Navigation Systems Design: An Application of Multi-rate Filtering Theory *

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Abstract

A new methodology is introduced for the design of multi-rate navigation systems for underwater vehicles. The design technique proposed borrows from Kalman filtering theory and leads naturally to multi-rate complementary filtering structures, the performance of which can be assessed using a frequency-like domain interpretation. An example illustrates the application of the new technique to the design of a multi-rate navigation system for an autonomous underwater vehicle (AUV).

1 Introduction

Currently, there is considerable interest in the development of navigation systems to provide robotic vehicles with the capability to perform complex missions autonomously. See [1, 6] and the references therein for in depth presentations of navigation systems for aircraft and [4, 5, 17] for an overview of similar systems and related research issues in the underwater robotics area.

Traditionally, navigation system design is done in a stochastic setting using Kalman-Bucy filtering theory [2]. In the case of nonlinear systems, design solutions are usually sought by resorting to Extended Kalman filtering techniques [2]. The stochastic setting requires a complete characterization of process and observation noises, a task that may be difficult, costly, or not suited to the problem at hand. This issue is argued at great length in [3], who points out that in a great number of practical applications the filter design process is entirely dominated by constraints that are naturally imposed by the sensor bandwidths. In this case, a design method that explicitly addresses the problem of merging information provided by a given sensor suite over distinct, yet *complementary* frequency regions is warranted.

Motivated by these considerations, this paper offers a new approach to navigation system design that relies on *complementary filtering theory* [3]. The set-up adopted leads naturally to the design of linear Kalman filters whereby the covariances of process and observation noises are viewed as tuning knobs that shape the characteristics of the operators that map measured to estimated variables.

In the case of attitude estimation, all sensors are sampled at the same rate and the corresponding filter operators are linear and time-invariant. This leads to a fruitful interpretation of the filters in the frequency domain. In the case of linear position and velocity estimation, however, the characteristics of the sound channel imply that the position measurements (obtained from a Long Base-Line system) are available at a rate that is lower than that of the remaining sensors. To deal with this problem, this paper proposes a new approach to navigation system design that relies on multi-rate complementary Kalman filtering theory. The design methodologies for such types of multi-rate systems are discussed, and the properties normally associated with single rate complementary filters are shown to be preserved. In particular, it is shown that the multi-rate filters can be viewed as input-output operators exhibiting "frequency-like" properties that are the natural generalization of those obtained for the single rate case. This is done by quantifying the performance of the filters in a weighted induced operator norm setting, and by exploring well known theoretical relationships between multi-rate and periodic systems. These properties play a key role in assessing the stability of combined guidance, control, and navigation systems even when multi-rate sensor systems are present [4].

The organization of the paper is as follows. Section 2 reviews some background material on linear time-varying systems, induced operator norms, and generalizes the definitions of low and high pass systems to a time-varying setting. Section 3 sets the motivation for the sections that follow: a simple filtering problem is formulated and its solution in terms of standard complementary filter structures is presented. Section 4 formulates a multi-rate Kalman filtering problem and describes its solution by exploring a relationship between multi-rate and periodic systems. Analysis tools are provided to evaluate the performance of the resulting filters using frequency-like domain interpretations. Finally, section 5 describes the navigation problem that is the main subject of this paper and describes its solution in terms of a complementary multi-rate filtering structure. The performance of the

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resulting navigation system is assessed in simulation.

Due to space limitations, this paper focuses on the key ideas that are relevant to the design of multi-rate navigation systems. The reader is referred to [13] for complete technical details and a description of the software package that was developed for navigation system design.

2 Mathematical background. Basic definitions.

The main objective of this section is to extend the usual concepts of low and high pass filters to a linear time-varying setting, The necessary basic background material is briefly summarized below for the continuous-time case. The extension to the discrete-time case is straightforward. See [13, 18] and the references therein for complete details.

