ON THE DESIGN OF MULTIRATE COMPLEMENTARY FILTERS FOR AUTONOMOUS MARINE VEHICLE NAVIGATION [1]

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Abstract. A new methodology is proposed for the design of navigation systems for autonomous marine vehicles. Using simple kinematic relationships, the problem of estimating the velocity and position of an autonomous vehicle based on motion sensor data available at different rates is solved by resorting to complementary multirate time-varying filters. The set-up adopted for filter design and analysis builds on Linear Matrix Inequalities (LMIs) and on efficient numerical analysis tools that borrow from convex optimization techniques. The paper describes the key steps involved in the design of a multirate navigation system for a prototype autonomous marine vehicle. Results of tests at sea illustrate the performance of the system developed.

Keywords. Autonomous Underwater Vehicles, Navigation Systems, Multirate Filters, Linear Matrix Inequalities.

1. INTRODUCTION

There is currently considerable interest in the development of navigation systems to provide robotic vehicles with the capability to perform complex missions at sea. See (Stambaugh and Thibault, 1992; Fryxell *et al.*, 1994) and the references therein for in-depth presentations of navigation systems for marine vehicles. See also (Lin, 1991; Kayton and Fried, 1969) for an overview of similar systems and related research issues in aircraft navigation. Navigation system design is usually done in a stochastic setting by resorting to Kalman-Bucy filtering theory (Brown and Hwang, 1992). The stochastic setting requires a complete characterization of process and observation noises, a task that may be difficult, costly, or not suited to the problem at hand. This issue is argued at length in (Brown, 1992), where the author points out that in a great number of practical applications filter design is entirely dominated by constraints that are naturally imposed by the sensor bandwidths. In this case, a design method that explicitly addresses the problem of merging information provided by a given sensor suite over distinct, yet *complementary* frequency regions is warranted.

When motion sensor data are available at exactly the same rate, the corresponding complementary filters are time invariant. This in turn leads to a fruitful interpretation of these filters in the frequency domain. In the

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case of linear position and velocity estimation, however, the characteristics of the sensor suites used are such that sensor data become available at different rates and the resulting filters become time-varying. This occurs for example in the case of AUV (Autonomous Underwater Vehicle) Navigation, when the sensor suite consists of a Doppler Velocity Log (DVL), an Attitude and Heading Reference (AHR) unit, and an acoustic based Longbaseline (LBL) system. Another example is the case of ASV (Autonomous Surface Vehicle) Navigation, where one typically resorts to motion sensor data available from a DVL, an AHR, and a Differential Global Positioning System (DGPS).

The main purpose of this paper is to show how, using simple kinematic relationships, the problem of estimating the linear velocity and position of an autonomous marine vehicle can be posed and solved by resorting to multirate complementary filters. These are the natural generalization (to a multirate setting) of linear time-invariant complementary filters that are widely used to properly merge sensor information available at low frequency with that available in the complementary region. The key results of the paper also imply that the resulting multirate input-output operators from measured to estimated variables exhibit "frequency-like" properties that are the generalization of those obtained for the single-rate case. This striking property plays a crucial role during the navigation system design phase, for it provides valuable insight into the selection of closed loop filter "bandwidths" so as to match the natural bandwidths of the sensors.

The paper exploits well known results that allow for the characterization of multirate filters as equivalent periodic ones. See (Oliveira, 2002) and the references therein. Once in a periodic setting, the filter design process builds on two main facts: i) filter performance can be evaluated by resorting to $\mathcal{H}_2$ and $\mathcal{H}_\infty$ criteria (the first is closely related to the criteria that are commonly used for filter design in a stochastic setting, whereas the latter capture constraints on estimation error bounds); ii) the computation of the above criteria can be done in an expedite manner by using the theory of Linear Matrix Inequalities (LMIs) (Boyd *et al.*, 1994), which have become the tool par excellence to deal with seemingly unwieldy dynamical system design problems. In this framework, filter design and "frequency-like" analysis is simply done by determining the feasibility of a related set of linear matrix inequalities. The latter problem is solved by resorting to commercially available numerical tools that borrow from convex optimization theory (MATLAB, 1997). In this paper, the new methodology proposed for filter design is applied to the development of a multirate navigation system for an autonomous surface craft. Results of tests at sea illustrate the performance of the system developed. Due to space limitations, only the key steps in the design procedure are briefly summarized here. The interested reader is referred to (Oliveira, 2002) for complete details.

