Coordinated Path Following Control of Multiple Vehicles subject to Bidirectional Communication Constraints

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Summary. The paper addresses the problem of making a set of vehicles follow a a set of given spatial paths at required speeds, while ensuring that they reach and maintain a desired formation pattern. Problems of this kind arise in a number of practical applications involving ground and underwater robots. The paper summarizes and brings together in a unified framework previous results obtained by the authors for wheeled robots and fully actuated underwater vehicles. The decentralized solution proposed does not require the concept of a leader and applies to a very general class of paths. Furthermore, it addresses explicitly the dynamics of the vehicles and the constraints imposed by the inter-vehicle bi-directional communications network. The theoretical machinery used brings together Lyapunov-based techniques and graph theory. With the set-up proposed, path following (in space) and inter-vehicle coordination (in time) can be viewed as essentially decoupled. Path following for each vehicle is formulated in terms of driving a conveniently defined generalized error vector to zero; vehicle coordination is achieved by adjusting the speed of each vehicle along its particular path, based on information on the position and speed of a number of neighboring vehicles, as determined by the communications topology adopted. The paper presents the problem formulation and summarizes its solution. Simulations with dynamics models of a wheeled robot and an underwater vehicle illustrate the efficacy of the solution proposed.

Key words: Coordinated Motion Control, Graph Theory, Path following, Wheeled Robots

1 Introduction

There is growing interest in the problem of coordinated motion control of multiple autonomous vehicles. Applications include aircraft and spacecraft formation flying control (Beard *et al.*, 2001), (Giuletti *et al.*, 2000), (Pratcher *et al.*, 2001), (Queiroz *et al.*, 2000), coordinated control of land robots (Desai *et al.*, 1998), (Ögren *et al.*, 2002) and control of multiple surface and underwater vehicles (Encarnação and Pascoal, 2001), (Lapierre *et al.*, 2003a), (Skjetne *et al.*, 2002), (Skjetne *et al.*, 2003), (Stilwell *et al.*, 2000). The work reported in the literature is by now quite vast and addresses a large class of topics that include, among others, leader/follower formation flying, control of the "center of mass" and radius of dispersion of swarms of vehicles, and attaining a moving formation. In the latter, the goal is for the vehicles to achieve and maintain desired relative positions and orientations with respect to each other, while evolving at a desired formation speed. Central to the problems stated is the fact that each vehicle can only exchange information with a subset of the group of vehicles.

At first inspection, the problem of coordinated motion control seems to fall within the domain of decentralized control. However, as clearly pointed out in (Fax and Murray, 2002), (Fax and Murray, 2003), it possesses several unique aspects that are at the root of new challenges to system designers. Among these, the following are worth stressing:

i) except for some cases in the area of aircraft control, the motion of one vehicle does not directly affect the motion of the other vehicles, that is, the vehicles are dynamically decoupled; the only coupling arises naturally out of the specification of the tasks that they are required to accomplish together.

ii) there are strong practical limitations to the flow of information among vehicles, which is severely restricted by the nature of the supporting communications network. In marine robotics, for example, underwater communications rely on the propagation of acoustic waves. This sets tight limits on the communication bandwidths that are achievable. Thus, as a rule, possibly no vehicle will be able to communicate with the entire formation (Fax and Murray, 2003). Furthermore, a reliable vehicle coordination scheme should exhibit some form of robustness against certain kinds of vehicle failures or loss of inter-vehicle communications.

A rigorous methodology to deal with some of the above issues has emerged from the work reported in (Fax and Murray, 2002), (Fax and Murray, 2003), which addresses explicitly the topics of information flow and cooperation control of vehicle formations simultaneously. The methodology proposed builds on an elegant framework that involves the concept of Graph Laplacian (a matrix representation of the graph associated with a given communication network). In particular, the results in (Fax and Murray, 2003) show clearly how the Graph Laplacian associated with a given inter-vehicle communication network plays a key role in assessing stability of the behaviour of the vehicles in a formation. It is however important to point out in that work that: i) the dynamics of the vehicles are assumed to be linear, time-invariant, and ii) the information exchanged among vehicles is restricted to linear combinations of the vehicles' state variables.

Inspired by the progress in the field, this paper tackles a problem in coordinated vehicle control that departs slightly from mainstream work reported in the literature. Specifically, we consider the problem of *coordinated path following* where *multiple vehicles are required to follow pre-specified spatial paths while keeping a desired inter-vehicle formation pattern in time.* This mission scenario occurs naturally in underwater robotics (Pascoal *et al.*, 2000). Namely, in the operation of multiple autonomous underwater vehicles for fast acoustic coverage of the seabed. In this important case, two or more vehicles are required to fly above the seabed at the same or different depths, along geometrically similar spatial paths, and map the seabed using copies of the same suite of acoustic sensors. By requesting that the vehicles traverse identical paths so as to make the acoustic beam coverage overlap along the seabed, large areas can be covered in a short time. This imposes constraints on the inter-vehicle formation pattern. Similar scenarios can of course be envisioned for land and air vehicles.

To the best of our knowledge, previous work on coordinated path following control has essentially been restricted to the area of marine robotics. See for example (Lapierre *et al.*, 2003a), (Lapierre *et al.*, 2003b), (Skjetne *et al.*, 2002) and the references therein. However, the solutions developed so far for underactuated vehicles are restricted to two vehicles in a leader-follower type of formation and lead to complex control laws. Even in the case of fully actuated vehicles, the solutions presented do not address communication constraints explicitly. There is therefore a need to re-examine this problem to try and arrive at efficient and practical solutions.