2.1 Induced operator norms

The symbol L_2 denotes the Hilbert space of Lebesgue measurable functions from $\mathcal{R}_+$ to $\mathcal{R}^p$ endowed with the usual operator norm, while L_{2e} denotes the corresponding extended space. An input-output system $\mathcal{G}$ is identified with an operator $\mathcal{G} : L_{2e} \to L_{2e}$. A causal system $\mathcal{G} : L_2 \to L_2$ is *(finite-gain) stable* if the L_2 *induced operator norm* $||\mathcal{G}||_{2,i}$ (abbv. $||\mathcal{G}||$) is finite. In what follows we restrict ourselves to the class of linear time-varying (LTV) systems $\mathcal{G}$ with finite-dimensional state-space realizations $\Sigma_{\mathcal{G}} := \{A(t), B(t), C(t)\}$ of bounded, piece-wise continuous matrix functions of time. Often, we will use the same symbol $\mathcal{G}$ to denote both an LTV system and its particular realization $\Sigma_{\mathcal{G}}$, as the meaning will become clear from the context. We assume the reader is familiar with the concept of exponential stability of LTV systems. To simplify the exposition, we will henceforth refer to an exponentially stable system as *internally stable*, while a (finite-gain) stable system will be simply called stable. If $\mathcal{G} : L_{2e} \to L_{2e}$ has an internally stable realization, then $\mathcal{G}$ defines a stable operator from $L_2 \to L_2$.

Let $\mathcal{G}$ be a stable linear time invariant (LTI) system with a minimal realization $\Sigma_{\mathcal{G}} := \{A, B, C\}$, and let $G(s) = C(sI - A)^{-1}B$ denote the corresponding transfer matrix. Then, the induced operator norm $||\mathcal{G}||$ equals the $\mathcal{H}_\infty$ norm of G, denoted $||G||_\infty$, where $||G||_\infty := \sup\{\sigma_{max}(G^T(-j\omega)G(j\omega)) : \omega \in \mathcal{R}\}$ and $\sigma_{max}(.)$ denotes the maximum singular value of a matrix. Efficient numerical tools are available to compute the induced operator norm of an LTI system [8]. In the general case of LTV systems, however, its computation is far more difficult [18]. Interestingly enough, in the case of discrete time periodic systems, the induced operator norm is easy to compute, as will be explained later. This seemingly simple result plays a key role in the analysis of the multi-rate systems that will be introduced.

2.2 Low and high pass filters.

The concept of low pass and high pass filters is well understood in the case of linear time-invariant systems. We now extend these concepts to the class of linear time-varying systems. The new concepts will play a major role in assessing the performance of the linear multi-rate complementary filters that will be introduced later.

Definition. Low pass property. Let $\mathcal{G}$ be a linear, internally stable time-varying system and let $\mathcal{W}_\omega^n$ be a low-pass, linear time-invariant Chebyschev filter of order n and cutoff frequency ω. The system $\mathcal{G}$ is said to satisfy a low pass property with indices (ϵ, n) over $[0, \omega_c]$ if

$$||(\mathcal{G} - I)\, W_{\omega_c}^n|| < \epsilon.$$

Definition. Low pass filter with bandwidth ω_c. A linear, internally stable time-varying system $\mathcal{G}$ is said to be an (ϵ, n) *low pass filter* with bandwidth ω_c if

- $\lim_{\omega\to 0} ||(\mathcal{G} - I)\, W_\omega^n||$ is well defined and equals 0.
- $\omega_c := \sup\{\omega : ||(\mathcal{G} - I)\, W_\omega^n|| < \epsilon\}$, i.e. $\mathcal{G}$ satisfies a low pass property with indices (ϵ, n) over $[0, \omega]$ for all $\omega \in [0, \omega_c)$ but fails to satisfy that property whenever $\omega \geq \omega_c$.
- For every $\delta > 0$, there exists $\omega^* = \omega^*(\delta)$ such that $||\mathcal{G}(I - W_\omega^n)|| < \delta$ for $\omega > \omega^*$.

Definition. High Pass Filter with break frequency ω_c. A linear, internally stable time-varying system $\mathcal{G}$ is said to be an (ϵ, n) high pass filter with break frequency ω_c if $(I - \mathcal{G})$ is an (ϵ, n) low pass filter with bandwidth ω_c.

The conditions in the definition of low pass filters generalize the following facts that are obvious in the linear time-invariant case: i) the filter must provide a gain equal to one at zero frequency, ii) there is a finite band of frequencies over which the system behaviour replicates very closely that of an identity operator, and iii) the system gain rolls-off to zero at high frequency. Notice the role played by the weighting operator $\mathcal{W}_\omega^n$, which was arbitrarily selected as a Chebyschev filter. In practice, the order of the filter can be made arbitrarily large so that the filter will effectively select the "low frequency components" of the input signal.

