymbol{\Delta} = -\frac{1}{V_S}\mathbf{H}_R\mathbf{d} \tag{4}$$

Define also $\mathbf{H}_Q \in \mathbb{R}^{2\times 2}$ as

$$\mathbf{H}_Q = \frac{1}{V_S}\mathbf{H}_R^T\mathbf{H}_R$$

which is assumed to be non-singular. This turns out to be a weak hypothesis as it is always true if, at least, 3 receivers are mounted in noncolinear positions. In those conditions $\mathbf{H}_R$ has maximum rank and so does $\mathbf{H}_Q$. Then,

$$\mathbf{d} = -\mathbf{H}_Q^{-1}\mathbf{H}_R^T\boldsymbol{\Delta} \tag{5}$$

which directly relates the direction of the target to the TDOA vector.

4. CONTROLLER DESIGN

In this section an integrated nonlinear closed loop guidance and control law is derived that solves the homing problem stated earlier in Section 2. Assuming there are no ocean currents, the idea behind the control strategy proposed here is to steer the vehicle directly towards the emitter. The synthesis of the guidance and control law resorts extensively to the Lyapunov's direct method and backstepping techniques.

Consider a first error variable z_1 defined as the angle θ between the vehicle's x axis and the direction of the target $\mathbf{d}$, i.e,

$$z_1 := \theta$$

This error variable is directly obtained, using the 4-quadrant inverse tangent, from the direction of the target, which in turn is directly related to the TDOA vector through (5). To force the vehicle to move towards the target, consider a second error variable defined as

$$z_2 := [1,\ 0]\mathbf{v} - V_d$$

where V_d is a positive constant that corresponds to the desired velocity during the homing stage. When z_1 and z_2 converge to zero, the vehicle is moving with positive surge speed V_d and its x axis is pointing to the target. However, these two conditions are not sufficient to ensure that the vehicle is driven towards the target during the homing stage as the sway speed is left free. Nevertheless, it will be shown that, with the control law based upon these two error variables, the sway velocity converges to zero, which suffices, in conjunction with the convergence of z_1 and z_2 to zero, to achieve the desired behavior of the vehicle during the homing stage.

To synthesize the control law, define the Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$$

The time derivative of V_1 is given by

$$\dot{V}_1 = z_1\dot{z}_1 + z_2\dot{z}_2 = z_1\dot{\theta} + z_2[1,\ 0]\dot{\mathbf{v}}$$

The derivative $\dot{\mathbf{v}}$ is directly obtained from the dynamics of the vehicle (2). The derivative of θ can be written, after a few algebraic manipulations, as

$$\dot{\theta} = -\omega + \frac{\mathbf{dS}(1)\mathbf{v}}{r}$$

where r is the range to the target as measured by the USBL sensor. Notice that the first term corresponds to the rotation of the vehicle and the second to the induced rotation due to the linear velocity of the vehicle. Now, $\dot{V}_1$ can be written as

$$\dot{V}_1 = z_1\left(-\omega + \frac{\mathbf{dS}(1)\mathbf{v}}{r}\right) + z_2[1,\ 0]\mathbf{M}^{-1}\left(\mathbf{l}u_{\mathbf{v}} - [\mathbf{S}(\omega)\mathbf{M}\mathbf{v} + \mathbf{D}_{\mathbf{v}}(\mathbf{v})\mathbf{v}]\right)$$

Setting $u_{\mathbf{v}}$ equal to

$$u_{\mathbf{v}} = \frac{[1,\ 0]\mathbf{M}^{-1}[\mathbf{S}(\omega)\mathbf{M}\mathbf{v} + \mathbf{D}_{\mathbf{v}}(\mathbf{v})\mathbf{v}] - k_2z_2}{[1,\ 0]\mathbf{M}^{-1}\mathbf{l}} \tag{6}$$

where $k_2 > 0$ is a control gain, and ω equal to ω_d,

$$\omega_d := k_1z_1 + \frac{\mathbf{dS}(1)\mathbf{v}}{r}$$

where k_1 is a second positive scalar control gain, $\dot{V}_1$ becomes negative definite. Although $u_{\mathbf{v}}$ is a real control variable, the same cannot be said about ω, which was regarded here as a virtual control variable. Following the standard backstepping technique, define a third error variable

$$z_3 = \omega - \omega_d$$

and the augmented Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_3^2 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2$$