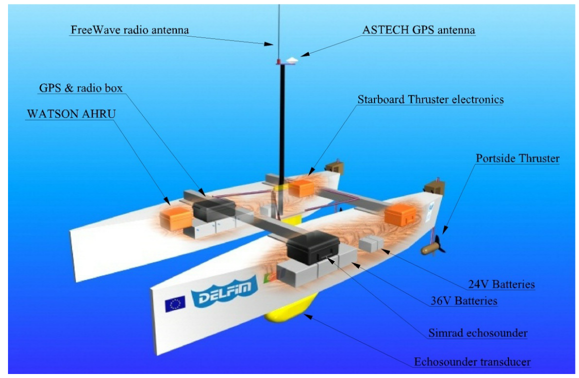

Fig. 1. The DELFIM autonomous marine vehicle.

The paper is organized as follows. Section 2 motivates the problem of multirate navigation system design for marine vehicles. Section 3 formulates the corresponding filtering problem, outlines its solution, and includes results of tests at sea with an autonomous surface vehicle (ASV). The paper concludes with a short discussion of the results obtained and of issues that warrant further research.

2. PROBLEM MOTIVATION. A NAVIGATION SYSTEM FOR THE DELFIM ASV.

This section motivates the problem of multirate filtering by presenting the objectives that were set in the design of a navigation sensor for a prototype autonomous surface vehicle named DELFIM, see figure 1. Notice however that the methodology for navigation system design presented in the paper applies to the case of other marine vehicles, including autonomous underwater vehicles (AUVs).

The DELFIM ASV was designed and built by the Institute for Systems and Robotics of the Instituto Superior Técnico to carry out automatic marine data acquisition and to serve as an acoustic relay between submerged craft and a support vessel (ASIMOV, 2000). The DELFIM ASV can also be used as a stand-alone unit, capable of maneuvering autonomously and performing precise path following while carrying out automatic marine data acquisition and transmission to an operating center installed on-board a support vessel or on-shore. This is in line with the current trend to develop systems that will lower the costs and improve the efficiency of operation of oceanographic vessels at sea.

The DELFIM is a small Catamaran 3.5 m long and 2.0 m wide, with a mass of 320 kg. The propulsion system consists of two propellers driven by electrical motors. The vehicle is equipped with on-board resident systems for navigation, guidance, and control, as well as for mission control. Navigation is done by integrating motion sensor data obtained from an attitude reference unit, a Doppler log, and a DGPS (Differential Global Positioning System) receiver. Its guidance and control systems consist of simplified versions of the $s - y$ controller described in (Silvestre, 2000).

Transmissions between the vehicle, its support vessel, the fixed GPS station, and the on-shore control center installed on-shore are achieved by means of a radio link with a range of 80 km. The vehicle has a wing shaped central structure that is lowered during operations at sea. At the bottom of this structure a hydrodynamically shaped body is installed that carries all acoustic transducers, including those used to communicate with the underwater craft.