3 Complementary filters

Complementary filters arise naturally in the context of signal estimation based on measurements provided by sensors over distinct, yet complementary regions of frequency. Brown [3] was the first author to stress the importance of complementary filters in navigation system design. Since then, this subject has been studied in a number of publication that address theoretical as well as practical implementation issues; see for example [1, 9] and the references therein. The key ideas in complementary filtering are very intuitive, and can be simply introduced by referring to the example below.

Motivating example: Suppose it is required to determine the position p of a body that is restricted to move along a straight line, based on measurements p_m and v_m of p and $v = \dot{p}$ respectively, provided by a position and a velocity sensor. The measurements are corrupted by disturbances v_d and p_d. Let $p(s)$ and $v(s)$ denote the

Laplace Transforms of p and v, respectively. Then, for every $k > 0$, $p(s)$ admits the stable decomposition

$$p(s) = \frac{s+k}{s+k}\, p(s) = T_1(s)p(s) + T_2(s)p(s), \qquad (1)$$

where $T_1(s) = k/(s+k)$ and $T_2(s) = s/(s+k)$ satisfy the equality $T_1(s)+T_2(s) = I$. Using the relationship $v(s) = sp(s)$, it follows from the above equations that $p(s) = F_p(s)p(s) + F_v(s)v(s)$ where $F_p(s) = T_1(s) = k/(s+k)$ and $F_v(s) = 1/(s+k)$. This suggests a complementary filter with the structure $\hat{p} = \mathcal{F}_p p_m + \mathcal{F}_v v_m$ where $\mathcal{F}_p$ and $\mathcal{F}_v$ are linear time-invariant operators with transfer functions $F_p(s)$ and $F_v(s)$, respectively. Clearly, the filter admits the state space realization

$$\dot{\hat{p}} = -k\hat{p} + kp_m + v_m = v_m + k(p_m - \hat{p})$$

that is represented in figure 1.

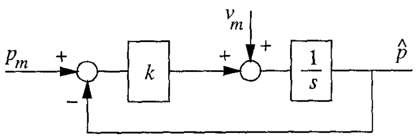

Figure 1: Structure of a first order complementary filter.

Let $\mathcal{T}_1$ and $\mathcal{T}_2$ denote linear time-invariant operators with transfer functions $T_1(s)$ and $T_2(s)$, respectively. Simple computations show that $\hat{p} = (\mathcal{T}_1 + \mathcal{T}_2)p + \mathcal{F}_p p_d + \mathcal{F}_v v_d$, that is, the estimate $\hat{p}$ consists of an undistorted copy $(\mathcal{T}_1 + \mathcal{T}_2)p = p$ of the original signal p, together with corrupting terms that depend on the measurement disturbances p_d and v_d. Notice the following important properties:

- $T_1(s)$ is low-pass: the filter relies on the information provided by the position sensor at low frequency only.
- $T_2(s) = I - T_1(s)$ is high-pass: the filter blends the information provided by the position sensor in the low frequency band with that available from the velocity sensor in the complementary band.
- the break frequency is simply determined by the choice of the parameter k.

The frequency decomposition induced by the complementary filter structure holds the key to its practical success, since in many cases it mimics the natural frequency decomposition induced by the physical nature of the sensors themselves: position sensors usually provide reliable information at low frequency only, whereas velocity sensors often exhibit biases and drift phenomena in the same frequency band and are therefore useful at higher frequencies.

Complementary filter design is then reduced to the computation of the gain k so as to meet a target break frequency that is entirely dictated by the physical characteristics of the sensors. From this point of view, the emphasis is shifted from a stochastic framework - that relies heavily on a correct description of process and measurement noise [3] and the minimization of filter errors - to a deterministic framework that aims at shaping the filter closed-transfer functions.

As convincingly argued in [3], the latter approach is best suited to tackle a large number of practical situations where the characterization of process and measurement disturbances in a stochastic context does not fit the problem at hand, the filter design process being entirely dominated by the constraints imposed by sensor bandwidths. Once this set-up is adopted, however, one is free to adopt any efficient design method, the design parameters being simply viewed as "tuning knobs" to shape the characteristics of the closed loop operators. In this context, filter design can be done using H_2 or H_∞ design techniques [2, 12, 15]. Filter analysis is easily carried out in the frequency domain using Bode plots.