The time derivative $\dot{V}_2$ can be written as

$$\dot{V}_2 = -k_1z_1^2 - k_2z_2^2 + z_3\left(\frac{1}{J}[-d_\omega(\omega)\omega + u_\omega] - \dot{\omega_d} - z_1\right)$$

Now, setting

$$u_\omega = d_\omega(\omega)\omega + J(\dot{\omega_d} + z_1 - k_3z_3) \tag{7}$$

where $k_3 > 0$ is a control gain, one obtains $\dot{V}_2$ negative definite. The time derivative $\dot{\omega}_d$ is not presented here for the sake of simplicity.

The following theorem states the main result of this section.

Theorem 1. Consider a vehicle with kinematics and dynamics given by equations (1) and (2), respectively, moving without ocean currents, and suppose the homing problem stated in Section 2 to be defined outside a ball of radius R_{min} and centered at the target's position. Further assume that

$$R_{min} > \frac{m_u}{d_v}V_d \tag{8}$$

Then, with the control law (6)-(7), the error variable $\mathbf{z} = [z_1, z_2, z_3]^T$ converges globally exponentially fast to zero. Moreover, the sway velocity converges to zero, thus solving globally the homing problem stated in Section 2.

PROOF. Before going into the details a sketch of the proof is first offered. The convergence of the error variable $\mathbf{z}$ is a straightforward application of the Lyapunov's second method. The analysis of the vehicle's sway equation of motion, when $\mathbf{z}$ converges to zero, allows to conclude the convergence to zero of the sway velocity.

The function V_2 is, by construction, continuous, radially unbounded, and positive definite. Moreover, with the control law (6)-(7), the time derivative $\dot{V}_2$ results in $\dot{V}_2 = -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2$, which is negative definite. Thus, the origin $\mathbf{z} = \mathbf{0}$ is a globally asymptotically stable equilibrium point. Furthermore, as there exists $\lambda > 0$ such that $\dot{V}_2 \leq -\lambda V_2$ it is straightforward to conclude, resorting to the Comparison Lemma, that $V_2(t) \leq V_2(0)e^{-\lambda t}$ which implies that the error variable $\mathbf{z}$ converges exponentially fast to zero.

To complete the stability analysis all that is left to do is to show that the sway velocity converges to zero. The dynamics of the sway velocity can be written as

$$\dot{v} = -\frac{m_u}{m_v} u\omega - \frac{d_v + d_{|v|v}|v|}{m_v} v$$

Taking the limit of the angular velocity when $\mathbf{z}$ converges to zero yields

$$\lim_{\mathbf{z}\to\mathbf{0}} \omega = -\frac{v}{r}$$

On the other hand, u converges to the desired velocity V_d. Therefore, when $\mathbf{z}$ converges to zero, the dynamics of the sway velocity can be written as

$$\dot{v} = -\frac{d_v + d_{|v|v}|v| - m_u \frac{V_d}{r}}{m_v} v$$

During the homing stage the vehicle is operating outside a ball of radius R_{min} centered at the target's position. Thus, $r > R_{min}$ which, in conjuction with assumption (8), allows to write

$$d_v + d_{|v|v}|v| - m_u \frac{V_d}{r} > 0$$

from which follows that the sway velocity converges to zero, thus completing the proof.