In what follows, $\{I\}$ denotes a fixed reference frame located at the origin of a pre-specified mission area and $\{S\}$ is a body-fixed coordinate that moves with the ASV. The vehicle motion is subject to the influence of a constant unknown current ${}^{I}\mathbf{v}_{_W} = [{}^{I}u_{_W}\, {}^{I}v_{_W}\, 0]^T$ expressed in $\{I\}$. The symbol $\{W\}$ denotes a coordinate frame that moves along with the current. The following additional notation is required:

${}^{I}\mathbf{p}_S := [{}^{I}x_s\ {}^{I}y_s\ {}^{I}z_s]^T$ - position of the origin of $\{S\}$ measured in $\{I\}$;

${}^{I}\mathbf{v}_S := [{}^{I}u_s\, {}^{I}v_s\, {}^{I}w_s]^T$ - velocity of the origin of $\{S\}$ with respect to the fixed frame $\{I\}$;

$\mathbf{v}_{S,W} := [u_{s,w}\, v_{s,w}\, w_{s,w}]^T$ - relative velocity of the origin of $\{S\}$ with respect to coordinate frame $\{W\}$;

$\lambda := [\phi\ \ \theta\ \ \psi]^T$ - vector of of roll, pitch, and yaw angles that parametrize locally the orientation of $\{S\}$ relative to $\{I\}$;

${}^{I}_{S}R\,(\lambda)$ - rotation matrix from $\{S\}$ to $\{I\}$.

With this notation, the relevant kinematics of the ASC can be written in compact form as

$$\frac{d}{dt}{}^{I}\mathbf{p}_S = {}^{I}_{S}R\,(\lambda)^{S}(\mathbf{v}_{S,W}) +^{I} \mathbf{v}_{_W}, \qquad (1)$$

where ${}^{S}(\mathbf{v}_{S,W})$ is the vector $\mathbf{v}_{S,W}$ expressed in $\{S\}$ and

$${}^{I}_{S}R\,(\lambda)^{S}(\mathbf{v}_{S,W}) := {}^{I}(\mathbf{v}_{S,W}) = [{}^{I}u_{s,w}\ \ {}^{I}v_{s,w}\ \ {}^{I}w_{s,w}]^T$$

is the velocity of the origin of $\{S\}$ with respect to the water, expressed in $\{I\}$.

Given a variable x, its measurement will be denoted x_m or $(x)_m$. The ASV sensor suite yields the following measurements:

(1) $({}^{I}\mathbf{p}_S)_m := [({}^{I}x_s)_m\ ({}^{I}y_s)_m\ ({}^{I}z_s)_m]^T$ - obtained by a Differential Global Positioning System with the mobile segment on board the ASV.
(2) $\boldsymbol{\lambda}_m$ - vector of roll, pitch, and yaw angles provided by the attitude reference unit.
(3) $({}^{I}\mathbf{v}_{S,W})_m := [({}^{I}u_{s,w})_m\ \ ({}^{I}v_{s,w})_m\ \ ({}^{I}w_{s,w})_m]^T$ - obtained by rotating the Doppler log measurements ${}^{S}(\mathbf{v}_{S,W})_m$ through matrix ${}^{I}_{S}R\,(\lambda_m)$.

In this example, the interrogation rates for the GPS unit and the Doppler sonar are $2\,Hz$ and $4\,Hz$, respectively. The sampling rate of vector λ is much larger and does not play a significant role in the development below. Consider the following guidelines for the design of a navigation system for the ASV:

(1) Obtain accurate estimates

$${}^{I}\hat{\mathbf{p}}_S = \left[\, {}^{I}\hat{x}_s\ \ {}^{I}\hat{y}_s\ \ {}^{I}\hat{z}_s \,\right]^T$$

and

$$\hat{\mathbf{v}}_S = \left[\, {}^{I}\hat{u}_s\ \ {}^{I}\hat{v}_s\ \ {}^{I}\hat{w}_s \,\right]^T$$

of the vehicle's position and velocity vectors, respectively;
(2) Achieve a settling time of approximately 240 s on the estimate of the water current ${}^{I}\mathbf{v}_{_W}$.
(3) Achieve a settling time of approximately 6 s on the position estimate.

It is now easy to derive the the model for the design of a navigation system to complement the data acquired by the GPS unit with that obtained by the Doppler log. See (Fryxell *et al.*, 1994) and the references therein. Straightforward manipulations of the ASV kinematic equations lead to three sets of decoupled equations that correspond to the three linear coordinates x, y, and z. See figure 2 for the design model that captures the discretized motion of the ASV along the coordinate $x-$coordinate.