In the simple case described here, and in preparation for the sections that follow, the underlying discretized process model can be written as

$$\begin{aligned} p(k+1) &= p(k) + h\,(v_m(k) + v_d(k)) \\ p_m(k) &= p(k) + p_d(k) \end{aligned}$$

where v_d and p_d play the roles of process and measurement disturbances, respectively. Notice the important fact that v_m (the measured value of v) is an input to the system. In an H_2 setting, v_d and p_d are viewed as stochastic, stationary white gaussian processes with zero mean and covariances $q \geq 0$ and $r > 0$, respectively and the objective is to minimize the covariance of the estimation error $p - \hat{p}$. This leads directly to the position estimator

$$\hat{p}(k+1) = \hat{p}(k) + hv_m(k) + \mathrm{k}(p_m(k) - \hat{p}(k)), \qquad (2)$$

where k is the stationary Kalman filter gain. Clearly, the above filter exhibits a complementary structure. Furthermore, the covariances q and r can be viewed as design parameters to vary the break frequency of the resulting filter.

In practice, the simple complementary structure described above can be modified to meet additional constraints. For example, to achieve steady state rejection of the velocity sensor bias, the filter must be augmented with an integrator to obtain a new complementary filter structure. See [13] for details. In view of the discussion above, we henceforth adopt a deterministic framework for complementary filter design and analysis where the objective is to shape the filter transfer functions to obtain desired bandwidths.

4 Multi-rate filters: synthesis and analysis

This section summarizes basic results on multi-rate filtering and describes the mathematical tools that are used to assess their performance using frequency-like criteria. The key results used explore the equivalence between a very general class of multi-rate systems and the class of periodic systems [11], as well as the isomorphism between periodic and time-invariant systems using the concept of lift operator [10].

4.1 Synthesis

Following the basic definitions in [11], let $\mathcal{G}$ be a discrete time system with m inputs and p outputs. Let $i_i h; i = 1, .., m$ and $o_j h; j = 1, .., p$ be the sampling periods for the i^{th} input and j^{th} output respectively, where i_i and o_j are positive integers, and h is the system's *shortest time period*. Denote by Mh the *basic time period*, where $M = lcm\{\{i_i\}_{i=1}^m, \{o_j\}_{j=1}^p\}$. If $\mathcal{G}$ is finite-dimensional, linear, and causal, then it admits the state space representation

$$\begin{aligned} \mathbf{x}(k+1) &= A(k)\mathbf{x}(k) + B(k)\mathbf{u}(k) \\ \mathbf{y}(k) &= C(k)\mathbf{x}(k) + D(k)\mathbf{u}(k) \end{aligned}$$

where $\mathbf{x} \in \Re^n$, $\mathbf{u} \in \Re^m$, and $\mathbf{y} \in \Re^p$ denote the state, input, and output vectors respectively, and the time index k denotes the instant of time kh. Under very general assumptions, the state space representation is easily seen to be periodic with period M, that is, $A_{k+M} = A_k$, $B_{k+M} = B_k$, $C_{k+M} = C_k$ and $D_{k+M} = D_k$ [11]. Thus, the problem of multi-rate filtering can be easily cast and solved by resorting to periodic filtering theory. In a stochastic setting, this leads to periodic Kalman filters. In this set-up, one is given a discrete-time periodic system described by

$$\begin{aligned} \mathbf{x}(k+1) &= A(k)\mathbf{x}(k) + B(k)\mathbf{u}(k) + G(k)\mathbf{w}(k), \\ \mathbf{y}(k) &= C(k)\mathbf{x}(k) + D(k)\mathbf{u}(k) + \mathbf{v}(k). \end{aligned} \tag{3}$$

The state and measurements are corrupted by white, Gaussian noise $\mathbf{w}$ and $\mathbf{v}$ respectively, with zero mean and noise covariance matrices

$$\begin{aligned} E[\mathbf{w}(k)\mathbf{w}(j)^T] &= Q(k-j)\delta(k-j) \\ E[\mathbf{v}(k)\mathbf{v}(j)^T] &= R(k-j)\delta(k-j) \end{aligned}$$

where $\delta(k-j)$ is the Kronecker operator. The corresponding periodic Kalman filter is described by the equations

$$\hat{\mathbf{x}}(k+1) = A(k)\hat{\mathbf{x}}(k) + B(k)\mathbf{u}(k) + K(k)[\mathbf{z}(k) - C(k)\hat{\mathbf{x}}(k)], \tag{4}$$

where the Kalman gain is defined as

$$K(k) = P(k)C^T(k)\left(C(k)P(k)C^T(k) + R(k)\right)^{-1} \tag{5}$$

and the covariance of the error $P(k)$ verifies, at each step, the Riccati equation with periodic parameters

$$\begin{aligned} P(k+1) &= A(k)P(k)A^T(k) + G(k)Q(k)G^T(k) \\ &\quad - A(k)P(k)C^T(k)\left(C(k)P(k)C^T(k)\right. \\ &\quad \left. + R(k)\right)^{-1} C(k)P(k)A^T(k) \end{aligned}$$