5. CONTROL IN THE PRESENCE OF OCEAN CURRENTS

Consider that the vehicle is moving with water relative velocity $\mathbf{v}_r$ in the presence of a known ocean current $\mathbf{v}_c$, both expressed in body-fixed coordinates. It is further assumed that the current velocity is constant in the inertial frame. The dynamics of the vehicle can then be rewritten as

$$\begin{cases} \mathbf{M}\dot{\mathbf{v}}_r = -\mathbf{S}(\omega)\mathbf{M}\mathbf{v}_r - \mathbf{D}_{\mathbf{v}_r}(\mathbf{v}_r)\mathbf{v}_r + \mathbf{l}u_{\mathbf{v}} \\ J\dot{\omega} = -d_\omega(\omega)\omega + u_\omega \end{cases} \quad (9)$$

and the vehicle's velocity relative to the inertial frame, expressed in body-fixed coordinates, is $\mathbf{v} = \mathbf{v}_r + \mathbf{v}_c$. Under these conditions the strategy synthesized in Section 4 does not solve the homing problem in the presence of currents, as the new control objective is to align the velocity of the vehicle relatively to the inertial frame towards the target instead of the x axis of the vehicle.

Consider the vehicle reference relative velocity $\mathbf{v}_R := [V_d,\ 0]^T$. The error variable z_2, which accounts for the surge speed, is naturally modified to

$$z_2 := [1,\ 0]\mathbf{v}_r - V_d$$

The idea now is to redefine the error variable z_1 so that the effect of the ocean current is included. In order to do so, define a new coordinate system $\{E\}$ as follows: let the x axis of $\{E\}$ have direction $\mathbf{d}$ and the y axis have direction $\mathbf{S}(1)\mathbf{d}$, all expressed in the body-fixed frame. The rotation matrix from $\{E\}$ to $\{B\}$ is simply given by

$$^B_E\mathbf{R} = \left[\, \mathbf{d} \,|\, \mathbf{S}(1)\mathbf{d} \,\right]$$

where $\mathbf{d}$, using (5), is directly obtained from the TDOA of the USBL sensor. Notice that, in the new coordinate frame, $\{E\}$, the target's direction $\mathbf{d}$ is, by construction,

$$^E(\mathbf{d}) = [1,\ 0]^T \quad (10)$$

Denote by $^E\left(\mathbf{v}_r^O\right)$ the velocity of the vehicle relative to the water, expressed in $\{E\}$, when the vehicle is moving directly towards the target with speed V_d and zero sway velocity. Then, the relationship

$$\frac{^E\left(\mathbf{v}_r^O\right) + {}^E(\mathbf{v}_c)}{\|\, ^E(\mathbf{v}_r^O) + {}^E(\mathbf{v}_c)\|} = {}^E(\mathbf{d})$$

is satisfied. Using (10), it is straightforward to conclude that

$$[0,\ 1]\ {}^E\left(\mathbf{v}_r^O\right) = -\,[0,\ 1]\ {}^E(\mathbf{v}_c)$$

Since $\left\|\, {}^E\left(\mathbf{v}_r^O\right)\right\| = V_d$, there are only two possible values left for the first component of ${}^E\left(\mathbf{v}_r^O\right)$. However, this component can be shown to be always positive under the assumption that $V_d > V_c$, which is a reasonable hypothesis. In fact, if this was not true, it could be impossible for the vehicle to approach the target as its relative velocity could be insufficient to counteract a direct opposing water current. Thus, the signal ambiguity is solved and ${}^E\left(\mathbf{v}_r^O\right)$ uniquely defined. This objective relative velocity plays now the role that direction $\mathbf{d}$ played in Section 4 and the error variable z_1 is now redefined as

$$z_1 := \theta_e$$

where θ_e is the angle between the vehicle's x axis and the desired relative velocity vector $\mathbf{v}_r^O$. When z_1 and z_2 are zero, the vehicle is moving directly towards the target if its sway velocity is zero. However, this velocity was left free but again, as in Section 4, it will be shown that with the control based upon these two error variables, the sway velocity also converges to zero.

To synthesize the control law, consider the same Lyapunov function as in Section 4

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$$

The time derivative of V_1 is given by

$$\dot{V}_1 = z_1\dot{z}_1 + z_2\dot{z}_2 = z_1\dot{\theta}_e + z_2[1,\ 0]\dot{\mathbf{v}}$$

After long but straightforward algebraic manipulations, the time derivative of θ_e can be written as

$$\dot{\theta}_e = -\omega + \frac{\left\|\mathbf{v}_r^O + \mathbf{v}_c\right\|}{V_d^2 D} \left(\mathbf{v}_r^O\right)^T \mathbf{S}(1) \left(\mathbf{v} - \frac{\left(\mathbf{v}_r^O\right)^T \mathbf{d}}{\left(\mathbf{v}_r^O\right)^T \mathbf{d}} \mathbf{d} \right)$$