The output integrator captures the relationship $\frac{d}{dt}{}^{I}\mathbf{p}_S = {}^{I}(\mathbf{v}_{S,W}) +^{I} \mathbf{v}_{_W}$. The input integrator was inserted to estimate the water current, which is assumed constant. Let $x_1 = {}^{I}\mathbf{p}_S$ and $x_2 = {}^{I}\mathbf{v}_{\mathcal{W}}$. Adopting the basic sampling period $h = 0.25\ s$, the design model admits the realization

$$\Sigma_{\mathcal{G}_2} = \begin{cases} \mathbf{x}(k+1) &= A(k)\mathbf{x}(k) + B_u(k)\mathbf{w}(k) \\ \mathbf{z}(k) &= C_z(k)\mathbf{x}(k) \\ \mathbf{y}(k) &= C_y(k)\mathbf{x}(k) \end{cases}, \qquad (2)$$

where $\mathbf{x} = \left[\, x_1\ x_2 \,\right]^T$, $\mathbf{w} = ({}^{I}u_{s,w})_m$, and $\mathbf{z} = \mathbf{y} = ({}^{I}x_s)_m$. Furthermore,

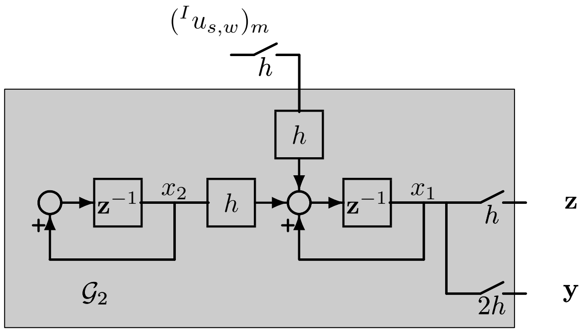

Fig. 2. Position estimation: filter design model.

$$A(k) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix}, \quad B_u(k) = \begin{bmatrix} h \\ 0 \end{bmatrix},$$

$$C_y(k) = \begin{cases} \begin{bmatrix} 1 & 0 \end{bmatrix}^T & \text{if } k\ MOD\ M = 0 \\ \begin{bmatrix} 0 & 0 \end{bmatrix}^T & \text{if } k\ MOD\ M = 1 \end{cases},$$

and $C_z(k) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$, where $M = 2$. *Notice how this multirate model can be simply viewed as a periodic system* with period $M = 2$, the periodic nature of the system being clear from the analysis of matrix $C_y(k)$. The next section addresses the problem of periodic filter design given the design model described above.

3. FILTER DESIGN. EXPERIMENTAL RESULTS.

The general setup for estimation design for periodically time-varying, discrete-time systems consists of the interconnections presented in figure 3, and is by now standard.

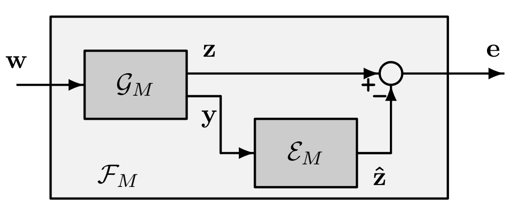

Fig. 3. General setup for periodic filtering synthesis.

The design model $\mathcal{G}_M$ is a linear, periodically time-varying discrete-time system with realization

$$\Sigma_{\mathcal{G}_M} = \begin{cases} \mathbf{x}(k+1) = A(k)\mathbf{x}(k) + B_w(k)\mathbf{w}(k) \\ \mathbf{z}(k) = C_z(k)\mathbf{x}(k) + D_{zw}(k)\mathbf{w}(k) \\ \mathbf{y}(k) = C_y(k)\mathbf{x}(k) + D_{yw}(k)\mathbf{w}(k) \end{cases}, \quad (3)$$