In general, the solution to the above Riccati equation is not periodic. However, as in the time-invariant case, the periodic stationary positive semidefinite solution for an infinite prediction interval is usually sought. See [16], and references therein for results on the existence and uniqueness of solutions of periodic Riccati equations. See also [13] for a description of different methods for the off-line computation of periodic stationary solutions to periodic Riccati equations.

4.2 Analysis

In the section that follows, a complementary multi-rate filter will be designed to estimate the position of an autonomous underwater vehicle given measurements of its position and velocity. The results presented thus far show that this problem can be tackled in the framework of periodic filtering theory. If one is to follow closely the key ideas in complementary filter design, however, the process and measurement noise covariances must be viewed as tuning knobs to "match the frequency properties of the resulting filters to the characteristics of the sensors used". At this point, the key concepts of low and high pass filters introduced in section 2 can be used to characterize the "equivalent" bandwidths of multi-rate filters. Given the equivalence between a general class of multi-rate systems and periodic systems, the problem is reduced to that of computing the induced operator norm of a periodic system. This is done as follows.

Let $\mathcal{G}$ be a linear, discrete-time, $m-$ input, $p-$ input, $M-$ periodic causal system. Then, using the concept of lift operator introduced in [10], $\mathcal{G}$ can be shown to be equivalent to an nM-input, pM-output linear time invariant system $\overline{\mathcal{G}}$ that is usually referred to as the *lifted version* of $\mathcal{G}$. The equivalence is norm preserving in the following sense: if $\mathcal{G}$ has finite induced operator norm $||\mathcal{G}||$, then the LTI system $\overline{\mathcal{G}}$ has finite induced operator norm $||\overline{\mathcal{G}}|| = ||\mathcal{G}||$. The reader will find in [13] a thorough discussion of this circle of ideas, as well as a description of the computational procedures that can be used to compute the LTI lifted version of a periodic system and its induced operator norm.

5 Navigation system design

This section contains a simple example that illustrates the design of a multi-rate navigation system to estimate the position of an autonomous underwater vehicle. In what follows, $\{U\}$ is a fixed reference frame and $\{B\}$ is a body-fixed coordinate that moves with the AUV. The vehicle motion is subject to the influence of a constant, unknown current $\mathbf{v}_w = (u_w, v_w, w_w)^T$ expressed in $\{U\}$. The following notation is required: $\mathbf{p} := (x, y, z)^T$ - position of the origin of $\{B\}$ measured in $\{U\}$; ${}^U\mathbf{v}_r := (u_r, v_r, w_r)^T$ - relative velocity of the origin of $\{B\}$ with respect to $\mathbf{v}_w$, measured in $\{U\}$; ${}^B\mathbf{v}_r := (u, v, w)^T$ - relative velocity of the origin of $\{B\}$ with respect to $\mathbf{v}_w$, measured in $\{U\}$ and expressed in $\{B\}$ (i.e., body-fixed relative linear velocity); $\boldsymbol{\lambda} := (\phi, \theta, \psi)^T$ - vector of of roll, pitch, and yaw angles that parametrize locally the orientation the attitude of $\{B\}$ relative to $\{U\}$; ${}^U_B R(\boldsymbol{\lambda})$ - rotation matrix from $\{B\}$ to $\{U\}$.