Now, the derivative of V_1 reads as

$$\dot{V}_1 = z_1 \frac{\left\|\mathbf{v}_r^O + \mathbf{v}_c\right\|}{V_d^2 D} \left(\mathbf{v}_r^O\right)^T \mathbf{S}(1) \left(\mathbf{v} - \frac{\left(\mathbf{v}_r^O\right)^T \mathbf{d}}{\left(\mathbf{v}_r^O\right)^T \mathbf{d}} \mathbf{d} \right) - z_1\omega$$
$$+ z_2 [1,\ 0] \mathbf{M}^{-1} \left(\mathbf{l} u_{\mathbf{v}} - [\mathbf{S}(\omega)\mathbf{M}\mathbf{v}_r + \mathbf{D}_{\mathbf{v}_r}(\mathbf{v}_r)\mathbf{v}_r] \right)$$

Setting

$$u_{\mathbf{v}} = \frac{[1,\ 0]\mathbf{M}^{-1} \left[\mathbf{S}(\omega)\mathbf{M}\mathbf{v}_r + \mathbf{D}_{\mathbf{v}_r}(\mathbf{v}_r)\right] - k_2 z_2}{[1,\ 0]\mathbf{M}^{-1}\mathbf{l}} \qquad (11)$$

where k_2 is a positive control gain, and ω equal to ω_d,

$$\omega_d := k_1 z_1 + \frac{\left\|\mathbf{v}_r^O + \mathbf{v}_c\right\|}{V_d^2 D} \left(\mathbf{v}_r^O\right)^T \mathbf{S}(1) \left(\mathbf{v} - \frac{\left(\mathbf{v}_r^O\right)^T \mathbf{d}}{\left(\mathbf{v}_r^O\right)^T \mathbf{d}} \mathbf{d} \right)$$

where k_1 is another positive control gain, $\dot{V}_1$ becomes negative definite. Since ω is not a real control variable, and using the same technique as in Section 4, consider a third error variable defined as $z_3 = \omega - \omega_d$ and the augmented Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_3^2 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2$$

The time derivative of V_2 can be written as

$$\dot{V}_2 = -k_1 z_1^2 - k_2 z_2^2$$
$$+ z_3 \left(\frac{1}{J} \left[-d_\omega(\omega)\omega + u_\omega\right] - \dot{\omega_d} - z_1 \right)$$

For the sake of simplicity, the derivative $\dot{\omega}_d$ is not presented here. Now, setting

$$u_\omega = d_\omega(\omega)\omega + J\left(\dot{\omega_d} + z_1 - k_3 z_3\right) \qquad (12)$$

where $k_3 > 0$ is a control gain, $\dot{V}_2$ is made negative definite.

The following theorem is the main result of this section.

Theorem 2. Consider a vehicle with kinematics and dynamics given by equations (1) and (9), respectively, moving in the presence of constant known ocean currents and suppose the homing problem stated in Section 2 to be defined outside a ball of radius R_{min} and centered at the target's position. Further assume that

$$R_{min} > \frac{m_u}{d_v} 2V_d \qquad (13)$$

Then, with the control law (11)-(12), the error variable $\mathbf{z} = [z_1, z_2, z_3]^T$ converges globally exponentially fast to zero. Moreover, the sway velocity converges to zero, thus solving globally the homing problem stated in Section 2 in the presence of constant known ocean currents.