where $\mathbf{x}(k) \in \mathbb{R}^n$ is the state vector, $\mathbf{w}(k) \in \mathbb{R}^m$ is the vector of external inputs, $\mathbf{z}(k) \in \mathbb{R}^p$ is the vector of outputs from the system, $\mathbf{y}(k) \in \mathbb{R}^q$ represents the measurement vector, and the remaining matrices have compatible dimensions. It can be shown, extending the results for the linear time-invariant case available in (Anderson and Moore, 1979) that the optimum estimator in the sense of providing the state estimate with minimum variance for the system (3) consists of a finite-dimensional linear, periodically time-varying estimator with realization

$$\Sigma_{\mathcal{E}_M} = \begin{cases} \hat{\mathbf{x}}(k+1) &= A(k)\hat{\mathbf{x}}(k) \\ & \quad +K(k)(\mathbf{y}(k) - C_y(k)\hat{\mathbf{x}}(k)) \\ \hat{\mathbf{z}}(k) &= C_z(k)\hat{\mathbf{x}}(k) \end{cases}, \quad (4)$$

where $K \in R^{n \times q}$ is a periodically varying observer gain to be determined and $\hat{\mathbf{x}}(k)$ and $\hat{\mathbf{z}}(k)$ have the dimensions of $\mathbf{x}(k)$ and $\mathbf{z}(k)$, respectively.

Let $\tilde{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}$ and $\mathbf{e} = \mathbf{z} - \hat{\mathbf{z}}$ be the state and output estimation errors, respectively. Using (3) and (4), the dynamics can be written as

$$\Sigma_{\mathcal{F}_M} = \begin{cases} \tilde{\mathbf{x}}(k+1) &= (A(k) - K(k)C_y(k))\tilde{\mathbf{x}}(k) \\ & \quad +(B_w(k) - K(k)D_{yw}(k))\mathbf{w}(k) \\ \mathbf{e}(k) &= C_z(k)\tilde{\mathbf{x}}(k) + D_{zw}(k)\mathbf{w}(k). \end{cases} \quad (5)$$

It now remains to chose the periodically varying gains so as to meet adequate performance criteria for the estimated variables. The work in (Oliveira, 2002) describes in detail how this choice can be done in a "frequency-like" setting that naturally captures the requirements that the "transfer function" from position measurement $({}^I x_s)_m$ to position estimate $\hat{x}_1$ be "low-pass", while that from the integral of ${}^I(u_{s,w})_m$ to $\hat{x}_1$ is high pass. See also (Pascoal *et al.*, 2000) and the references therein. The design tools used are well rooted in the theory of Linear Matrix Inequalities (Boyd *et al.*, 1994). Namely, they use the extensions of previous results for linear time-invariant filter design to a periodic setting, reported independently in (Oliveira, 2002) and (Bittanti and Cuzzola, 2001). Using this framework, filter design can be cast in the form of an optimization problem and solved using efficient numerical tools such as those described in (MATLAB, 1997).

The key results required for filter design involve the computation $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of closed loop periodic operators. It is assumed that the reader is familiar with these concepts. See for example (Boyd *et al.*, 1994) for formal definitions. In a stochastic setting, the $\mathcal{H}_2$ of a stable operator can be interpreted as the asymptotic output variance of its output when the input is excited by white noise input signals. In a deterministic setting, it is simply the "total output energy" of its impulsive response. The $\mathcal{H}_\infty$ norm of a stable operator captures the worst case "energy" amplification from input to output and admits (in the linear time-invariant case) important interpretations in the frequency domain. As an example, the following theorems derived in (Oliveira, 2002) allow for the computation of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of important closed loop operators using LMIs.