With this notation, the relevant kinematics of the AUV can be written in compact form as

$$\frac{d}{dt}\mathbf{p} = {}^U_B R(\boldsymbol{\lambda})^B\mathbf{v}_r + \mathbf{v}_w. \tag{6}$$

In the design example, the AUV is equipped with: i) a **L**ong **B**ase**L**ine *positioning system* that computes the round-trip travel times ($\Delta_{1..4}$) of the acoustic pulses that are emitted by the vehicle and returned by an array of four transponders, and ii) a *Doppler sonar* that provides on-board referenced measurements of the velocity ${}^B\mathbf{v}_r$ of the vehicle with respect to the water. An attitude reference unit is also available to provide accurate estimates of the vector $\boldsymbol{\lambda}$ of roll, pitch, and yaw angles. The LBL data are input to the iterative triangulation algorithm described in [13] to provide measurements $\mathbf{p}_m := (x_m, y_m, z_m)^T$ of $\mathbf{p}$. The Doppler data are simply converted from the body to the reference coordinate frame using the rotation matrix ${}^U_B R(\boldsymbol{\lambda})$ to obtain measurements $(u_{r_m}, v_{r_m}, w_{r_m})^T$ of $(u_r, v_r, w_r)^T$ (this is a simplyfing approach, in view of the existence of Doppler biases. See [14] for an in depth discussion of this problem). The interrogation rates for the LBL and Doppler units are $1Hz$ and $10Hz$, respectively. The former is mission dependent and is naturally imposed by the speed of sound in the water.

The design specifications for the navigation system require the development of a multi-rate complementary filter that will: i) obtain accurate estimates $(\hat{x}, \hat{y}, \hat{z})$ and $(\hat{u}_w, \hat{v}_w, \hat{w}_w)$ of the vehicle position and current velocity, respectively and ii) achieve an approximate bandwidth of $0.0314 rad/sec$ for the low pass filter that corresponds to the operator from position measurements to position estimates.

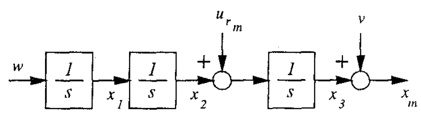

Figure 2: Position estimation: filter design model.

Following the guidelines introduced in section 3, the design model for the complementary navigation filter is easily obtained from the kinematic equations of the AUV, leading to three sets of decoupled equations that correspond to the three linear coordinates x, y, and z. See Figure 2 for the design model that captures the motion of the AUV along the coordinate x. The output integrator captures the relationship $\dot{x} = u_r + u_w$. The middle integrator was inserted to estimate the current bias. Finally, the input integrator was included to provide "faster roll-off" of the filter. Adopting a sampling period $h = 0.1s$, the corresponding discretized design model is given by

$$\begin{aligned} \mathbf{x}(k+1) &= A\mathbf{x}(k) + B\mathbf{u}(k) + \mathbf{w}(k) \\ \mathbf{y}(k) &= C(k)\mathbf{x}(k) + \mathbf{v}(k) \end{aligned}$$

where $\mathbf{x} = (x_1, x_2, x_3)^T$, $\mathbf{u} = u_{r_m}$, and $\mathbf{w}$ and $\mathbf{v}$ are white, gaussian, stochastic state and output noises, respectively. Furthermore,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ h & 1 & 0 \\ \frac{h^2}{2} & h & 1 \end{bmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix}$$

and

$$C_k = \begin{cases} (0,0,1) & \text{if} \quad k\, MOD\, M = 0 \\ (0,0,0) & \text{if} \quad k\, MOD\, M \neq 0. \end{cases}$$

where $M = 10$ is the basic time period period and the output of the operator $()MOD()$ is defined as the remainder of the division of the first argument

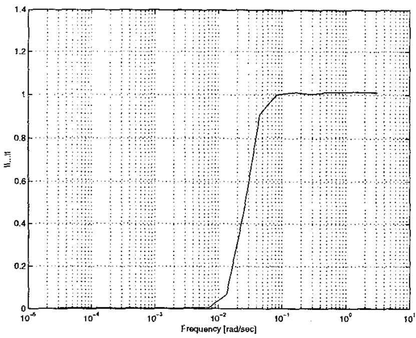

Figure 3: Induced weighted operator norm - position measurement to position estimate.

by the second one. Clearly, the matrix C_k is periodic. Based on the filter requirements, the noise covariances were selected as $E[\mathbf{w}(k)\mathbf{w}(j)^T] = Q\delta_{(k-j)\, MOD\, M}$, $Q = diag(0.1, 10^{-7}, 10^{-4})$ and $E[\mathbf{v}(k)\mathbf{v}(j)^T] = R\delta_{(k-j)\, MOD\, M}$; $R = 10^9$ to arrive at a $(0.6, 4)$ low pass multi-rate filter $\mathcal{G}_{\hat{x}, x_m}$ (from position measurement x_m to position estimate $\hat{x}$) with a bandwidth of $0.03 rad/s$. The corresponding Kalman gain is

$$K(k) = \begin{cases} \begin{pmatrix} 2.012 \times 10^{-4} \\ 1.996 \times 10^{-2} \\ 9.90 \times 10^{-7} \end{pmatrix} & \text{if} \quad k\, MOD\, M = 0 \\ (0,0,0)^T & \text{if} \quad k\, MOD\, M \neq 0 \end{cases}$$