A possible strategy is to consider similar problems for wheeled robots in the hope that the solutions derived for this simpler case may shed some light into the problem of coordinated path following for the more complex case of air and marine robots. Preliminary steps in this direction were taken in (Ghabcheloo *et al.*, 2004a), where the problem of coordinated path following of multiple wheeled robots was solved by resorting to linearization and gain scheduling techniques. The solutions obtained are conceptually simple and embody in themselves a straightforward mechanism that allows for the decoupling of path following (in space) and vehicle synchronization (in time). The price paid for the simplicity of the solutions is the lack of global results, that is, attractivity to so-called trimming paths and to a desired formation pattern can only be guaranteed locally, when the initial vehicle formation is sufficiently close to the desired one. The present paper overcomes this limitation and yields global results that allow for the consideration of arbitrary paths, formation patterns (compatible with the paths), and initial conditions. The solution adopted for coordinated path following is well rooted in Lyapunov-based theory and addresses explicitly the vehicle dynamics as well as the constraints imposed by the topology of the inter-vehicle communications network. The latter are tackled in the framework of graph theory (Godsil and Royle, 2001), which seems to be the tool par excellence to study the impact of communication topologies on the performance that can be achieved with coordination.

Once again, using this set-up, path following (in space) and inter-vehicle coordination (in time) are essentially decoupled. Path following for each vehicle amounts to reducing a conveniently defined error variable to zero. In its simplest form, vehicle coordination is achieved by adjusting the speed of each of the vehicles along its path, according to information on the relative position of the other vehicles, as determined by the communications topology adopted. No other kinematic or dynamic information is exchanged among the robots. The coordination system is simple and holds great potential to be extended and applied to the case of air and marine robots.

The paper is organized as follows. Section 2 introduces the basic notation required, describes the simplified model of a wheeled robot, and offers a novel solution to the problem of path following for a single vehicle. The main contribution of the paper is summarized in Section 3, where a strategy for multiple vehicle coordination is proposed that builds on Lyapunov and graph theory. The proofs required are given in Section **??**. Section **??** examines the convergence properties of the solutions of the combined path following and coordination algorithms. Section **??** revisits the coordination problem and provides added insight into the case where the formation pattern is time-varying. Section 4 describes the results of simulations. Finally, Section 5 contains the main conclusions and describes problems that warrant further research.

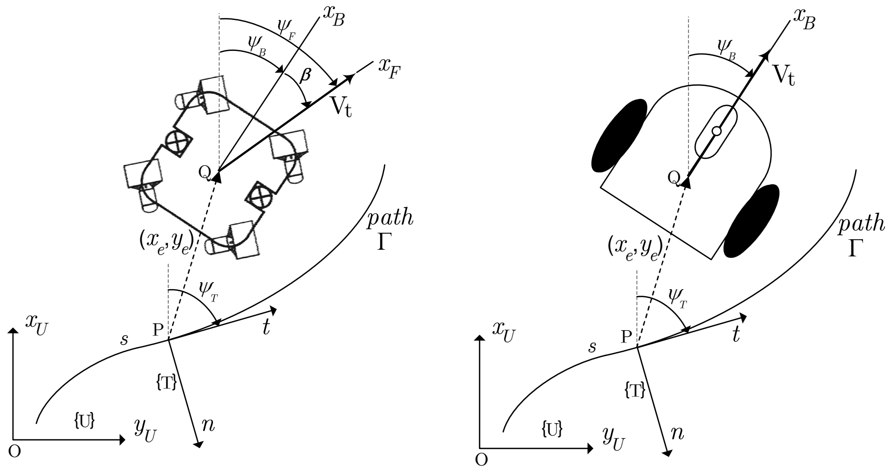
2 Path Following

This section describes a novel solution to the problem of path following for a single wheeled robot, as well as for a single fully actuated marine vehicle. The solution was first introduced in (Ghabcheloo *et al.*, 2005b) for wheeled robots and in (Ghabcheloo *et al.*, 2005c) for marine vehicles. Because of similarities, we first derive the equations of motion (kinematics and dynamics) for a single fully actuated marine vehicle and then we simplify them to arrive in the equations of motion of a wheeled robot. The control laws as well are derived in the same way.

Consider a fully actuated autonomous underwater vehicle depicted in Figure 1(a), together a spatial path Γ in the $x - y$ plane that must be followed. The vehicle propeller arrange is such that two forces in surge and sway and a torque in yaw can be generated independently. The problem of path following can now be briefly stated as follows:

Given a spatial path Γ, develop feedback control laws for the surge and sway forces and (yaw) torque acting on the vehicle so that its center of mass converges asymptotically to the path while its total speed and heading angle track desired temporal profiles. The latter requirement is equivalent to controlling the side-slip angle.

Consider Figure 1(a) where P is an arbitrary point on the path to be followed and Q is the center of the mass of the vehicle. Associated with P, consider the Serret-Frenet {T}. The signed curvilinear abscissa of P along the path is denoted by s. Clearly, Q can be expressed either as $\overrightarrow{OQ} = (x, y)$ in the inertial reference frame {U}, or as (x_e, y_e) in {T}. Let $\overrightarrow{OP}$ be the position



(a) Marine vehicle (b) Wheeled robot

Fig. 1. Frames and error variables

of P in {U} and define two frames with their origin at the center of mass of the vehicle: i) the *body-fixed frame* denoted {B} with its x-axis along the main axis of the body, and ii) the *flow frame* denoted {F} with its x-axis along the total speed V_t of the vehicle. Further let ${}^U_T R$, and ${}^U_F R$ denote the rotation matrices from {T} to {U} and from {F} to {U} parameterized by ψ_T and ψ_F, respectively. The yaw angle of the vehicle will be denoted ψ_B. Define the variables u and v as the surge and sway linear speeds, respectively and $r = \dot{\psi}_B$ as the angular speed of the vehicle. From the figure, it follows that $\overrightarrow{OQ} = \overrightarrow{OP} + {}^U_T R \begin{pmatrix} x_e \\ y_e \end{pmatrix}$. Taking derivatives and expressing the result in frame {U} yields

$$
{}^U_F R \begin{pmatrix} V_t \\ 0 \end{pmatrix} = {}^U_T R \begin{pmatrix} \dot{s} \\ 0 \end{pmatrix} + {}^U_T \dot{R} \begin{pmatrix} x_e \\ y_e \end{pmatrix} + {}^U_T R \begin{pmatrix} \dot{x}_e \\ \dot{y}_e \end{pmatrix},
$$

where

$$
\begin{pmatrix} V_t \\ 0 \end{pmatrix} = {}^F_B R \begin{pmatrix} u \\ v \end{pmatrix}. \tag{1}
$$