Theorem 3.1. Consider the periodically time-varying discrete time system $\mathcal{F}_M : \mathbf{w} \rightarrow \mathbf{e}$ composed of a nominal system $\mathcal{G}_M : \mathbf{w} \rightarrow [\mathbf{z}^T \mathbf{y}^T]^T$ and an estimator $\mathcal{E}_M : \mathbf{y} \rightarrow \hat{\mathbf{z}}$ interconnected as described in figure 3, with realization (5). The $\mathcal{H}_2$ norm of such a system, from the input $\mathbf{w}$ to the output estimation error $\mathbf{e}$, is such that $\|F_M\|_2 < \gamma$ if and only if there exists a set of symmetric, positive definite matrices $P(i) \in \mathbb{R}^{n\times n}$, $i = 0, \cdots, M-1$, a set of auxiliary variables $X(i) \in \mathbb{R}^{m\times m}$, $i = 0, \cdots, M-1$ and a set of auxiliary variables $Y(i) \in \mathbb{R}^{n\times q}$, $i = 0, \cdots, M-1$ verifying $Y(i) = P(i+1)K(i)$, such that

$$\begin{bmatrix} P(i) & * & * \\ P(i+1)A^T(i) - Y(i)C_y(i) & P(i+1) & * \\ C_z(i) & 0 & I_p \end{bmatrix} > 0,$$
$i = 0, ..., M-1;$
$$\begin{bmatrix} X(i+1) & * \\ P(i+1)B_w(i) - Y(i)D_{yw}(i) & P(i+1) \end{bmatrix} > 0,$$
$i = 0, ..., M-1;$
$$\sum_{i=0}^{M-1} \mathbf{tr}(X(i)) + \mathbf{tr}(D^T(i)D(i)) < M\gamma^2.$$

Theorem 3.2. Consider the discrete-time system $\mathcal{F}_M : \mathbf{w} \rightarrow \mathbf{e}$, composed of a nominal system $\mathcal{G}_M : \mathbf{w} \rightarrow [\mathbf{z}^T \mathbf{y}^T]^T$ and an estimator $\mathcal{E}_M : \mathbf{y} \rightarrow \hat{\mathbf{z}}$ interconnected as described in figure 3, with realization (5). The $\mathcal{H}_\infty$ norm from the input $\mathbf{w}$ to the output estimation error $\mathbf{e}$ verifies $\|F_k\|_\infty < \gamma$ if and only if there exists a symmetric, positive definite set of matrices $P(i) \in \mathbb{R}^{n\times n}$, $i = 0, \cdots, M-1$ and a set of auxiliary variables $Y(i) \in \mathbb{R}^{n\times q}$, $i = 0, \cdots, M-1$ verifying $Y(i) = P(i+1)K(i)$, such that each LMI given by

$$\begin{bmatrix} -P(i) & \star & \star & \star \\ 0 & -\gamma^2 I_m & \star & \star \\ P(i+1)A(i) & P(i+1)B_w(i) & -P(i+1) & \\ -Y(i)C_y(i) & -Y(i)D_{yw}(i) & & \star \\ C_z(i) & D_{zw}(i) & 0 & -I_p \end{bmatrix} \quad (6)$$

is negative definite, for i=0, ..., M-1.

To cast the problem of navigation system design as an optimization problem, define

$T_{x_m \rightarrow e}$ - operator from position measurement $({}^I x_s)_m$ to estimate error $e = ({}^I x_s)_m - {}^I \hat{x}_s$;

$T_{u_m \rightarrow \hat{x}}$ - operator from velocity measurement $({}^I u_{s,w})_m$ to position estimate ${}^I \hat{x}_s$;

$T_{x_m \rightarrow \hat{x}}$ - operator from the position measurement $({}^I x_s)_m$ to the position estimate ${}^I \hat{x}_s$.