Figure 3 shows the evolution of the corresponding induced weighted operator norm $||(\mathcal{G}_{\hat{x}, x_m} - I)\, W^4_{\omega_c}||$ as a function of the cutoff frequency ω_c of the weighing Chebyschev filter. Notice how the norm approaches zero as the cutoff frequency tends do zero. For $\omega = 0.03 rad/sec$, the norm is equal to 0.6.

The performance of the navigation system was evaluated *in simulation* with a nonlinear model of the vehicle under closed loop control, subject to the influence of a constant current of $0.5 m/sec$ in the positive y direction. See [4] for a description of the vehicle model and the setup available for simulation. The reference command for linear position is an $\mathcal{U}$ - shaped trajectory that descends smoothly along the depth coordinate z. Its projection on the horizontal plane consists of two straight lines joined by a semi-circumference with a radius of $60\, m$. The projection on the vertical plane consists roughly of two horizontal lines, at the beginning and end of the maneuver, and two straight lines with a slope of -10%. In the simulation, the LBL system used four transponders located at positions $\{-40, 0, 160\}, \{130, 0, 150\}, \{-40, 190, 170\}$ and $\{140, 190, 135\}$. The reference and "real" trajectories are depicted in figure 4. The solid line in Figure 5 (top part) shows the vehicle position along the y direction, during the period from $220 - 280s$. The dotted line shows the estimated position that is obtained from the multirate complementary filter. The dashed line shows the outputs of the LBL triangulation algorithm. Notice that in spite of the LBL data being available only every second, the estimated position is updated every $0.1s$. Fi-

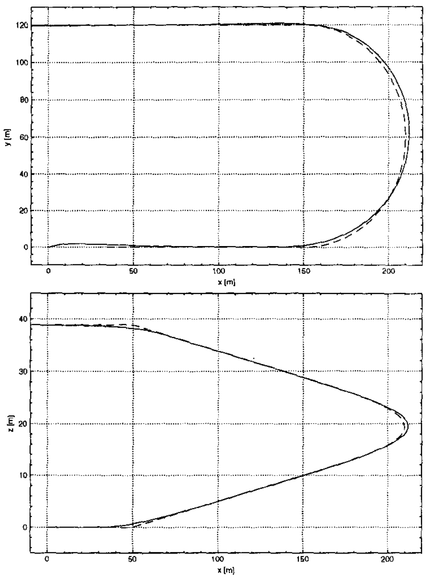

Figure 4: Commanded (-) and real (–) trajectory - horizontal and vertical planes.

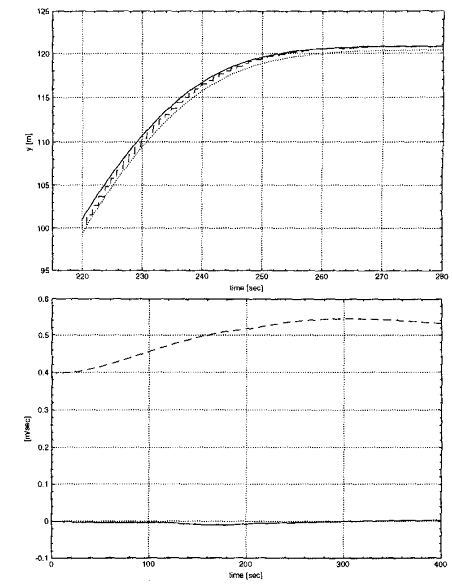

Figure 5: AUV position and current velocity estimates.

nally, the bottom part of Figure 5 shows the evolution of the estimated value of the current velocity along the $y-$direction.

Conclusions

The paper introduced a new methodology for the design of multirate navigation systems for underwater vehicles. Its key contribution was the extension of the concept of complementary filters to a multi-rate setting. The efficiency of the method was shown with a design example that brings forth the multirate characteristics of realistic navigation problems.

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