From the above expression, simple calculations lead to the kinematics of the vehicle in the (x_e, y_e) coordinates as

$$
\text{Kinematics} \begin{cases} \dot{x}_e = (y_e c_c(s) - 1)\dot{s} + V_t \cos\psi_e \\ \dot{y}_e = -x_e c_c(s)\dot{s} + V_t \sin\psi_e \\ \dot{\psi}_e = r - c_c(s)\dot{s} + \dot{\beta} \end{cases} \tag{2}
$$

where $\psi_e = \psi_F - \psi_T$ is the error angle, $\beta = \psi_F - \psi_B$ is the side-slip angle, and $c_c(s)$ is the path's curvature at P determined by s, that is $\dot{\psi}_T = c_c(s)\dot{s}$.

Notice how the kinematics are driven by V_t, r, and the term $\dot{s}$ that plays the role of an extra control parameter.

As shown in (Ghabcheloo *et al.*, 2005c) the dynamics of the vehicle can be rewritten in flow frame in terms of (V_t, β, r) as

$$\begin{aligned} \dot{V}_t &= f_{V_t}(V_t, \beta, r) + \tau_{V_t} \\ \dot{\beta} &= f_\beta(V_t, \beta, r) + \tau_\beta \\ \dot{r} &= f_r(V_t, \beta, r) + \frac{1}{m_r}\tau_r \end{aligned} \tag{3}$$

where τ_r is the torque control signal, and (τ_{V_t}, τ_β) are one-to-one functions of surge and sway forces. With this set-up, the problem of path following can be mathematically formulated as follows:

Problem 1 [Path following, Marine vehicle] *Given a spatial path Γ and desired time profiles $V_d(t)$ and $\beta_d(t)$ for the vehicle total speed V_t and side-slip angle β, respectively, derive a feedback control law for τ_{V_t}, τ_β, τ_r and $\dot{s}$ to drive x_e, y_e, ψ_e, $\beta - \beta_d$ and $V_t - V_d$ asymptotically to* 0.

Driving the speed V_t and β to their desired values is trivial to do with the simple control laws $\tau_{V_t} = -f_{V_t} + \dot{V}_d - k_0(V_t - V_d)$ and $\tau_\beta = -f_\beta + \dot{\beta}_d - k_0(\beta - \beta_d)$, which make the errors $V_t - V_d$ and $\beta - \beta_d$ decay exponentially to zero. Controlling V_t and β is therefore decoupled from the control of the other variables, and all that remains is to find suitable control laws for τ_r and for $\dot{s}$ to drive x_e, y_e, ψ_e to zero, no matter what the evolutions of $V_t(t)$ and β are. The only technical assumptions required are that the path be sufficiently smooth and that $\lim_{t\to\infty} V_t(t) \neq 0$. The main result of this Section is stated next.

Proposition 1 [Path following, Marine vehicle]. *Let Γ be a path to be followed by a fully actuated underwater vehicle. Further let the kinematic and dynamic equations of motion of the vehicle be given by (2) and (3), respectively. Assume $V_t(t)$ is uniformly continuous on $[0, \infty)$ and $\lim_{t\to\infty} V_t(t) \neq 0$. Define $\sigma = \sigma(y_e) = -\frac{2\psi_a}{\pi}\mathrm{sign}(V_t)\arctan y_e$ and $\phi = -\dot{\beta} + c_c\dot{s} + \dot{\sigma} - k_1(\psi_e - \sigma)$ for some $k_1 > 0$ and $0 < \psi_a \leq \pi/2$. Let the control laws for τ_r and $\dot{s}$ be given by*

$$\tau_r = m_r(-f_r + \dot{\phi} - k_2(r - \phi) - (\psi_e - \sigma)) \tag{4}$$

$$\dot{s} = V_t \cos\psi_e + k_3 x_e \tag{5}$$

for some $k_2, k_3 > 0$. Then, x_e, y_e, and ψ_e are driven asymptotically to zero from any initial condition.

See (Ghabcheloo *et al.*, 2004b) for a proof. Under the above conditions $\dot{s}$ tends to V_t, that is, the speed of the virtual target approaches V_t asymptotically. Furthermore, r approaches $c_c\dot{s} = c_cV_t$ as t increases.

Now, consider a wheeled robot of the unicycle type depicted in Figure 1(b), together a spatial path Γ in horizontal plane to be followed. The vehicle has two identical parallel, nondeformable rear wheels and a steering front wheel. The contact between the wheels and the ground is pure rolling and non-slipping. Each rear wheel is powered by a motor which generates a control

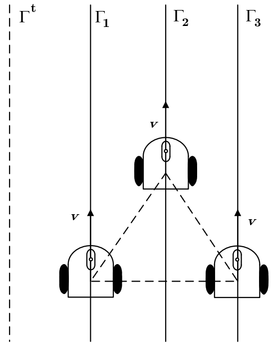


Fig. 2. Coordination: triangle formation

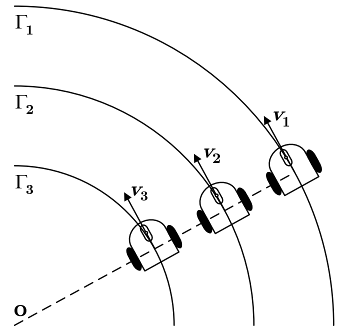


Fig. 3. Coordination: in-line formation

torque. This will in turn generate a control force and a control torque applied to the vehicle. Therefore a wheeled robot is an under actuated vehicle and only control force τ_{V_t} and (yaw) torque τ_r are available, however the side-slip angle is zero and obviously stable. Therefore making $V_t = u$, $v \equiv 0$, and $\beta \equiv 0$, we get the equations of the kinematics and the dynamics of a wheeled robot. Notice that the functions f_{V_t} and f_r are different for wheeled robot and for a simplified model of a wheeled robot can be considered zero, see for example (Ghabcheloo *et al.*, 2005b). Furthermore, driving the speed u to the desired value is trivial (as explained before for marine vehicle) and is therefore decoupled from the control of the other variables. Therefore the control laws τ_r and $\dot{s}$ for wheeled robot are derived in the same way using those given in (4) and (5).