PROOF. The proof of the theorem follows the same steps of the proof of Theorem 1. The Lyapunov function V_2 is continuous, radially unbounded, positive definite, and its time derivative, with the control law (11)-(12), is made negative definite. Moreover, there exists $\lambda > 0$ such that $\dot{V}_2 \leq -\lambda V_2$. Thus, the error variable $\mathbf{z}$ converges exponentially fast to zero. Taking the limit of the angular velocity when $\mathbf{z}$ converges to zero yields

$$\lim_{\mathbf{z}\to\mathbf{0}} \omega = -\frac{\left\|\mathbf{v}_r^O + \mathbf{v}_c\right\|}{V_d} \frac{v}{r}$$

Since the surge velocity converges to V_d, the dynamics of the sway velocity can be written (when $\mathbf{z}$ to zero) as

$$\dot{v}_r = -\frac{d_v + d_{|v|v}|v_r| - m_u \frac{\left\|\mathbf{v}_r^O + \mathbf{v}_c\right\|}{r}}{m_v} v_r$$

Now, notice that $\left\|\mathbf{v}_r^O + \mathbf{v}_c\right\| < 2V_d$ which, in conjunction with assumption (13), allows to write

$$d_v + d_{|v|v}|v_r| - m_u \frac{\left\|\mathbf{v}_r^O + \mathbf{v}_c\right\|}{r} > 0$$

from which follows that the sway velocity also converges to zero, thus completing this proof.

6. SIMULATION RESULTS

In this section a computer simulation is presented to illustrate the performance of the proposed solutions. The simulation was carried out with a simplified model of the horizontal plane of the SIRENE vehicle, assuming it is directly actuated in force and torque (Silvestre *et al.*, 1998).

In this simulation the vehicle has to counteract a constant ocean current with velocity $[-0.5,\ -0.5]^T$ m/s, expressed in the inertial frame. The vehicle starts at position $[0,\ 500]^T$ m and the acoustic transponder is located at position $[500,\ 500]^T$ m. The control parameters were chosen as follows: $k_1 = 0.025$, $k_2 = 0.04$ and $k_3 = 20$. The desired velocity was set to $V_d = 2$ m/s, and a semi-spherical symmetric USBL sensor with seven receivers was placed on the vehicle's nose. Figure 2 shows the trajectory described by the vehicle, whereas Figures 3 and 4 display the evolution of the vehicle's velocities and control inputs, respectively. From the figures it can be concluded

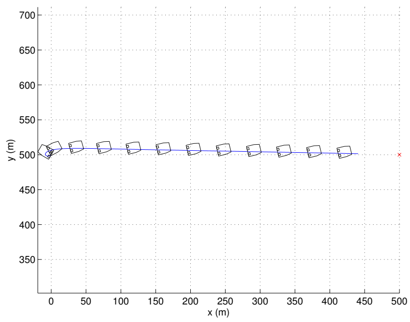

Fig. 2. Trajectory described by the vehicle in the presence of currents

that the vehicle is driven towards the target describing a smooth trajectory. The control inputs are smooth and the angular and sway velocities converge to zero, as expected.

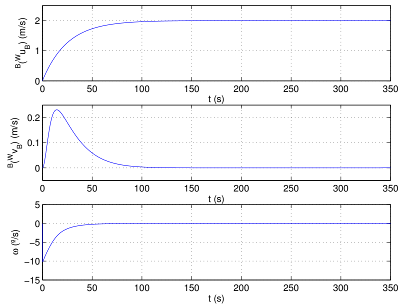

Fig. 3. Time evolution of body-fixed velocities of the vehicle in the presence of currents

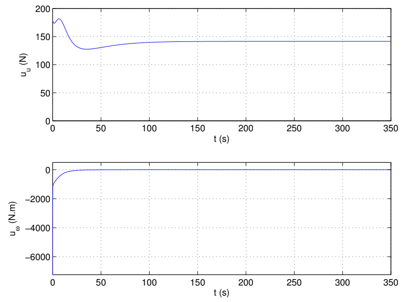

Fig. 4. Time evolution of control inputs in the presence of currents

7. CONCLUSIONS

The paper presented a new homing sensor based integrated guidance and control law to drive an underactuated AUV to a fixed target in 2D using the information provided by an USBL positioning system. The guidance and control laws were firstly derived for the vehicle's kinematics expressed as TDOAs and range to the target as measured by the USBL sensor and then extended to the dynamics of an AUV resorting to backstepping techniques. Global asymptotic stability was achieved for the guidance and control law in the presence (and absence) of known ocean currents. Simulations were presented and discussed to illustrate the performance of the proposed solutions.

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