Then, as argued in (Oliveira, 2002) the simplest problem of ASV navigation can be formulated as

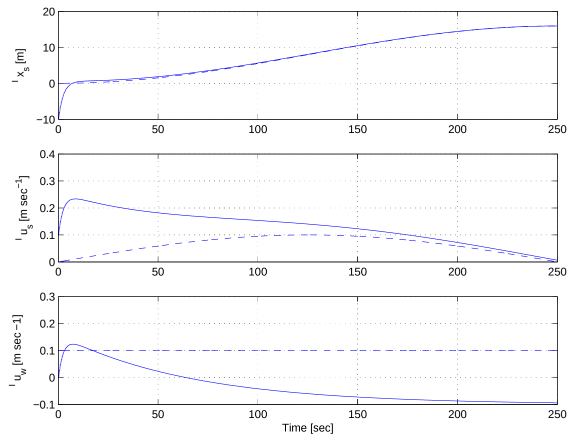

Fig. 4. Position ${}^I x_s$ (solid line) and position estimate ${}^I \hat{x}_s$ (dotted line) on top; Velocity ${}^I u_s$ and velocity estimate, in the middle; current ${}^I u_W$ and symmetric of current estimate, at the bottom.

$$\begin{array}{ll} & min \quad \|T_{x_m \rightarrow e}\|_2 \\ \text{subject to:} & \\ & \|T_{u_m \rightarrow \hat{x}}\|_2 < \gamma_v \\ & \|T_{x_m \rightarrow \hat{x}}\|_\infty < \gamma_p \end{array} \quad (7)$$

The first minimization objective captures the classical objective that the $\mathcal{H}_2$ norm from position measurements to estimation errors be small. The two other constraints aim at meeting adequate time and "frequency-like" design specifications. The positive variables γ_v and γ_p play the role of tuning knobs in the filter design process. See (Oliveira, 2002) for a more elaborate setting where "frequency-like" weights are directly incoporated in the criteria above.

A design exercise was carried out and a navigation filter was designed for the ASV to meet the design requirements introduced above, as well as other requirements in the frequency domain (Oliveira, 2002). This led to the the bounds $\gamma_v = 0.65$ and $\gamma_p = 1.8$ and to the multirate filter gains

$$K(k) = \begin{cases} \begin{bmatrix} 0.1890 & 0.0027 \end{bmatrix}^T & \text{if } k \; MOD \; M = 0 \\ \begin{bmatrix} 0 & 0 \end{bmatrix}^T & \text{if } k \; MOD \; M = 1 \end{cases}$$

yeilding a minimum value of $\|T^*_{x_m \rightarrow e}\|_2 = 1.0651$.

The performance of the navigation system was first evaluated *in simulation* for an initial $10\,m$ error on the position estimate. The unknown water current was set to $0.1 m/s$ in the $x-$ direction while its initial estimate was set to $0\,m/s$. The temporal evolution of the estimates is shown in figure 4.

This navigation system was tested at sea with the DELFIM catamaran. Figure 5 shows the actual path of the vehicle during a mission off the coast of Setubal, Portu-

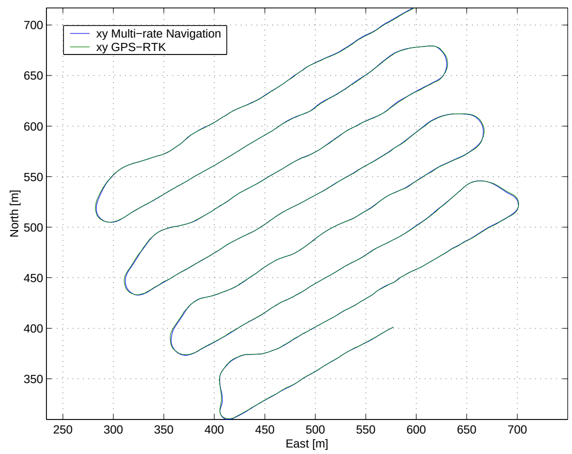

Fig. 5. Trajectory of the Delfim catamaran during a bathymetric survey at sea, in the Setubal canyon.