3 Coordination

Equipped with the results obtained in the previous section, we now consider the problem of coordinated path following control that is the main contribution of the present paper. In the most general set-up, one is given a set of $n \geq 2$ wheeled robots (or fully actuated marine vehicle) and a set of n spatial paths Γ_k; $k = 1, 2, ..., n$ and require that robot k follow path Γ_k. We further require that the vehicles move along the paths in such a way as to maintain a desired formation pattern compatible with those paths. The speeds at which the vehicles are required to travel can be imposed in a number of ways; for example, by nominating one of the vehicles as a formation leader, assigning it a desired speed, and having the other vehicles adjust their speeds accordingly. Figures 2 and 3 show the simple cases where 3 vehicles are required to follow the straight paths or circumferences $\Gamma_i; i = 1, 2, 3$ while keeping a desired "triangle" or "in-line" formation pattern.

In the simplest case, the paths Γ_i may be obtained as simple parallel translations of a "template" path Γ^t – Figure 2. A set of paths can also be

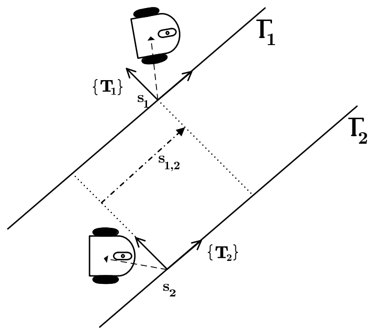


Fig. 4. Along-path distances: straight lines

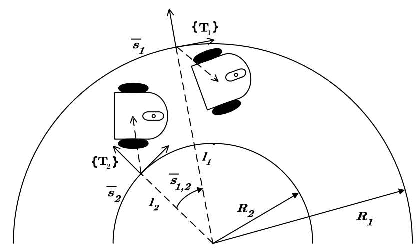


Fig. 5. Along-path distances: circumferences

obtained by considering the case of scaled circumferences with a common center and different radii R_i – Figure 3.

Assuming that separate path following controllers have been implemented for each vehicle, it now remains to coordinate (that is, synchronize) them in time so as to achieve a desired formation pattern. As will become clear, this will be achieved by adjusting the speeds of the vehicles as functions of the "along-path" distances among them. To better grasp the key ideas involved in the computation of these distances, consider for example the case of in-line formations maneuvering along parallel translations of straight lines. For each robot i, let s_i denote the signed curvilinear abscissa of the origin of the corresponding Serret-Frenet frame $\{\mathrm{T}_i\}$ being tracked, as introduced in the previous section. Since each vehicle's flow frame $\{\mathrm{F}_i\}$ tends asymptotically to $\{\mathrm{T}_i\}$, it follows that the vehicles are (asymptotically) synchronized if

$$s_{i,j}(t) := s_i(t) - s_j(t) \rightarrow 0, t \rightarrow \infty; i = 1, .., n; i < j \leq n. \tag{6}$$

In the case of wheeled robots, notice that body-frame and flow frame are coincide. This shows that in the case of translated straight lines $s_{i,j}$ is a good measure of the along-path distances among the robots. Similarly, in the case of scaled circumferences an appropriate measure of the distances among the robots is

$$\bar{s}_{i,j} := \bar{s}_i - \bar{s}_j; i = 1, .., n; i < j \leq n \tag{7}$$

where $\bar{s}_i = s_i/R_i$. See Figures 4 and 5.

Notice how the definition of $\bar{s}_{i,j}$ relies on a normalization of the lengths of the circumferences involved and is equivalent to computing the angle between vectors l_i and l_j directed from the center of the circumferences to origin of the Serret-Frenet frames $\{\mathrm{T}_i\}$ and $\{\mathrm{T}_j\}$, respectively. In both cases, we say that the vehicles are coordinated if the corresponding along path distance is zero, that is, $s_i - s_j = 0$ or $\bar{s}_i - \bar{s}_j = 0$. The extension of these concepts to a more general setting requires that each path Γ_i be parameterized in terms of a parameter ξ_i that is not necessarily the arc length along the path. An

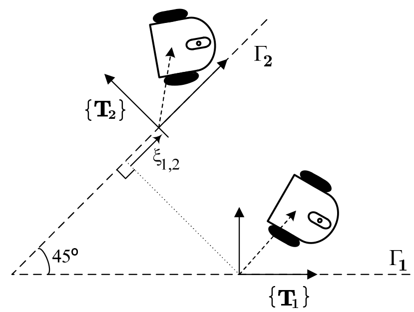


Fig. 6. A general coordination scheme

adequate choice of the parameterization will allow for the conclusion that the vehicles are synchronized iff $\xi_i = \xi_j$ for all i, j. For example, in the case of two robots following two circumferences with radii R_1 and R_2 while keeping an in-line formation pattern, $\xi_i = s_i/R_i; i = 1, 2$. This seemingly trivial idea allows for the study of more elaborate formation patterns. As an example, consider the problem depicted in Figure 6 where vehicles 1 and 2 must follow paths Γ_1 and Γ_2 while maintaining vehicle 2 "to-the-left-and-behind" vehicle 1, that is, along straight lines that make an angle of 135 degrees with the positive direction of path Γ_1. Let $\xi_1 = s_1$ and $\xi_2 = s_2\sqrt{2}$. It is clear the vehicles are synchronized if $\xi_1 - \xi_2 = 0$. Since the objective of the coordination is to synchronize ξ_i's, we sometimes refer to them as *coordination states*.

The above considerations motivate the mathematical development that follows. We start by computing the coordination error dynamics, after which a decentralized feedback control law is derived to drive the coordination error to zero asymptotically. In the analysis, graph theory - as the mathematical machinery par excellence to deal with inter-vehicle communication constraints - will play a key role.

3.1 Coordination error dynamics

To simplify the notation, we adopt a slightly different one in this section. In the sequel, the dynamics of total speed (3) of the i'th vehicle will be defined as

$$\dot{v}_i = F_i \tag{8}$$

where $F_i = \tau_{V_{ti}} + f_{V_{ti}}$ and v_i stands for the total speed V_{t_i}.

As before, we let the path Γ_i be parameterized by ξ_i and denote by $s_i = s_i(\xi_i); i = 1, 2, ..., n$ the corresponding arc length. We define $R_i(\xi_i) = \partial s_i / \partial \xi_i$ and assume that $R_i(\xi_i)$ is positive and uniformly bounded for all ξ_i. In particular, s_i is a monotonically increasing function of ξ_i. We further assume that all $R_i(\xi_i)$ is bounded away from 0 and that $\partial R_i / \partial \xi_i$ is uniformly bounded. The symbol $R_i(.)$ is motivated by the nomenclature adopted before for the case of paths that are nested arcs of circumferences. Using equation (5), it is straightforward to show that the evolution of ξ_i is given by

$$\dot{\xi}_i = \frac{1}{R_i(\xi_i)}(v_i \cos\psi_{ei} + k_{3i}x_{ei}) \tag{9}$$

which can be re-written as

$$\dot{\xi}_i = \frac{1}{R_i(\xi_i)}v_i + d_i \tag{10}$$

where

$$d_i = \frac{1}{R_i(\xi_i)}(\cos\psi_{ei} - 1)v_i + k_{3i}x_{ei}. \tag{11}$$

Notice from the previous section that $d_i \to 0$ asymptotically as $t \to \infty$, if v_i is bounded. It can be shown that this assumption is indeed met, see (Ghabcheloo *et al.*, 2004b). Suppose one vehicle, henceforth referred to as vehicle $\mathcal{L}$, is elected as a "leader" and let the corresponding path $\Gamma_{\mathcal{L}}$ be parameterized by its length, that is, $\xi_{\mathcal{L}} = s_{\mathcal{L}}$. In this case, $R_{\mathcal{L}}(\xi_{\mathcal{L}}) = 1$. It is important to point out that $\mathcal{L}$ can always be taken as a "virtual" vehicle that is added to the set of "real" vehicles as an expedient to simplify the coordination strategy. Let $v_{\mathcal{L}} = v_{\mathcal{L}}(t)$ be a desired speed profile assigned to the leader in advance, that is $\dot{\xi}_{\mathcal{L}} = v_{\mathcal{L}}$, and known to all the other vehicles. Notice now that in the ideal steady situation where the vehicles move along their respective paths while keeping the desired formation, we have $\xi_i - \xi_{\mathcal{L}} = 0$ and therefore $\dot{\xi}_i = v_{\mathcal{L}}$ for all $i = 1, .., n$. Thus, $v_{\mathcal{L}}$ becomes the desired speed of each of the vehicles, expressed in ξ_i coordinates. As such, *one can proceed without having to resort to the concept of an actual or virtual leader vehicle, thus making the coordination scheme truly distributed.*

From (10), making $d_i = 0$, it follows that the desired inertial velocities of vehicles $1 \le i \le n$ equal $R_i(\xi_i)v_{\mathcal{L}}(t)$. This suggests the introduction of the speed-tracking error vector

$$\eta_i = v_i - R_i(\xi_i)v_{\mathcal{L}}, \;\; 1 \le i \le n. \tag{12}$$

Taking into account the vehicle dynamics (8), the derivative of (12) yields

$$\dot{\eta}_i = f_i = F_i - \frac{d}{dt}\left(R_i(\xi_i)v_{\mathcal{L}}\right). \tag{13}$$

Using (10), it is also easy to compute the dynamics of the origin of each Serret-Ferret frame $\{\mathrm{T}_i\}$ as

$$\dot{\xi}_i = \tfrac{1}{R_i}\eta_i + v_{\mathcal{L}} + d_i. \tag{14}$$

To write the above dynamic equations in vector form, define $\eta = [\eta_i]_{n\times 1}$, $\xi = [\xi_i]_{n\times 1}$, $f = [f_i]_{n\times 1}$, $d = [d_i]_{n\times 1}$ and $C = C(\xi) = \mathrm{diag}[1/R_i(\xi_i)]_{n\times n}$ to obtain

$$\begin{aligned}\dot{\eta} &= f\\ \dot{\xi} &= C\eta + v_{\mathcal{L}}\mathbf{1} + d\end{aligned} \tag{15}$$

where $\mathbf{1} = [1]_{n\times 1}$. In the above, $||d|| \to 0$ asymptotically as $t \to \infty$ and matrix C is positive definite and bounded, that is,

$$0 < c_1 I \leq C(\xi(t)) \leq c_2 I \tag{16}$$

for all t, where c_1 and c_2 are positive scalars and I the identity matrix. Notice that C is allowed to be (state-driven) time-varying, thus allowing for more complex formation patterns than those in the motivating examples of the previous section.