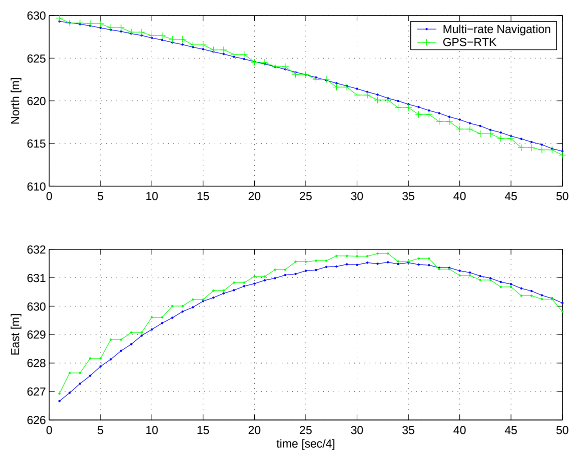

Fig. 6. Detail of the trajectory of the Delfim catamaran.

gal. Figure 6 shows a zoom in on position estimates over a period of 50 samples (= 12.5 s). The figures illustrate clearly the multirate characteristic of the navigation system as well as the "low-pass" characteristics from measured position to estimated position, which arise naturally from the complementary nature of the filter chosen.

4. CONCLUSIONS

The paper introduced a new methodology for the design of multirate navigation systems for autonomous marine vehicles. The set-up adopted for filter design and analysis builds on Linear Matrix Inequalities (LMIs) and on efficient numerical analysis tools that borrow from convex optimization techniques. Tests at sea showed good performance of the filtering algorithms derived. Future work will address the problems that arise from latency in sensor measurements.

5. REFERENCES

Anderson, B. and Moore, J. (1979). *Optimal Filtering,* Information and System Sciences Series, Prentice-Hall, Inc. Englewood Cliffs.

ASIMOV Project (2000). *Advanced System Integration for Managing the Coordinated Operation of Robotic Ocean Vehicles (ASIMOV),* Final EC Technical Report, ISR-IST, Lisbon, Portugal.

Bittant, S. and Cuzzola, F. (2001). "An LMI approach to periodic discrete-time unbiased filtering," *System and Control Letters*, Vol. 41, No. 1, pp. 21-35.

Boyd, S., El Ghaoui, L., Feron, E., Balakrishnan, B. (1994). *Linear Matrix Inequalities in Systems and Control Theory,* SIAM Studies in Applied Mathematics, Philadelphia.

Brown, R. and Hwang, P. (1992). *Introduction to Random Signals and Applied Kalman Filtering,* Second Edition, John Wiley and Sons, Inc.

Brown, R. (1972). "Integrated navigation systems and Kalman filtering: a perspective," *Journal of the Institute of Navigation*, Vol. 19, No. 4, pp. 355-362.

Fryxell, D., Oliveira, P., Pascoal, A. and Silvestre, C. (1994). "An integrated approach to the design and analysis of navigation, guidance and control systems for AUVs," *Proc. Symposium on Autonomous Underwater Vehicle Technology*, Cambridge, Massachusetts, USA.

Kayton, M. and Fried, W. (ed.) (1969). *Avionics Navigation Systems.* John Wiley and Sons, Inc., New York.

Lin, C. (1991). *Modern Navigation, Guidance, and Control Processing,* Prentice-Hall.

MATLAB (1997), LMI Control Toolbox ,*The Math Works Inc.*.

Oliveira, P.(2002). *Periodic and Non-linear Estimators with Applications to the Navigation of Ocean Vehicles*, Ph.D thesis, IST, Lisbon, Portugal.

Pascoal, A., Kaminer, I., and Oliveira, P. (2000). *Navigation System Design using Time-Varying Complementary Filters,* IEEE Transactions on Aerospace and Electronic Systems, October, pp. 1099-1115.

Silvestre, C. (2000). *Multi-Objective Optimization Theory with Applications to the Integrated Design of Controllers/Plants for Autonomous Vehicles*, Ph. D. thesis, IST, Lisbon, Portugal.

Stambaugh, J. and Thibault, R. (1992). "Navigation requirements for autonomous underwater vehicles," *Journal of the Institute of Navigation*, Vol. 39, n. 1, pp. 79-92.