The objective is to derive a control strategy for f to make $\xi_1 = ... = \xi_n$ or, equivalently, $(\xi_i - \xi_j) = 0$ for all i, j. At this point, however, two extremely important control design constraints must be taken into consideration. The first type of constraints is imposed by the topology of the inter-vehicle communications network (that is, by the types of links available for communication). The second type of constraints arises from the need to drastically reduce the amount of information that is exchanged over the communications network. In this paper, it will be assumed that the vehicles only exchange information on their positions and speeds. The case where only position information is exchanged leads to more complex coordination control laws and will not be examined here.

A possible control law is of the form

$$f_i = f_i(\eta_i, \xi_i, \eta_j, \xi_j : j \in J_i) \tag{17}$$

where J_i is the index set (of the neighbors) that determines what coordination states ξ_j and speeds η_j; $j \neq i$ are transmitted to vehicle i. With this control law, each vehicle i requires only access to its own speed and coordination state and to some or all of the coordination states of the remaining vehicles, as defined by the index set J_i. Throughout the paper, we assume that the *communication links are bidirectional*, that is, if vehicle i sends information to j, then j also sends information to i. Formally, $i \in J_j \Leftrightarrow j \in J_i$. See (Ghabcheloo *et al.*, 2005b) for the case of unidirectional communication links where some of the vehicles may only send or receive information. Clearly, the index sets capture the type of communication structure that is available for vehicle coordination. This suggests that the vehicles and the data links among them be viewed as a graph where the vehicles and the data links play the role of vertices of the graph and edges connecting those vertices, respectively. It is thus natural that the machinery of graph theory be brought to bear on the definition of the problem under study.

3.2 Graphs

We summarize below some key concepts and results of graph theory that are relevant to the paper. See for example (Biggs, 1996), (Godsil and Royle, 2001), (Balakrishnan and Ranganathan, 2000), and the references therein.

Basic Concepts and Results

An *undirected graph* or simply a graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ (abbv. $\mathcal{G}$) consists of a set of *vertices* $V_i \in \mathcal{V}(\mathcal{G})$ and a set of *edges* $\mathcal{E}(\mathcal{G})$, where an edge $\{V_i, V_j\}$ is an unordered pair of distinct vertices V_i and V_j in $\mathcal{V}(\mathcal{G})$. A *simple graph* is a graph with no edges from one vertex to itself. In this paper we only consider simple graphs, and will refer to them simply as graphs. As stated before, in the present work the vertices and the edges of a graph represent the vehicles and the data links among the vehicles, respectively. If $\{V_i, V_j\} \in \mathcal{E}(\mathcal{G})$, then we say that V_i and V_j are *adjacent* or *neighbors*. A *path* of length N from V_i to V_j in a graph is a sequence of $N+1$ distinct vertices starting with V_i and ending with V_j, such that two consecutive vertices are adjacent. The graph $\mathcal{G}$ is said to be *connected* if two arbitrary vertices V_i and V_j can be joined by a path of arbitrary length.

The adjacency matrix of a graph $\mathcal{G}$, denoted $\mathbb{A}$, is a square matrix with rows and columns indexed by the vertices, such that the i,j-entry of $\mathbb{A}$ is 1 if $\{V_i, V_j\} \in \mathcal{E}$ and zero otherwise. The degree matrix $\mathbb{D}$ of a graph $\mathcal{G}$ is a diagonal matrix where the i,i-entry equals the *valency* of vertex V_i, that is $|J_i|$ the cardinality of J_i. The Laplacian of a digraph is defined as $L = \mathbb{D} - \mathbb{A}$.

Given any arbitrary vector ξ, if $y = L\xi$, then the i'th element of y is

$$y_i = \sum_{j \in J_i} (\xi_i - \xi_j),$$

that is, y_i is a linear combination of the terms $(\xi_i - \xi_j)$, where j spans the set J_i of vehicles that i communicates with. This seemingly trivial point plays a key role in the computation of a decentralized coordination control law that takes into consideration the a priori existing communication constraints, as will become clear later.

3.3 Coordination. Problem formulation and solutions

Equipped with the above machinery we now state the coordination problem that is the main focus of this section. First, however we comment on the type of communication constraints considered in the paper. It is assumed that: i) the *communications are bidirectional (L is symmetric)* and ii) the *communications graph is connected*. Notice that if assumption (ii) is not verified, then there are two or more clusters of vehicles and no information is exchanged among the clusters. Clearly, in this situation no coordination is possible.

Problem 2 [Coordination]. *Consider the coordination system with dynamics (15) and assume that d tends asymptotically to $\mathbf{0}$. Further assume that each of the n vehicles has access to its own state and exchanges information on its path parameter (coordination state) ξ_i and speed η_i with some or all of the other vehicles. Let $\mathcal{G}$ be a graph with n vertices and ϵ edges, where the presence of an edge between* *vertex i and j signifies that vehicle i and j communicate through a bidirectional link. Determine a feedback control law for u such that* $\lim_{t\to\infty}\eta = \mathbf{0}$ *and* $\lim_{t\to\infty}(\xi_i - \xi_j) = 0$ *for all* $i, j = 1, .., n$.

The next proposition offers a solution to the coordination problem, under the basic assumption that the communications graph $\mathcal{G}$ is connected.

Proposition 2 [Solution to the coordination problem]. *Consider the coordination problem described before and assume that the communications graph $\mathcal{G}$ is connected. Let L be the Laplacian of $\mathcal{G}$. Further let $A = \mathrm{diag}[a_i]_{n\times n}$ and $B = \mathrm{diag}[b_i]_{n\times n}$ be arbitrary positive definite diagonal matrices. Then, the control law*

$$f = -(A^{-1}L + A)C\eta - B\,\mathrm{sat}(\eta + A^{-1}L\xi), \tag{18}$$

where sat *is the saturation function*

$$\mathrm{sat}(x) = \begin{cases} x_m & x > x_m \\ x & |x| \le x_m \\ -x_m & x < -x_m \end{cases} \tag{19}$$

with $x_m > 0$ arbitrary, solves the coordination problem. Namely, the control law meets the communication constraints and yields input to state stability (ISS) from the input d to all the states [3].

Because of space limitation, the proof is omitted, see (Ghabcheloo *et al.*, 2004b). The control law adopted for vehicle i can be written as

$$f_i = -\frac{a_i}{R_i}\eta_i - \frac{1}{a_i}\sum_{j\in J_i}(\frac{1}{R_i}\eta_i - \frac{1}{R_j}\eta_j) - b_i\,\mathrm{sat}(\eta_i + \frac{1}{a_i}\sum_{j\in J_i}(\xi_i - \xi_j)). \tag{20}$$

We recall that J_i denotes the set of vehicles (vertices in the graph) that communicate with vehicle i. Notice how the control input of vehicle i is a function of its own speed and coordination state as well as of the coordination states and speeds of the other vehicles included in the index set J_i. Clearly, the control law is decentralized and meets the constraints imposed by the communications network, as required.

Matrices A and B play the role of tuning knobs aimed at shaping the behavior of the coordination system. Notice that the coordination vector ξ appears inside the s*at* function. From the form of control law, it is clear that the s*at* function affords the system designer an extra degree of freedom because as x_m increases, the control activity f becomes more "responsive" to vector ξ (intuitively, as x_m increases, the coordination dynamics become "faster"). Interestingly enough, the introduction of the s*at* function allows for a simple proof that $V_t(t)$ remains bounded when the path following and coordination systems are put together.

[3] See (Sontag, 1996) and (Khalil, 2002) for the definition of input to state stability of a system.

4 Simulations

This section contains the results of simulations that illustrate the performance obtained with the coordinated path following control laws developed in the paper for both wheeled robot and marine vehicle. Figure 7 illustrates the situation where 3 wheeled robots are required to follow paths that consist of parallel straight lines and nested arcs of circumferences (C piecewise constant). The figure corresponds to the case of an in-line formation pattern. In the simulation, vehicle 1 is allowed to communicate with vehicles 2 and 3, but the last two do not communicate between themselves directly. The reference speed $v_{\mathcal{L}}$ was set to $v_{\mathcal{L}} = 0.1$ [m s^{-1}]. Notice how the vehicles adjust their speeds to meet the formation requirements. Moreover, the coordination errors $\xi_{12} = \xi_1 - \xi_2$ and $\xi_{13} = \xi_1 - \xi_3$ and the path following errors decay to 0.

Figure 8 illustrates a different kind of coordinated maneuver in the $x - y$ plane: one robot is required to follow the $x-$axis, while the other must follow a sinusoidal path as the two maintain an in-line formation along the $y-$axis. In this case, C is time varying. Notice in Figure 8(b) how vehicle 1 adjusts its speed along the path so as to achieve coordination. As seen in Figures 8(c) and 8(d), the vehicles converge to the assigned paths and drive the error between their $x-$coordinates to 0.

Figure 9 corresponds to a simulation where 3 fully actuated marine vehicles were required to follow 3 straight parallel lines 3 meters apart, in the horizontal plane. In the simulation, the parameters of the SIRENE AUV were used (Aguiar, 2002). Moreover, they were required to keep an in-line formation pattern. As in the first simulation, vehicle 1 is allowed to communicate with vehicles 2 and 3, but the last two do not communicate between themselves directly. The reference speed $v_{\mathcal{L}}$ was set to $v_{\mathcal{L}} = 0.2$ [m s^{-1}] and the initial states to

$$\begin{aligned}
&(u, v) = (0.1, 0), (0.1, 0), (0.1, 0)[\text{m/s}] \\
&(x, y) = (0, 7), (0.5, 2), (0, -5)[\text{m}] \\
&\beta = 0, 0, 0; \text{and } \beta_d = 10, 0, -10[\text{deg}].
\end{aligned}$$

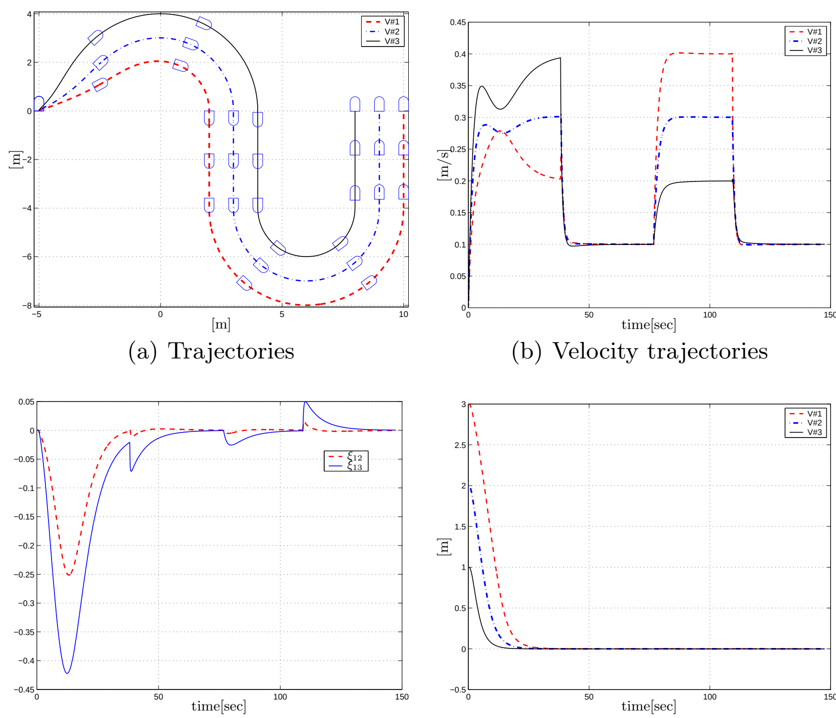
Figure 9(a) shows the evolution of the vehicles as they start from the initial points off the assigned paths and converge to them. Figure 9(b) is a plot of the vehicle speeds that ensure coordination along the paths. Finally, Figure 9(c) shows the coordination errors $\xi_1 - \xi_2$, $\xi_2 - \xi_3$ and $\xi_3 - \xi_1$ decaying to 0 and Figure 9(d) the side-slip angles converging to the desired values.
Controller parameters were set to

$$a = 2; b = 1; k_1 = 0.5; k_2 = 0.5; k_3 = 0.25; \psi_a = \pi/4, \tag{21}$$

in all the simulations.

5 Conclusions and suggestions for further research

The paper formulated and presented a solution to the problem of steering a fleet of wheeled robots along a set of given spatial paths, while keeping a desired inter-vehicle formation pattern. The solution adopted for coordinated path following builds on Lyapunov based techniques and addresses explicitly the constraints imposed by the topology of the inter-vehicle communications network. With this set-up, path following (in space) and inter-vehicle coordination (in time) are essentially decoupled. Path following for each vehicle amounts to reducing a conveniently defined error variable to zero. Vehicle coordination is achieved by adjusting the speed of each of the vehicles along its path, according to information on the position of the other vehicles, as determined by the communications topology adopted. The methodology proposed led to a decentralized control law whereby the exchange of data among the vehicles is kept at a minimum. Simulations illustrated the efficacy of the solution proposed. Further work is required to extend the methodology proposed to air and underwater vehicles. Namely, by addressing the problems of robustness against temporary communication failures.



(a) Trajectories

(b) Velocity trajectories

(c) Coordination errors

(d) Path following errors (y_e)

Fig. 7. Three wheeled robots, piecewise constant C

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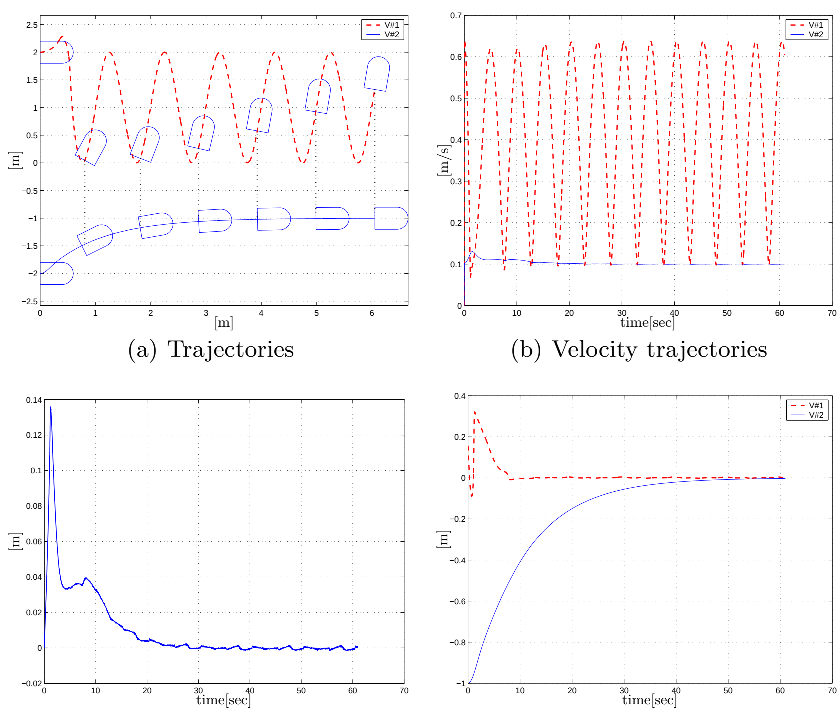

(a) Trajectories

(b) Velocity trajectories

(c) Coordination error

(d) Path following errors (y_e)

Fig. 8. Coordination of 2 wheeled robots, varying C

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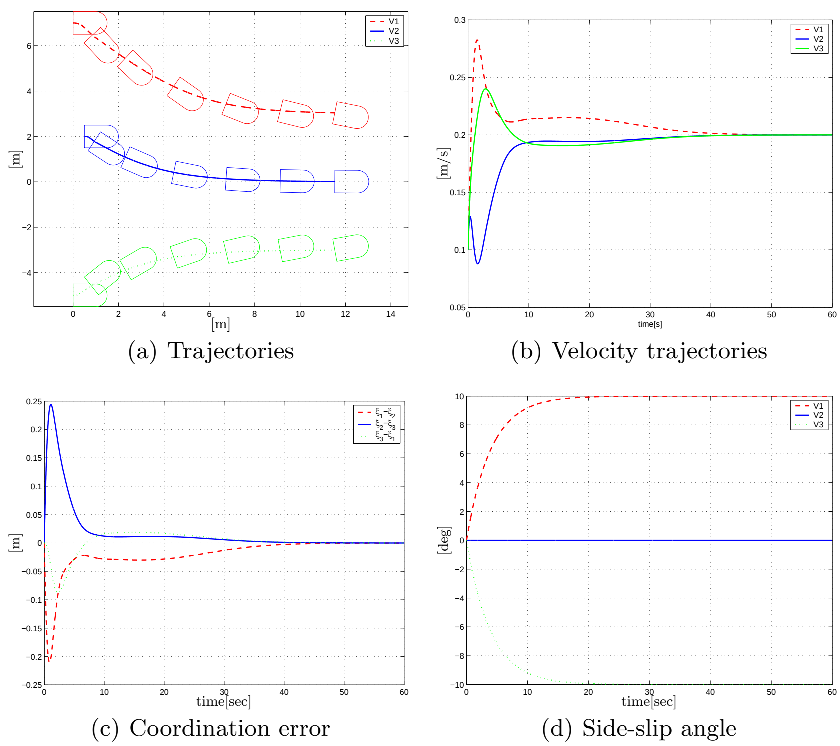

Fig. 9. Coordination of 3 marine vehicle along straight lines

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